DISCRETE MATHEMATICS, 2013-2014 SPRING, FINAL EXAM

110 minutes

May 23, 2014

Id	Fullname	Signature

Q1	Q2	Q3	Q4	Q5	Q6	Total
/15	/15	/15	/20	/15	/20	/100

No questions are allowed. Answer the questions to the best of your understanding. If you need to make extra assumptions, state them clearly. Make sure that all your answers are sufficiently (and mathematically) explained.

1. A binary string of length 12 is made up of 12 bits (that is, 12 symbols, each of which is a 0 or a 1). How many such strings either start with three 1's or end in four 0's?

2. Show that the greatest common divisor of two consecutive, non-zero, positive integers is always 1: $\forall n \in \mathbb{Z}^+[gcd(n, n+1) = 1]$

3. Let $S = \{2, 3, 4, 6, 8, 20, 24, 48, 100, 120\}$.

   (a) Draw the Hasse diagram that is obtained by using the $|$ relation ("divides") on S. Try to arrange the diagram such that lines between elements will not cross.

   (b) What are the minimal and maximal elements?

4. Let $\circ$ be defined as $a \circ b = a + b + ab$. For example: $3 \circ 5 = 3 + 5 + 3 \cdot 5 = 23$. Does the structure $< \mathbb{R}, \circ >$ form a group?

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5. What do the following functions calculate? Express them in simpler terms.

   (a) $f : \mathbb{N} \to \mathbb{N},\ f(x) = \begin{cases} 0 & if\, x = 0 \\ 2 + f(x-1) & otherwise \end{cases}$

   (b) $g : \mathbb{N} \times \mathbb{N} \to \mathbb{N},\ g(x, y) = \begin{cases} 1 & if\, x = 0 \\ y \cdot g(x-1, y) & otherwise \end{cases}$

6. A dog show is being judged from pictures of the dogs. The judges would like to see pictures of the following pairs of dogs next to each other for their final decision: Arfie and Fido, Arfie and Edgar, Arfie and Bowser, Bowser and Champ, Bowser and Dawg, Bowser and Edgar, Champ and Dawg, Dawg and Edgar, Dawg and Fido, Edgar and Fido, Fido and Goofy, Goofy and Dawg.

   (a) Draw a graph modeling this situation.

   (b) Suppose that it is necessary to put pictures of the dogs in a row on the wall so that each desired pair of pictures appear together exactly once. (There are many copies of each picture.) What graph-theoretic object is being sought?

   (c) Can the pictures be arranged on the wall in this manner? If so, how?