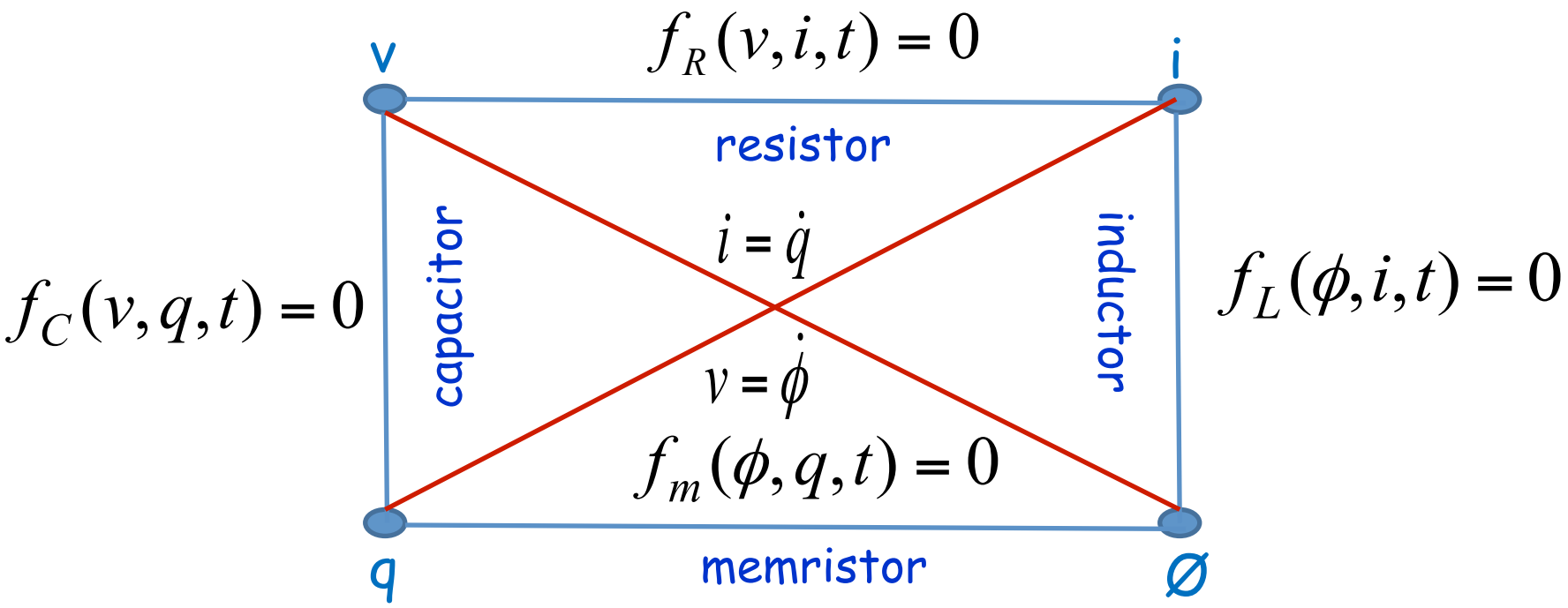
Element Equations

for 2-terminal elements



Resistor: An element that has an algebraic equation between v and i.

Inductor: An element that has an algebraic equation between Ø and i.

Capacitor: An element that has an algebraic equation between v and q.

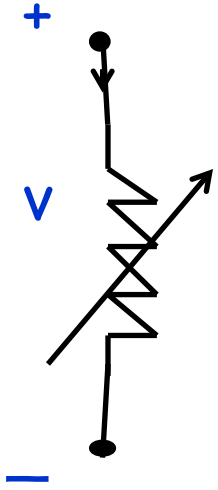
Memristor: An element that has an algebraic equation between Ø and q.

Algebraic equation: .....

Time-varying Resistors

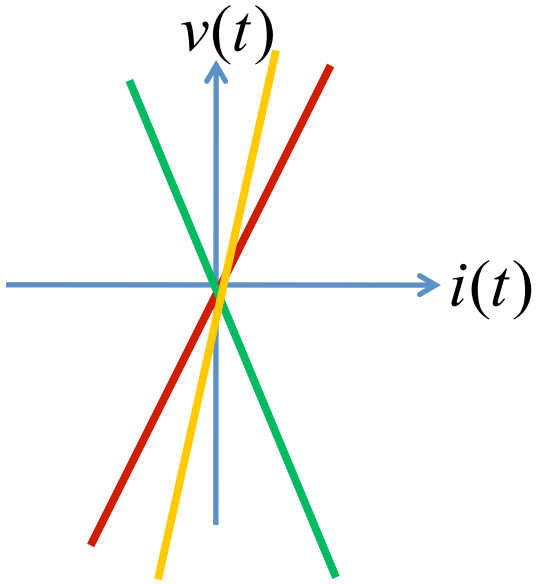
v-i characteristic of a time-varying resistor varies for different times. In other words, f(v,i,t)=0 depends on time.

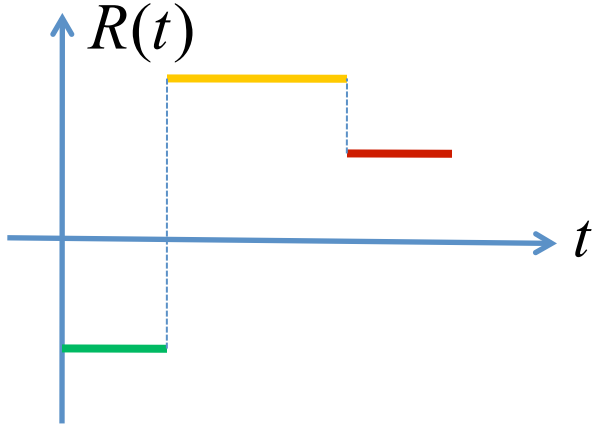
Linear time-varying resistor



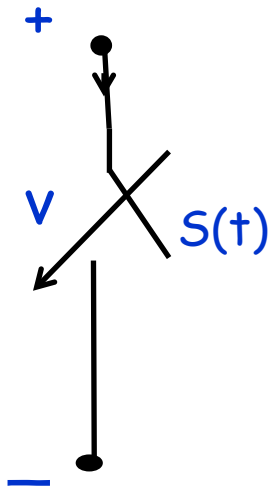
$$v(t) = R(t)i(t)$$

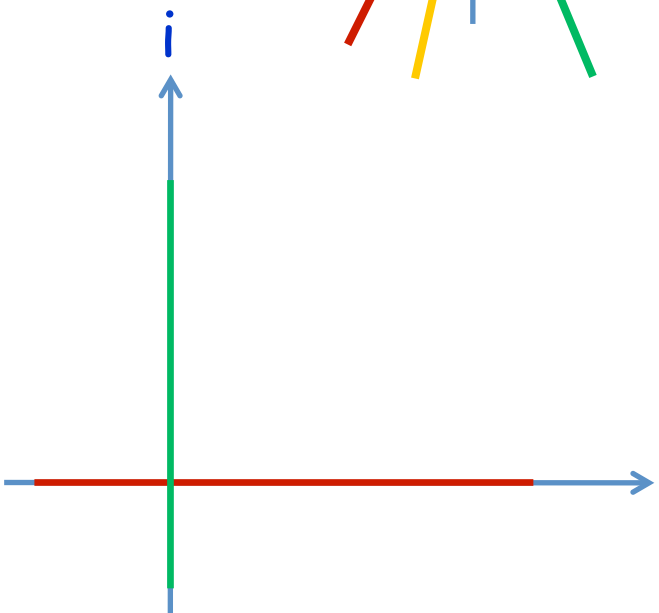
$$i(t) = G(t)v(t)$$

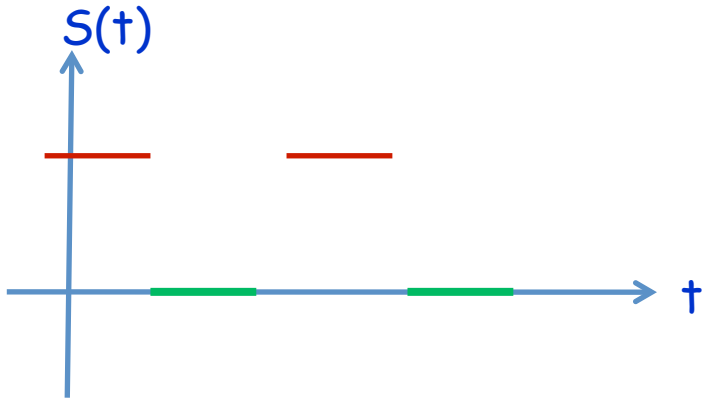




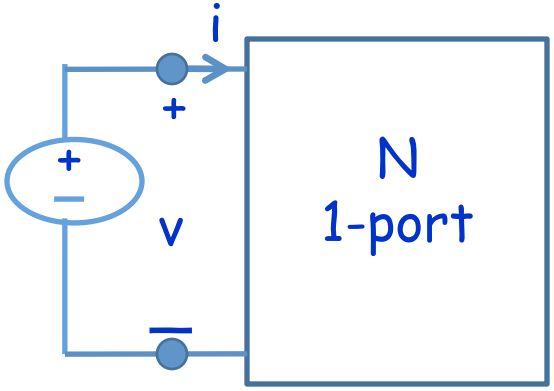
Switch





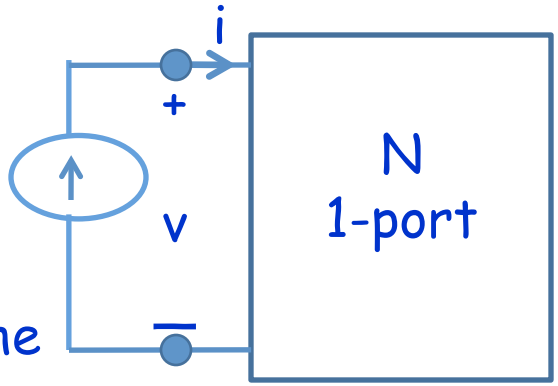


1-port (or 2-terminal) elements consisting of 2-terminal resistors

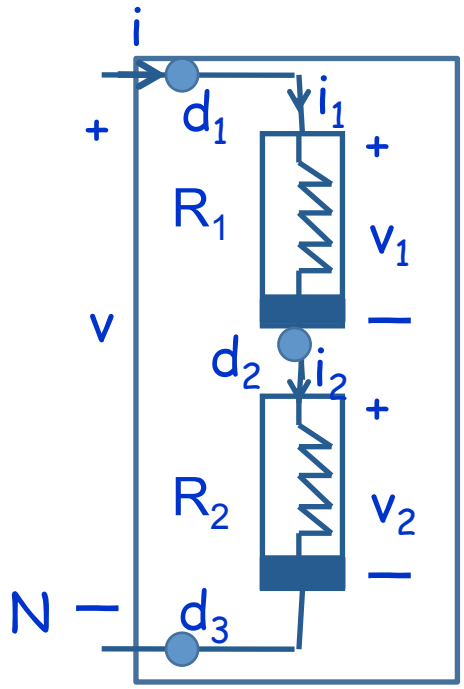


i: port corrent
v: port voltage

How can we find the element equation (or the v-i characteristic)?



Resistors connected in series



Element equations:
(current-controlled)

$$v_1 = \hat{v}_1(i_1) \qquad v_2 = \hat{v}_2(i_2)$$

KCL

$i = i_1$ 1. node

$i_1 = i_2$ 2. node

KVL

node sequence 1-2-3-1

$$v_1 + v_2 - v = 0$$

$$v_1 + v_2 = v$$

Aim: to find the current-controlled element equation for the 1-port:

$$v = \hat{v}(i)$$

KVL $\quad v = v_1 + v_2$

\+

KCL $\quad v = \hat{v}_1(i) + \hat{v}_2(i) \hat{=} \hat{v}(i)$

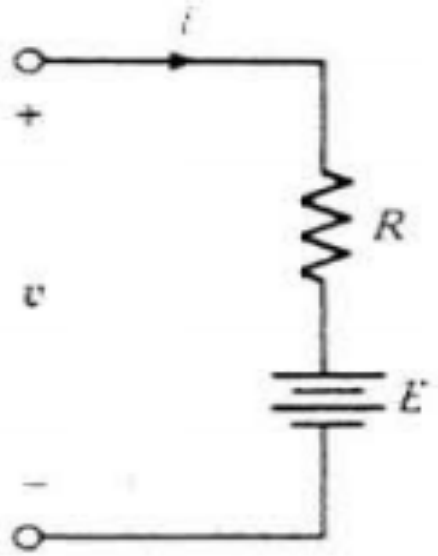
Result:

KCL → Port current is the same as the current of a resistor.

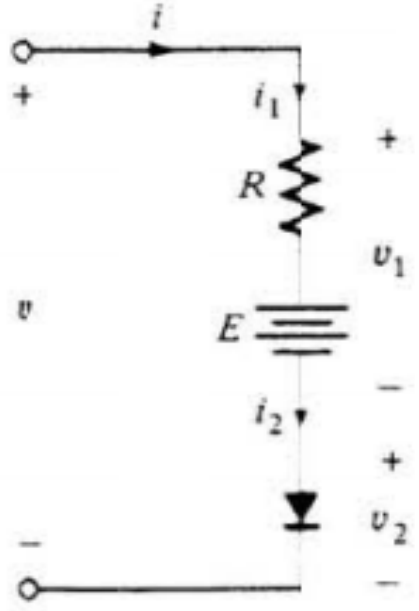
KVL → Port voltage is the sum of the voltages of resistors.

Since the resistors in series are current-controlled the 1-port is also current-controlled.

Example 1: Obtain the element equation for the 1-port.

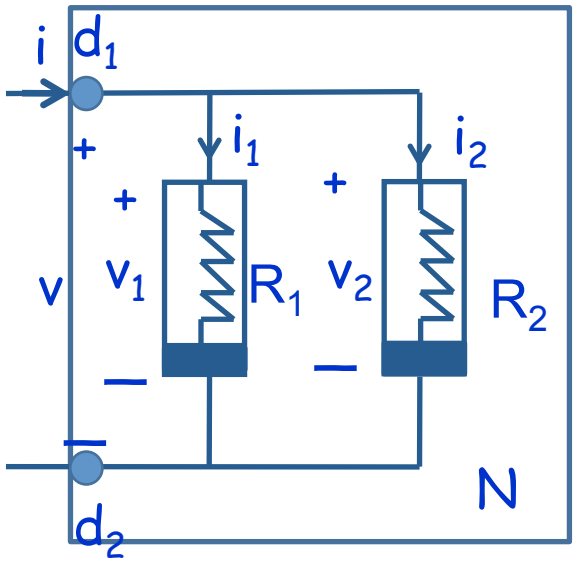


Example 2: Obtain the element equation for the 1-port.



L.O. Chua, C.A. Desoer, S.E. Kuh. "Linear and Nonlinear Circuits", Mc.Graw Hill, 1987, New York

Resistors connected in parallel



Element equations:
(voltage-controlled)

$$i_1 = \hat{i}_1(v_1) \qquad i_2 = \hat{i}_2(v_2)$$

KVL: $v = v_1 = v_2$

KCL: $i = i_1 + i_2$

Since the resistors in series are voltage-controlled the 1-port is also voltage-controlled.

Aim: to find the voltage-controlled element equation for the 1-port:

$$i = \hat{i}(v)$$

KCL → $i = i_1 + i_2$

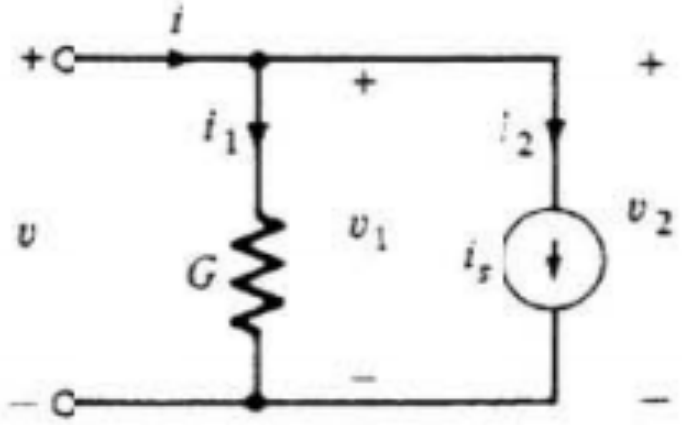
\+

EE → $i = \hat{i}_1(v_1) + \hat{i}_2(v_2)$

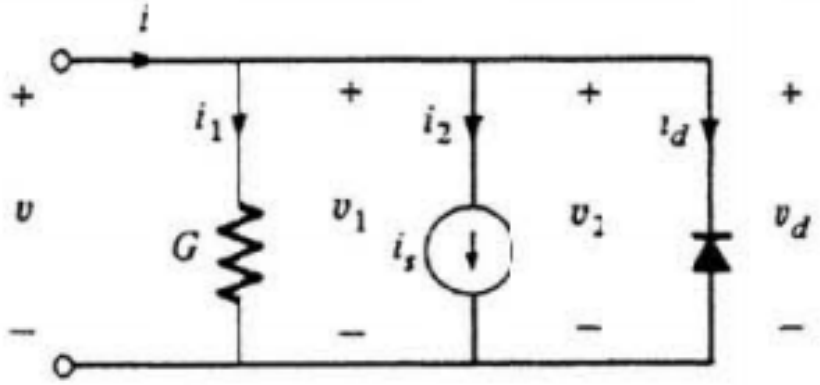
\+

KVL → $i = \hat{i}_1(v) + \hat{i}_2(v) \triangleq \hat{i}(v)$

Example 3: Obtain the element equation for the 1-port.



Example 4: Obtain the element equation for the 1-port.



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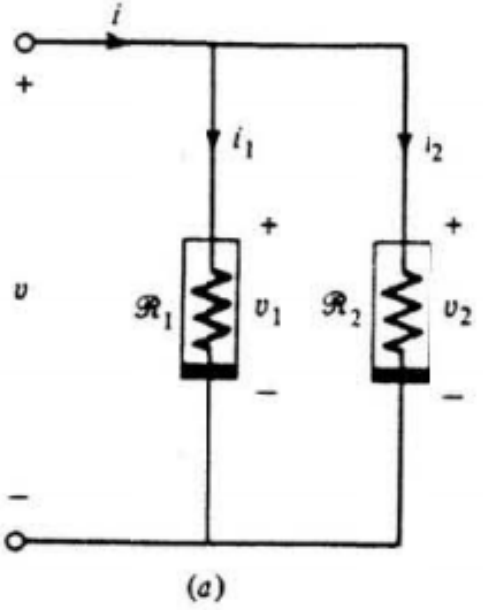
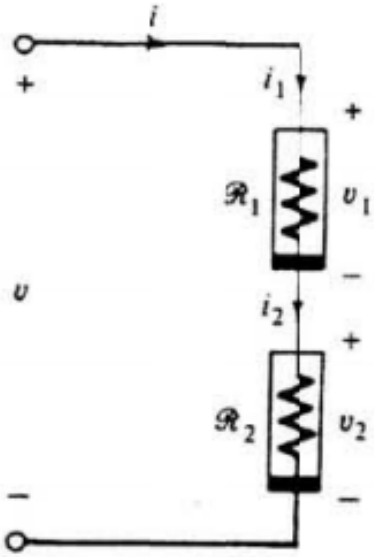
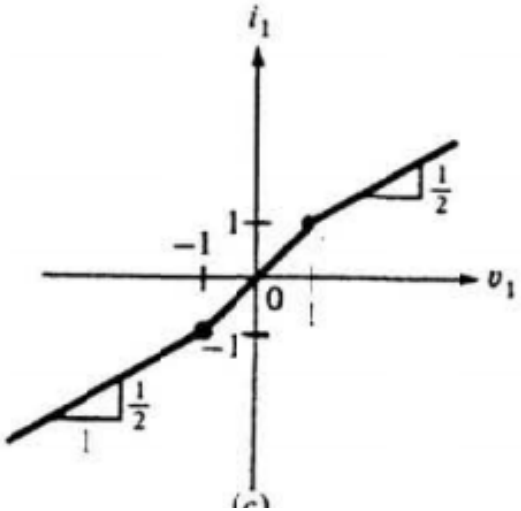
Other examples:

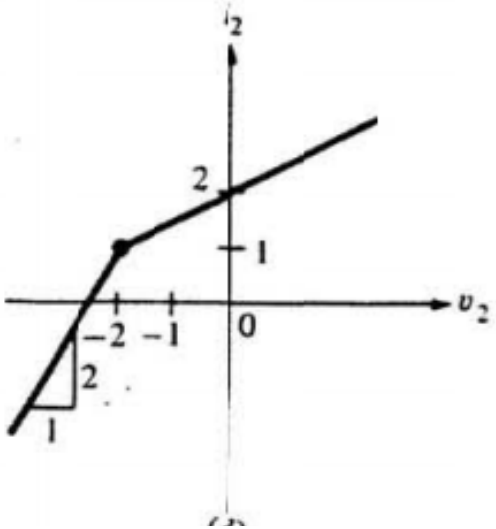


(a)



(b)





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