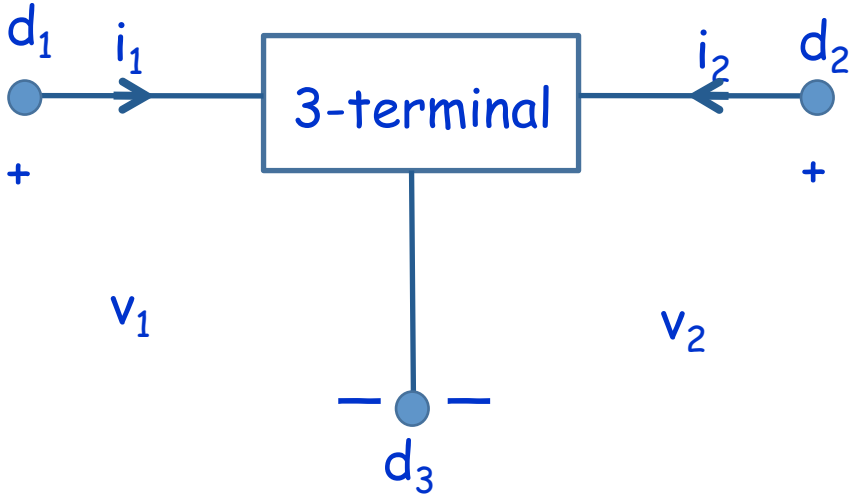
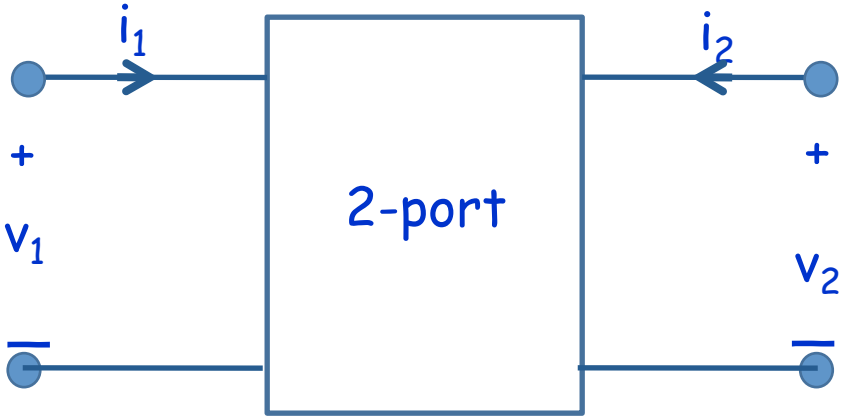
N-Terminal Resistors

3-terminal or 2-port



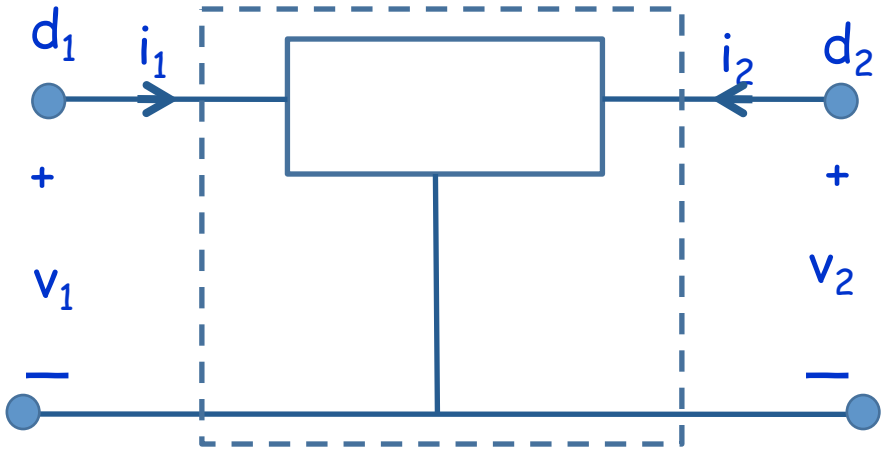


Terminal variables for a 3-terminal:

v_1 ,v_2 , i_1 , i_2

Terminal variables for a 2-port:

v_1 ,v_2 , i_1 , i_2



$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$R_R = \{(v_1, v_2, i_1, i_2) : f_1(v_1, v_2, i_1, i_2) = 0, f_2(v_1, v_2, i_1, i_2) = 0\}$$

Different Representations

- For a 1-port(2-terminal) there are two representations
- For a 2-port(3-terminal) there are ..... representations
- Why?

- Current-controlled: $v_1 =$

  $v_2 =$

- Voltage-controlled: $i_1 =$

  $i_2 =$

- Hybrid-1: $v_1 =$

  $i_2 =$

- Hybrid-2: $i_1 =$

  $v_2 =$

- Transmission-1: $v_1 =$

  $i_1 =$

- Transmission-2: $v_2 =$

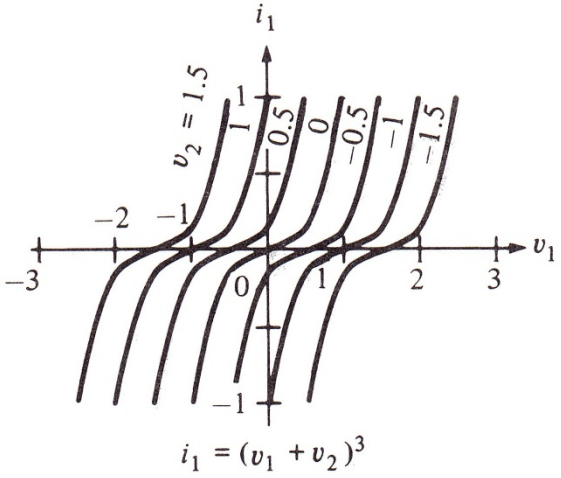
  $-i_2 =$

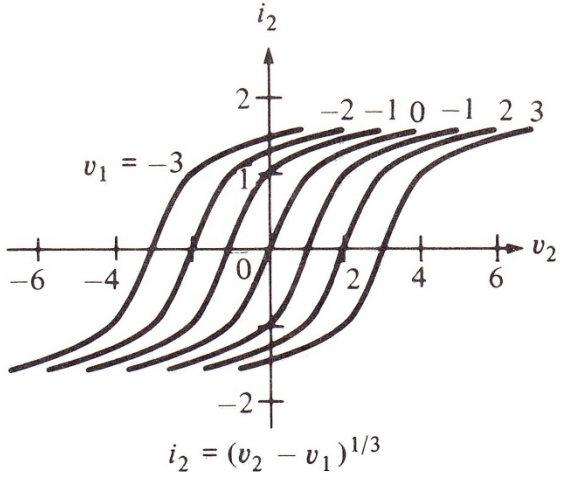
An Example

$$R_R = \{(v_1, v_2, i_1, i_2) : f_1(v_1, v_2, i_1, i_2) = 0, f_2(v_1, v_2, i_1, i_2) = 0\}$$

$$f_1(v_1, v_2, i_1, i_2) = i_1 + 2i_2 - (v_1 + v_2)^3 - 2(v_2 - v_1)^{1/3} = 0$$

$$f_2(v_1, v_2, i_1, i_2) = 2i_1 - i_2 - 2(v_1 + v_2)^3 + (v_2 - v_1)^{1/3} = 0$$

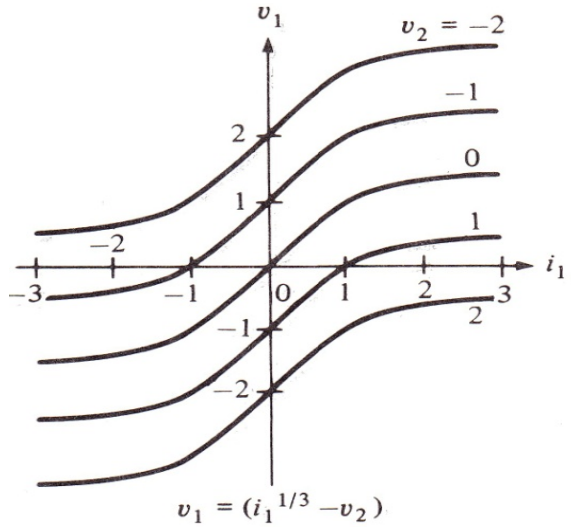


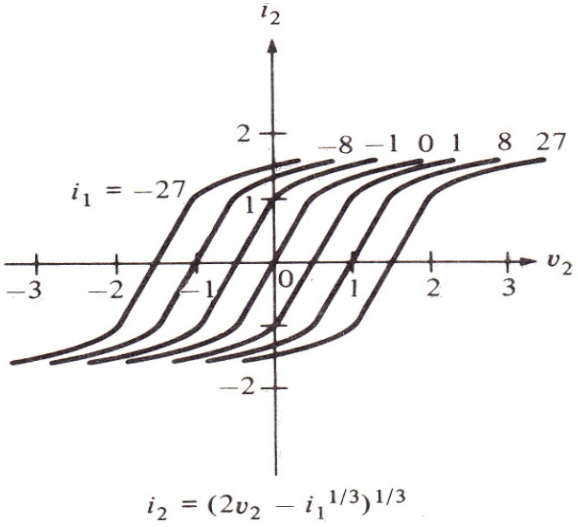


Voltage-controlled

$$i_1 = \hat{i}_1(v_1, v_2)$$

$$i_2 = \hat{i}_2(v_1, v_2)$$

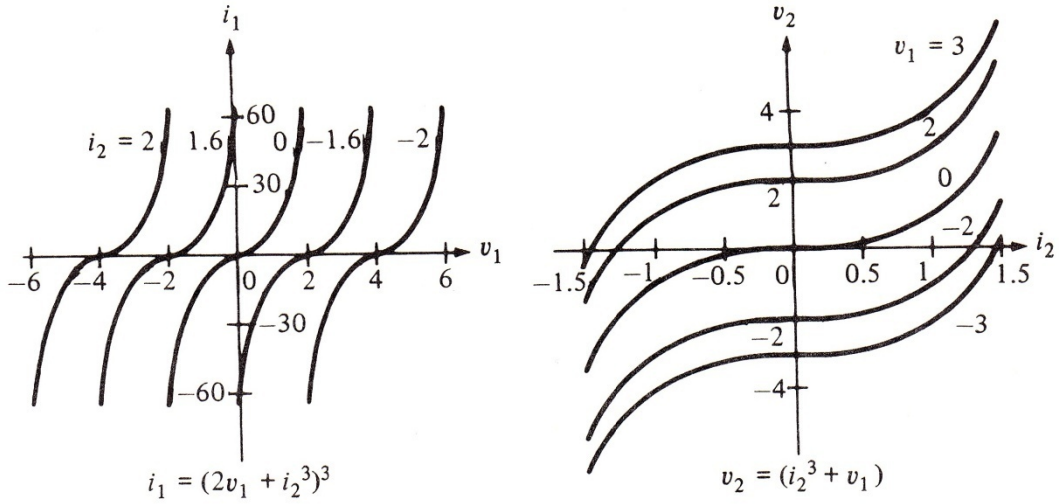




Hybrid-1

$$v_1 = \hat{v}_1(i_1, v_2)$$

$$i_2 = \hat{i}_2(i_1, v_2)$$

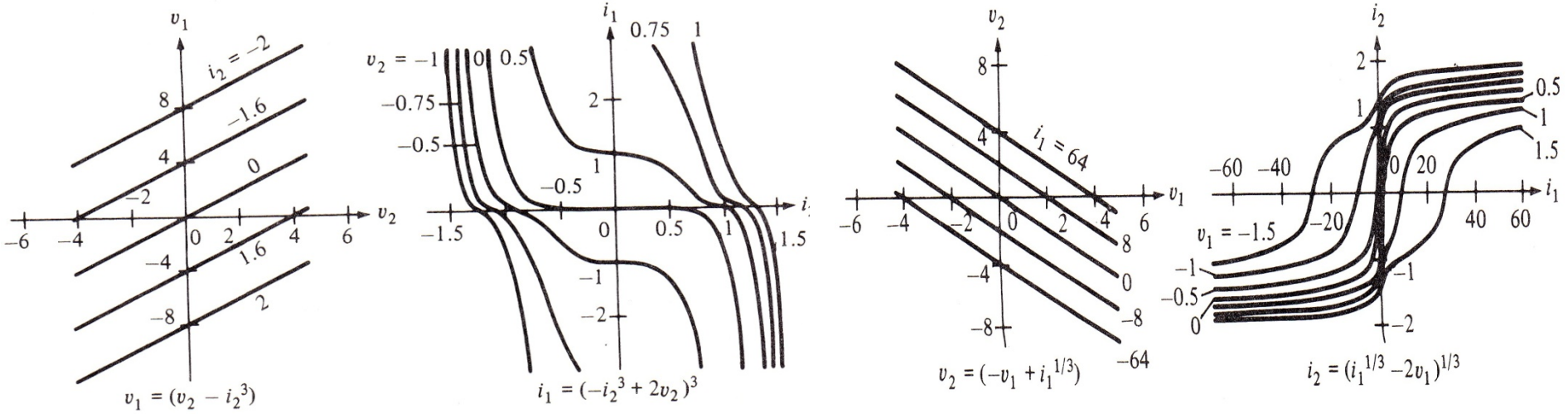


(d)

Hybrid-2

$$i_1 = \hat{i}_1(v_1, i_2)$$

$$v_2 = \hat{v}_2(v_1, i_2)$$



(f)

Transmission-1

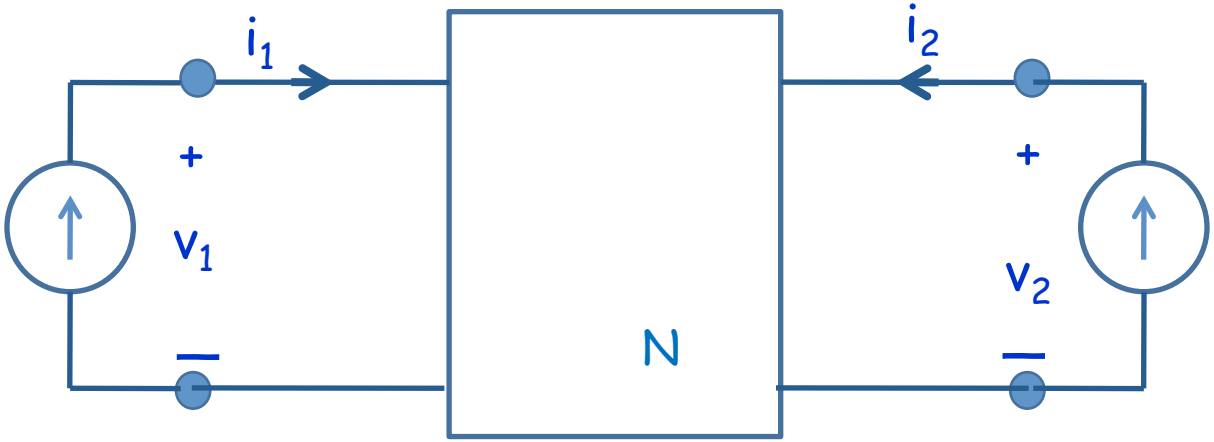
Transmission-2

L.O. Chua, C.A. Desoer, S.E. Kuh. "Linear and Nonlinear Circuits" Mc.Graw Hill, 1987, New York

Six Representations for Linear 2-ports

- If functions are linear then they act like matrices:

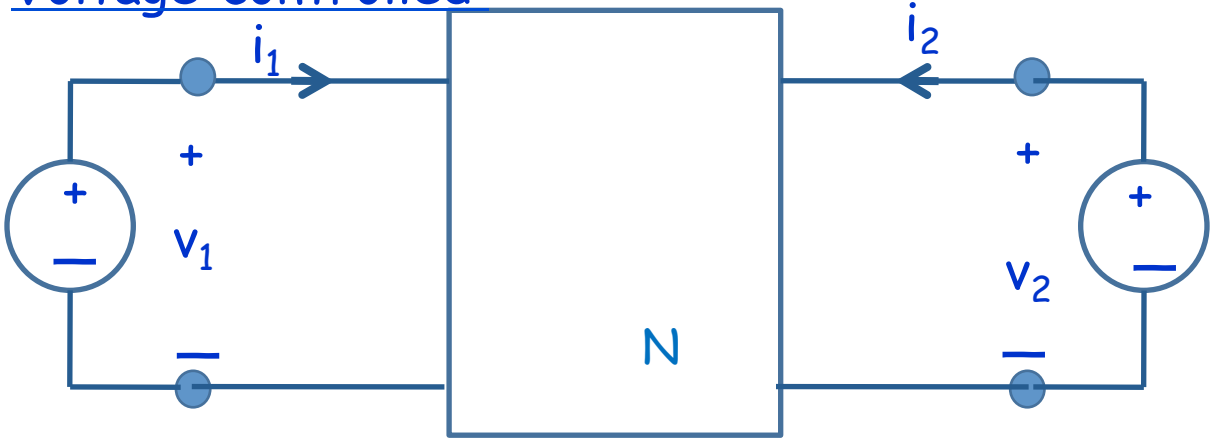
Current-controlled:



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$v = R\, i$$

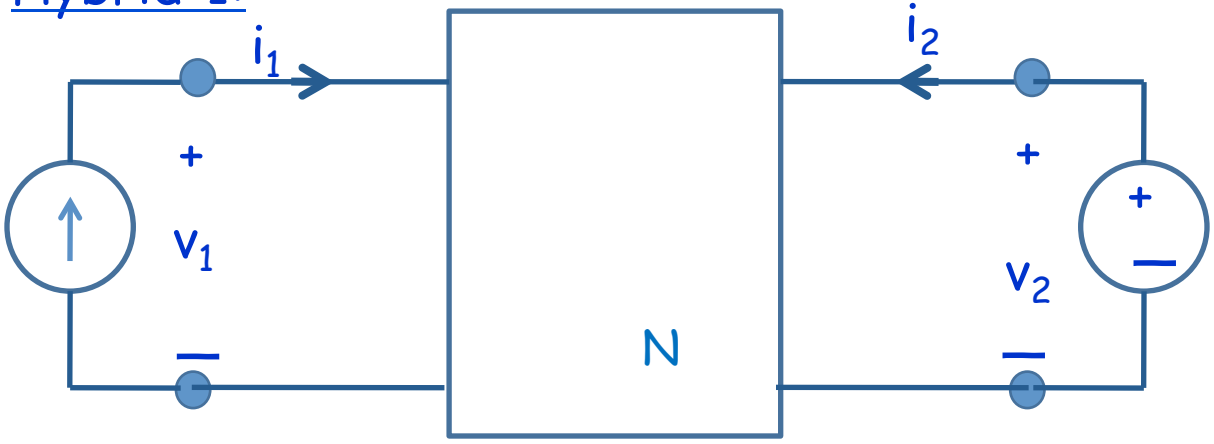
Voltage-controlled:



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$i = G\, v$$

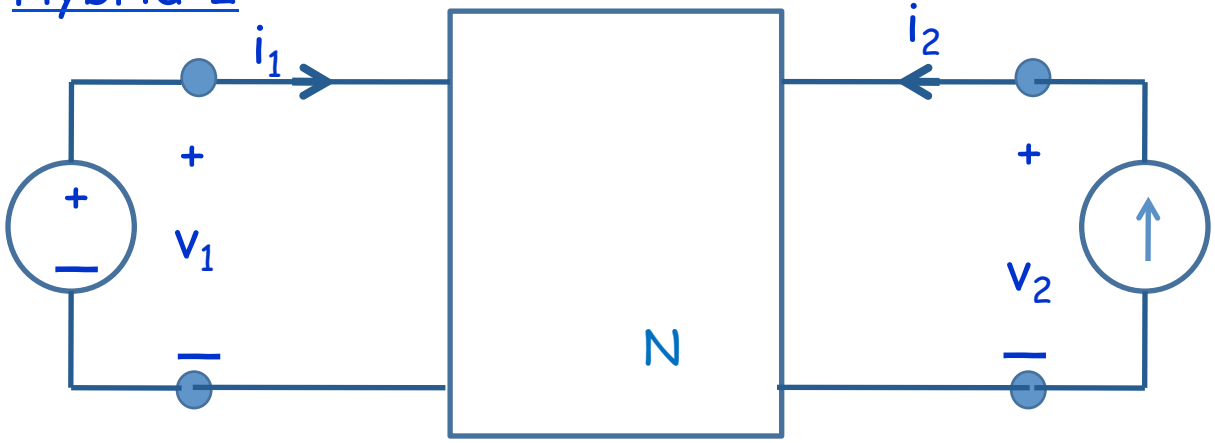
Hybrid-1:



$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = H \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

Hybrid-2



$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \tilde{h}_{11} & \tilde{h}_{12} \\ \tilde{h}_{21} & \tilde{h}_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \tilde{H} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

Transmission-1

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

Transmission-2

$$\begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} \\ \tilde{t}_{21} & \tilde{t}_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix}$$

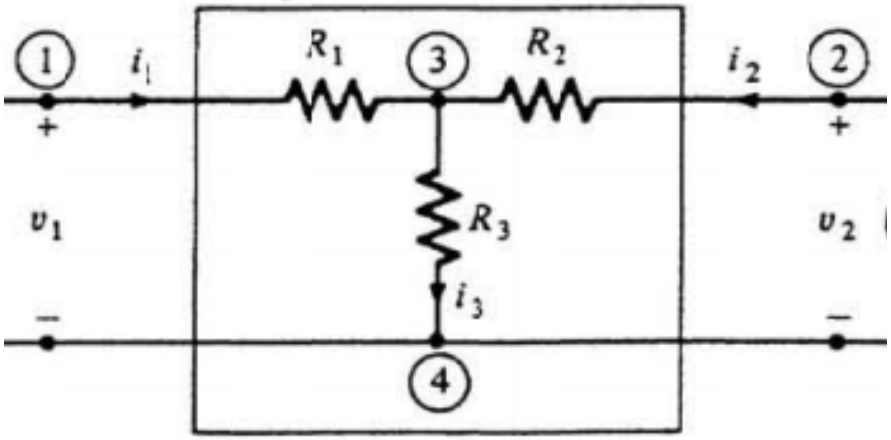
How To Obtain A Representation For Linear 2-ports

There are two ways: 1) All in one go 2) Each parameter seperately

1) All in one go: Connect appropriate sources to the ports according to the type of the representation. Write KCL and KVL equations and element equations for the elements in 2-port. Use these to find the representation.

EXAMPLE:

Find the current-controlled representation for the following 2-port:



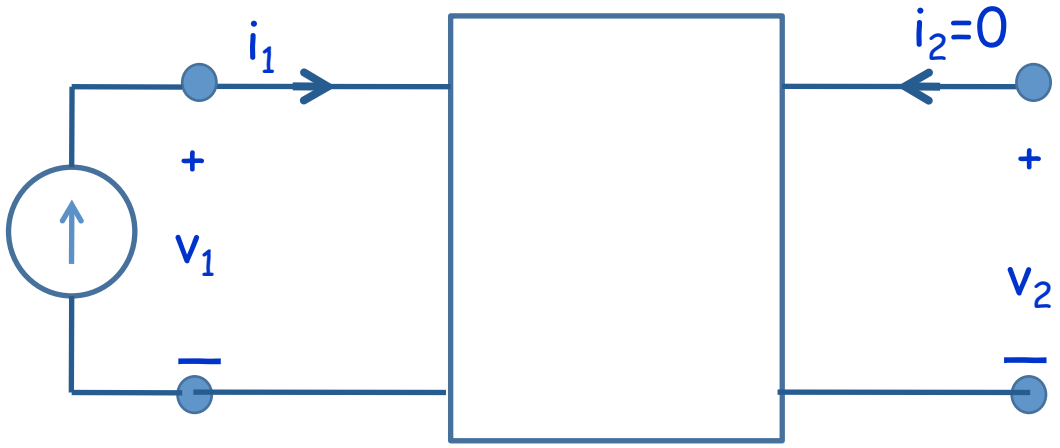
2) Each parameter seperately: Check the representation and express the parameter in terms of port variables. From this you will see that you need to connect a voltage or current source to one of the ports and the other port needs to be left either as a short-circuit or as an open circuit. Use this circuit to find the parameter, again using KVL,KCL and element equations.

For instance, for finding r_{11} .....

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$v_1 = r_{11} i_1 + r_{12} i_2$$

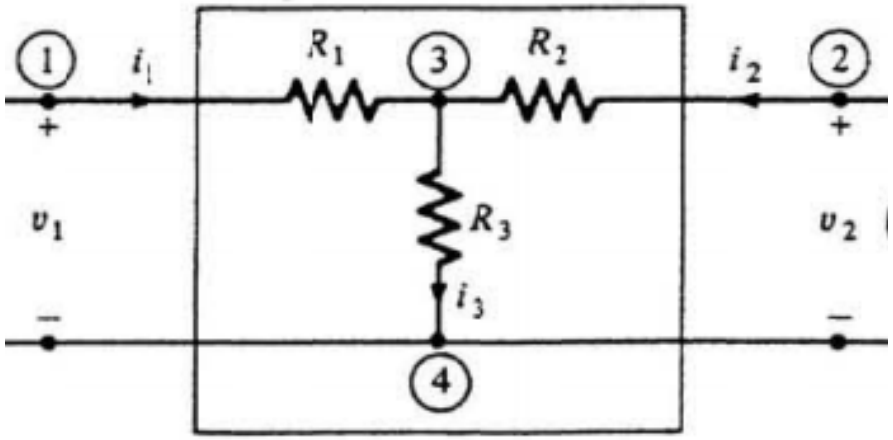
$$r_{11} = \left. \frac{v_1}{i_1} \right|_{i_2 = 0}$$



For finding g_{21} .....

EXAMPLE:

Find hybrid-2 representation for the following 2-port:



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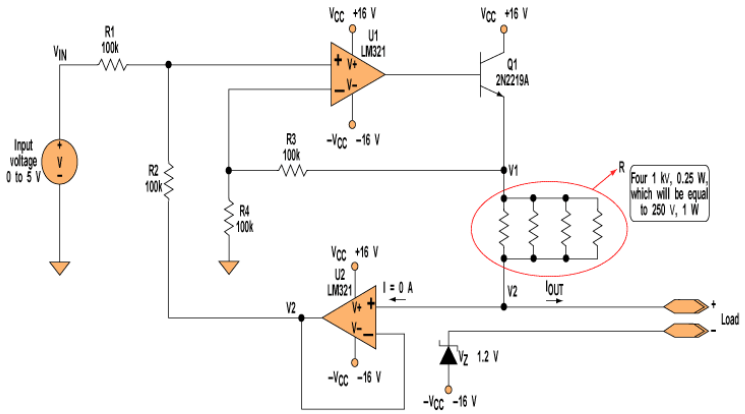
Some Linear 2-ports

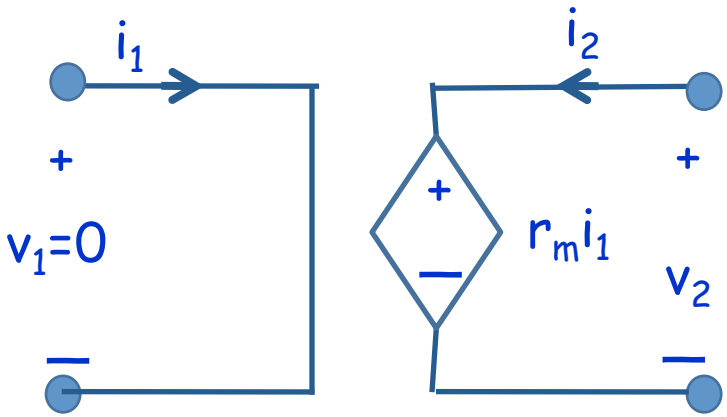
Voltage controlled current source



To achieve a low thermal coefficient, the current-sense resistor (R) used by this voltage-controlled current source consists of four individual resistors in parallel.

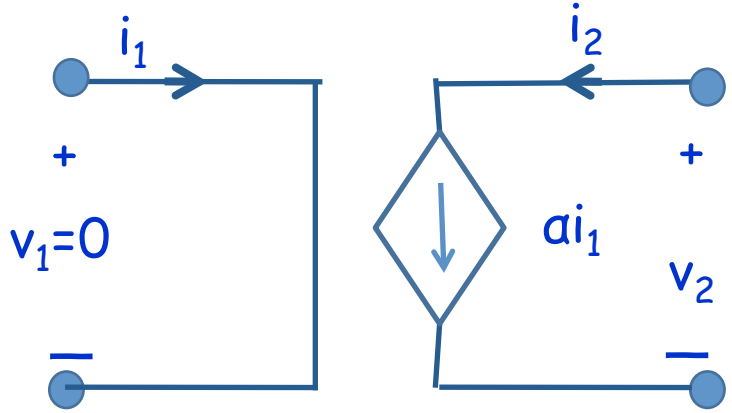
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Current controlled voltage source



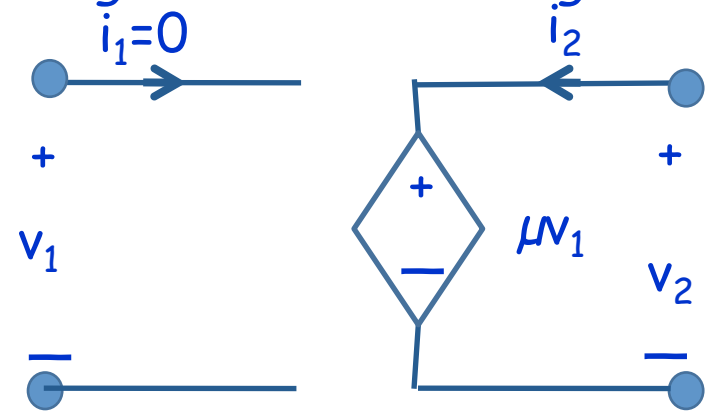
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r_m & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Current controlled current source



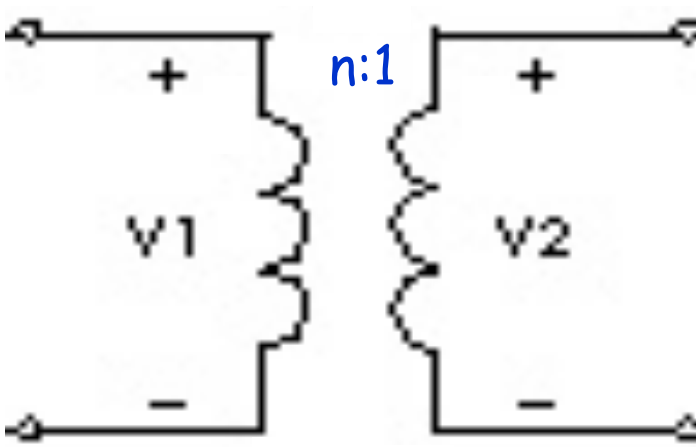
$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

Voltage controlled voltage source



$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

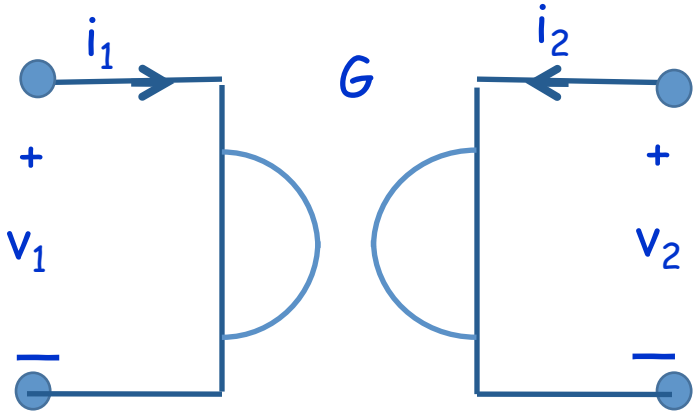
Ideal transformator



$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$\begin{aligned} p(t) &= v_1(t)i_1(t) + v_2(t)i_2(t) \\ &= v_1(t)i_1(t) - \frac{1}{n}v_1(t)ni_1(t) \\ &= 0 \end{aligned}$$

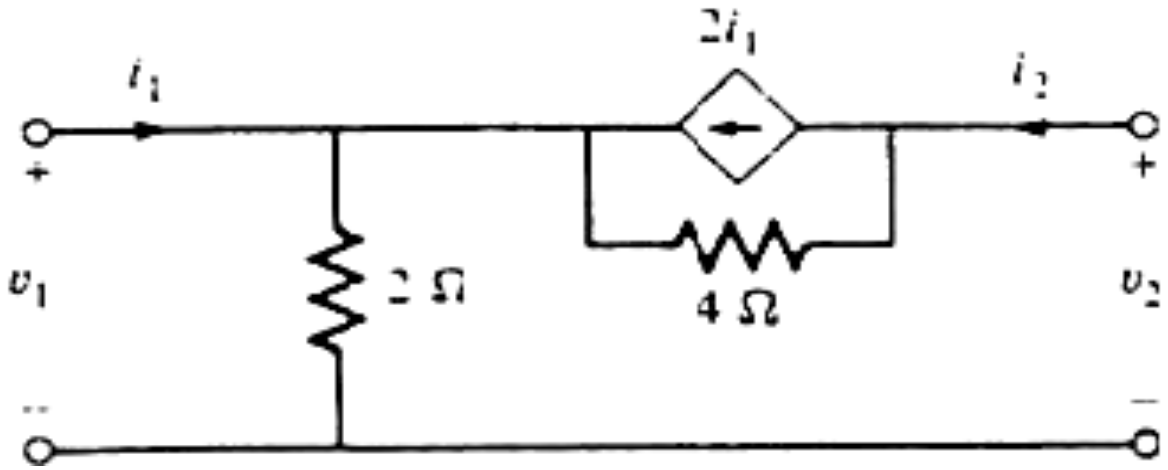
Gyrator



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$p(t) = \ldots\ldots$$

Obtain the element equations for the following 2-port!



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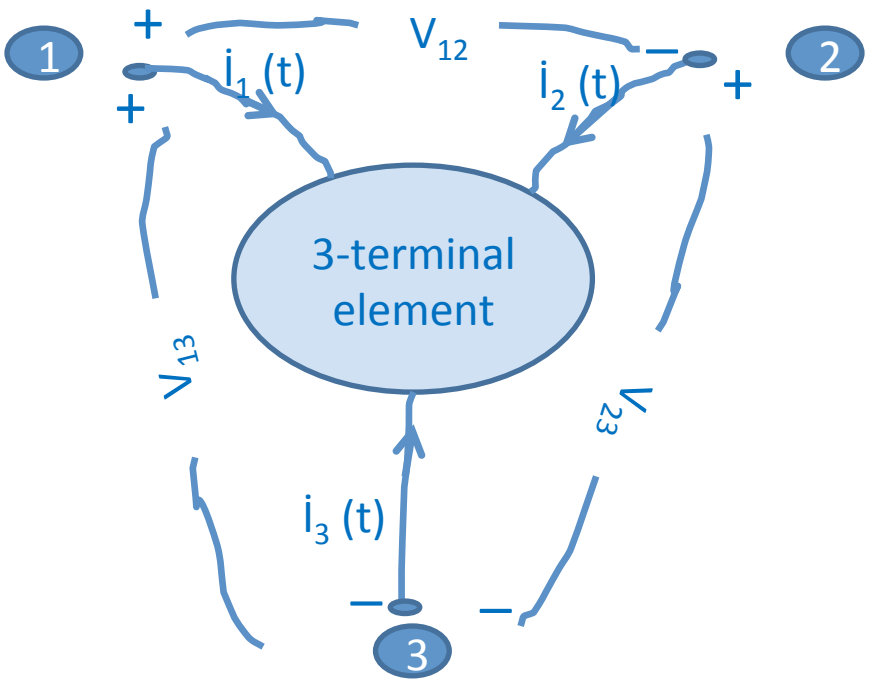

Figure 1

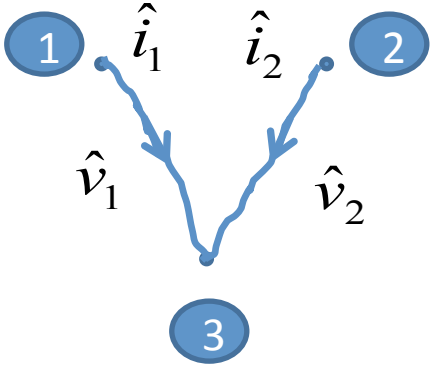

Figure 2

The element equation for the element in Figure 1 is given below:

$$v_{12} - 2i_1 + 3v_{32} = 0 \quad , \qquad 4i_1 - i_3 = 0$$

a) Find the element equation in terms of the quantities given in the element graph in Figure 2.

b) Construct a 3-terminal element that has this element equation using only 2-terminal linear resistors and some dependent sources.