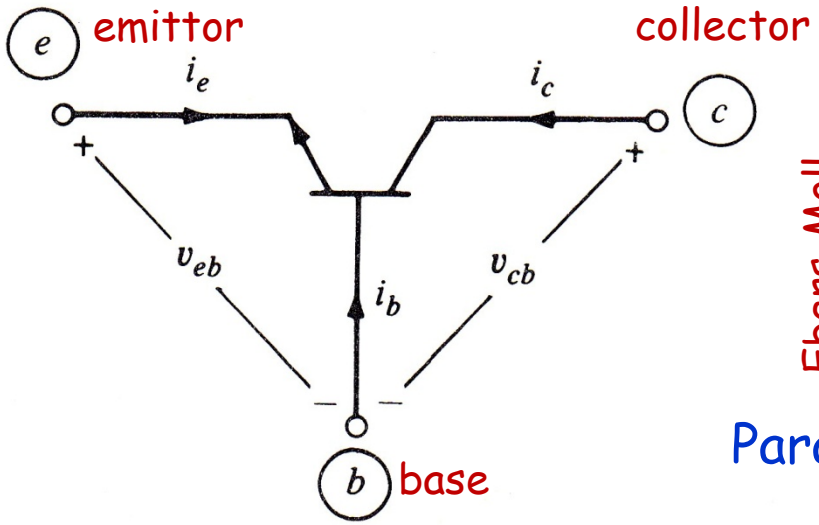
npn Bipolar Transistor



Low Frequency Characterization

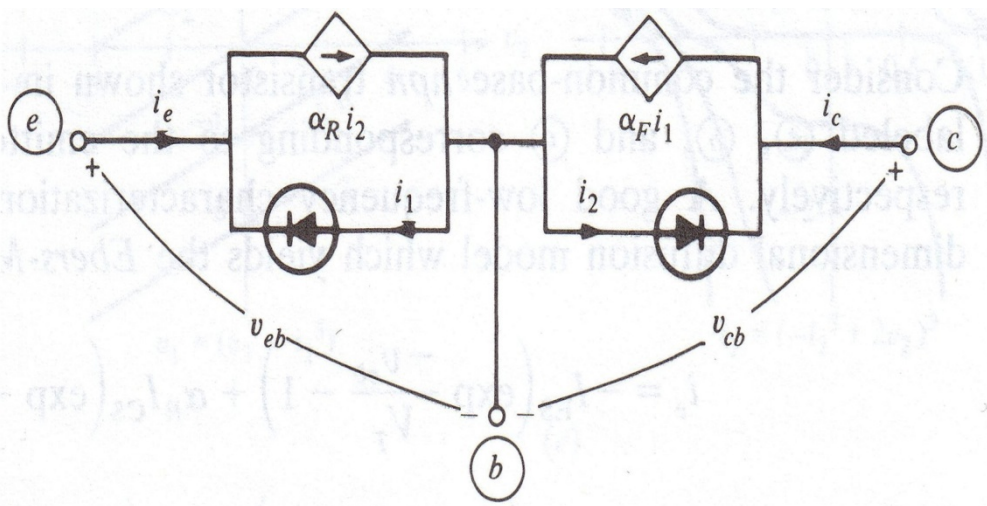
Ebers-Moll Equations

$$\begin{cases} i_e = -I_{ES}\left(e^{-\frac{v_{eb}}{v_T}} - 1\right) + \alpha_R I_{CS}\left(e^{-\frac{v_{cb}}{v_T}} - 1\right) \\ i_c = \alpha_F I_{ES}\left(e^{-\frac{v_{eb}}{v_T}} - 1\right) - I_{CS}\left(e^{-\frac{v_{cb}}{v_T}} - 1\right) \end{cases}$$

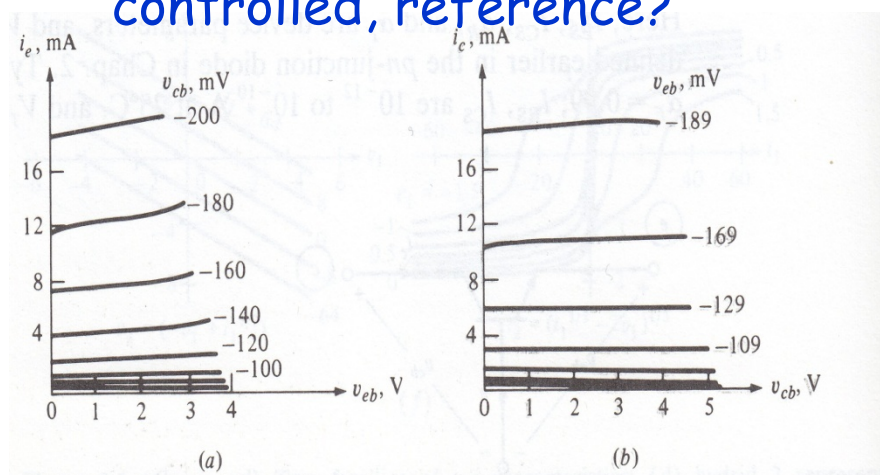
Parameters: $I_{ES}, I_{CS}, \alpha_F, \alpha_R, V_T$

$$I_{ES}, I_{CS} \in (10^{-12}, 10^{-10})A,\ \alpha_F = 0.99,\ \alpha_R\ \ 0.5 - 0.8,\ V_T \cong 25mV\ \ (25^\circ C)$$

Behaviour of the model



3-terminal element, voltage-controlled, reference?



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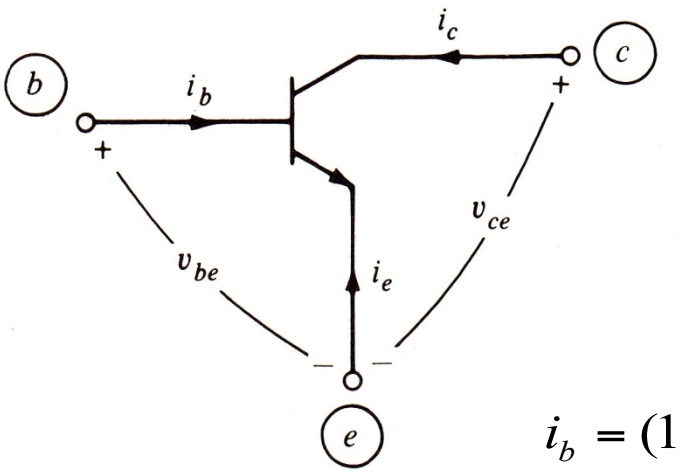
If the reference is chosen as the emittor...

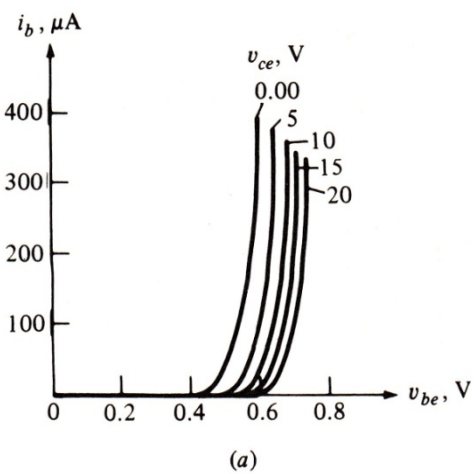


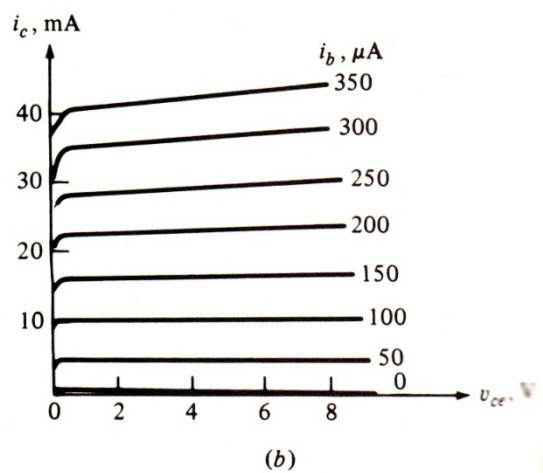
$$\left.\begin{aligned} v_{be} &= -v_{eb} \\ v_{ce} &= v_{cb} - v_{eb} \\ i_b &= -(i_e + i_c) \end{aligned}\right\}$$

Substitute in Ebers-Moll equations

$$i_b = (1-\alpha_F) I_{ES} (e^{\frac{v_{be}}{v_T}} - 1) + (1-\alpha_R) I_{CS} (e^{\frac{v_{be}-v_{ce}}{v_T}} - 1)$$

$$i_c = \alpha_F I_{ES} (e^{\frac{v_{be}}{v_T}} - 1) - I_{CS} (e^{\frac{v_{be}-v_{ce}}{v_T}} - 1)$$





$$v_{be} = \hat{v}_{be}(i_b, v_{ce})$$

$$i_c = \hat{i}_c(i_b, v_{ce})$$

Which representation is this?

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For many large-signal applications, e.g., digital circuits, these models are unnecessarily complicated and further simplifications are possible. In some approximate dc analysis, it suffices to represent the base emitter by a constant voltage source E_0 in series with an ideal diode and to represent the collector emitter by a CCCS together with an ideal diode as shown in Fig. 4.8; the corresponding characteristics are shown in Fig. 4.9. For silicon transistors at room temperature, the value of E_0 is usually between 0.6 and 0.7 V.

It is important to bear in mind that models are developed with specific applications in mind. Obviously, the simpler the model, the easier the circuit analysis. Thus, for some applications, if only an approximate solution is called for, we should use the simplest but valid model to get an idea of how the circuit functions. In other situations, e.g., in determining the precise operating points using a computer, we need to use a more precise model for the transistor than that of the Ebers-Moll model. Some complicated models have been developed using a combination of physical principles and experimental measurements (e.g., the Gummel-Poon model[3]). In small-signal analysis, we use a linear model which holds only approximately in the neighborhood of the operating point. Some of these will be discussed in the remaining portions of this section.

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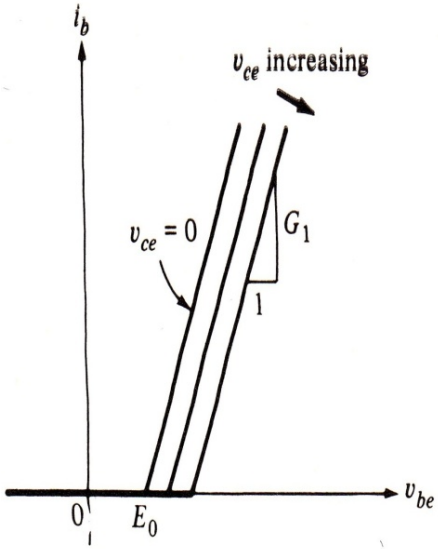
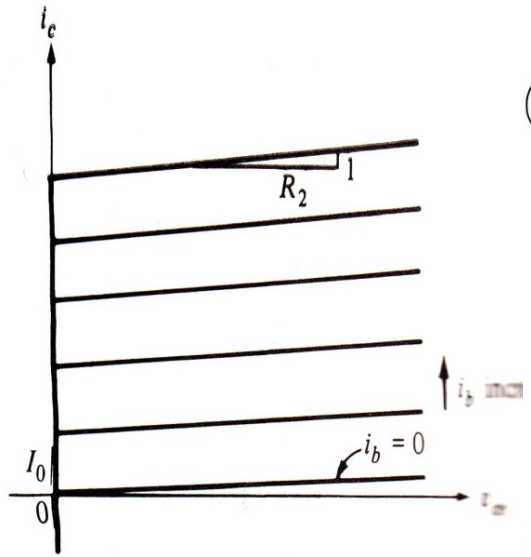
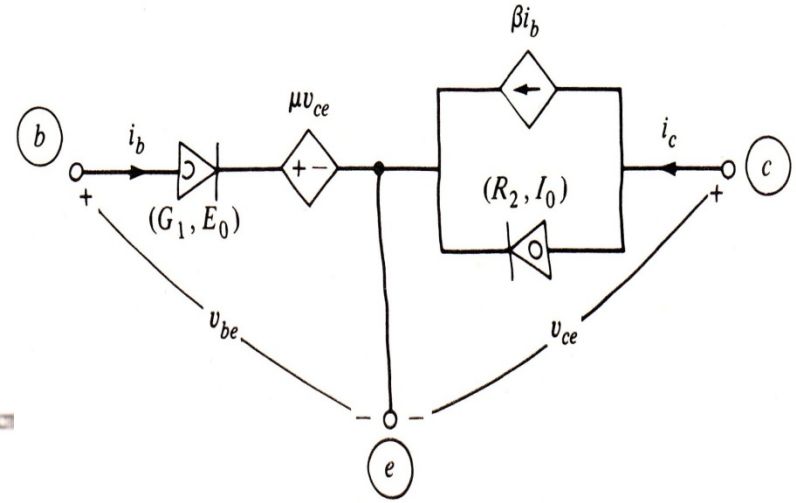
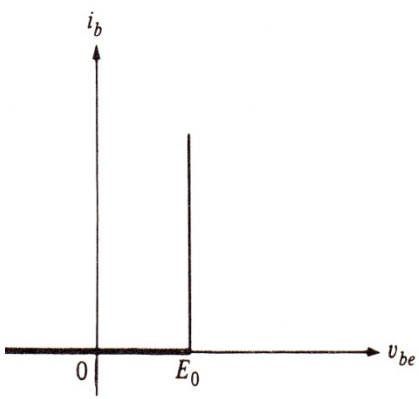
Piecewise-linear model

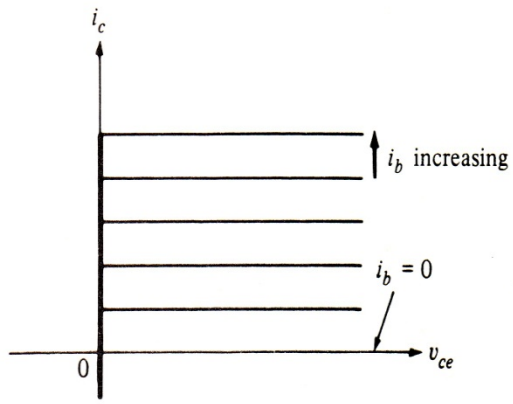


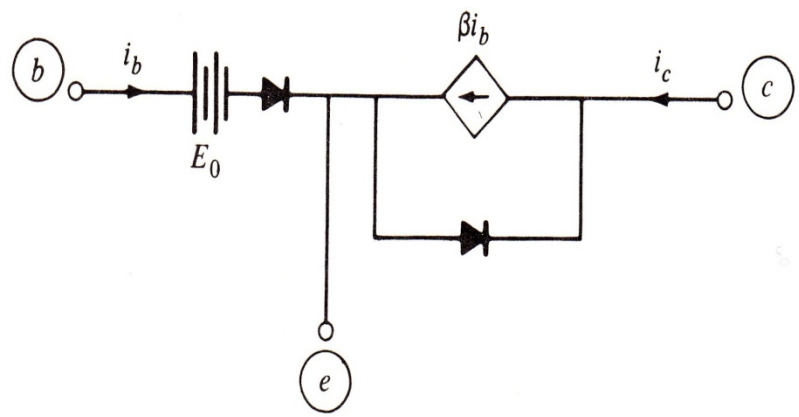




Ideal Model







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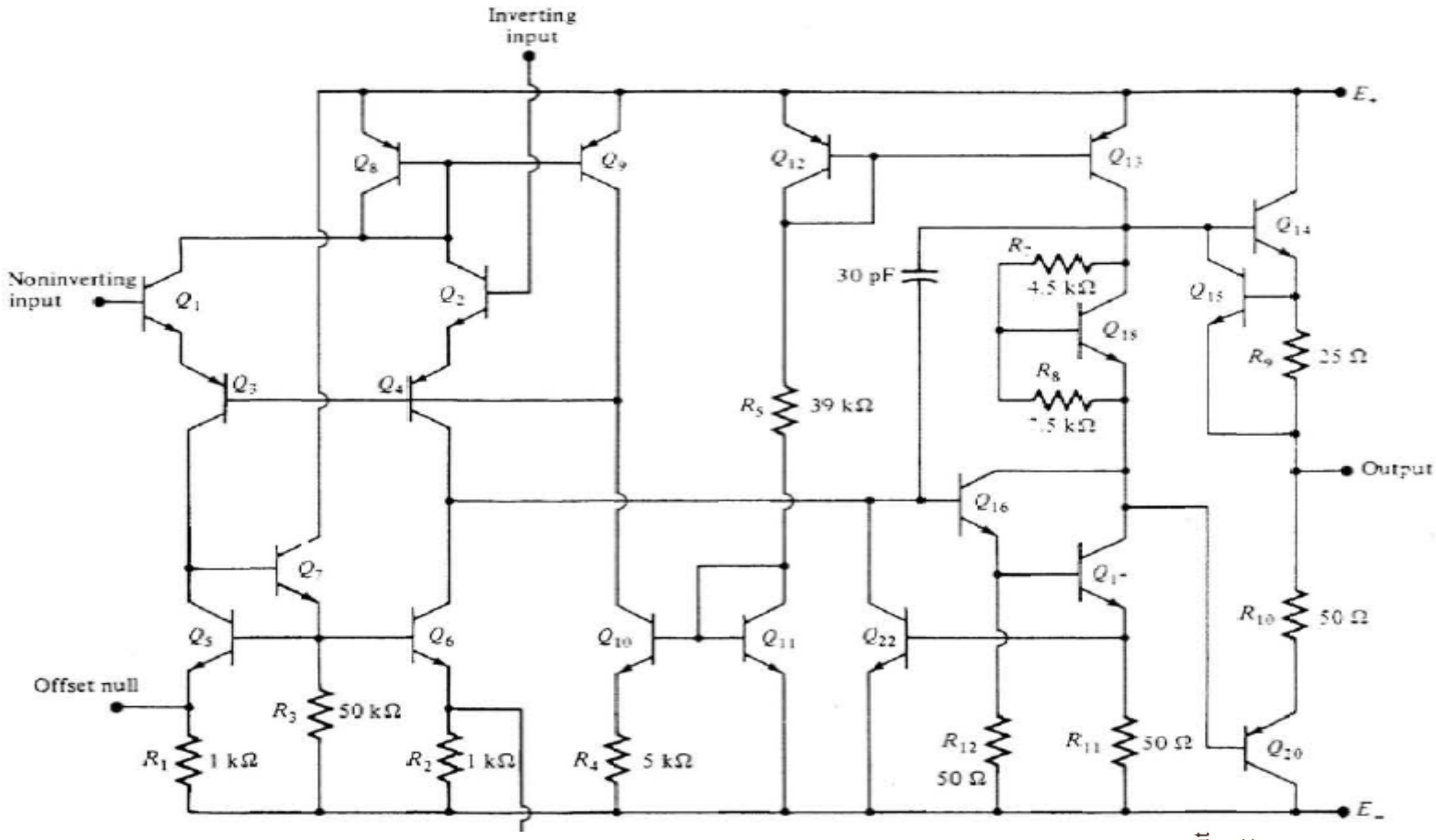
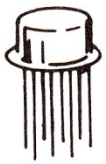
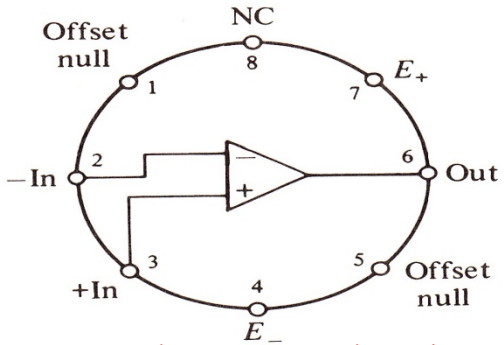
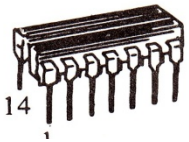
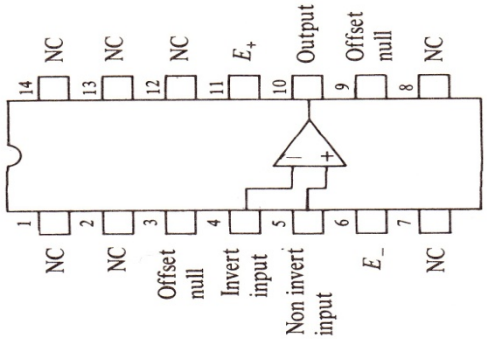
Operational Amplifier (Op-Amp)



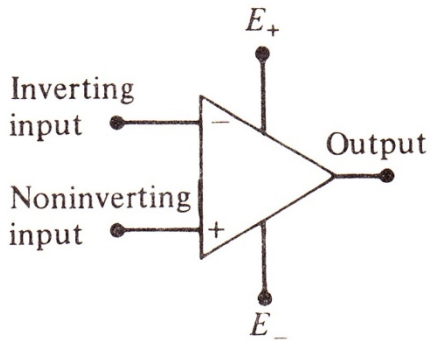




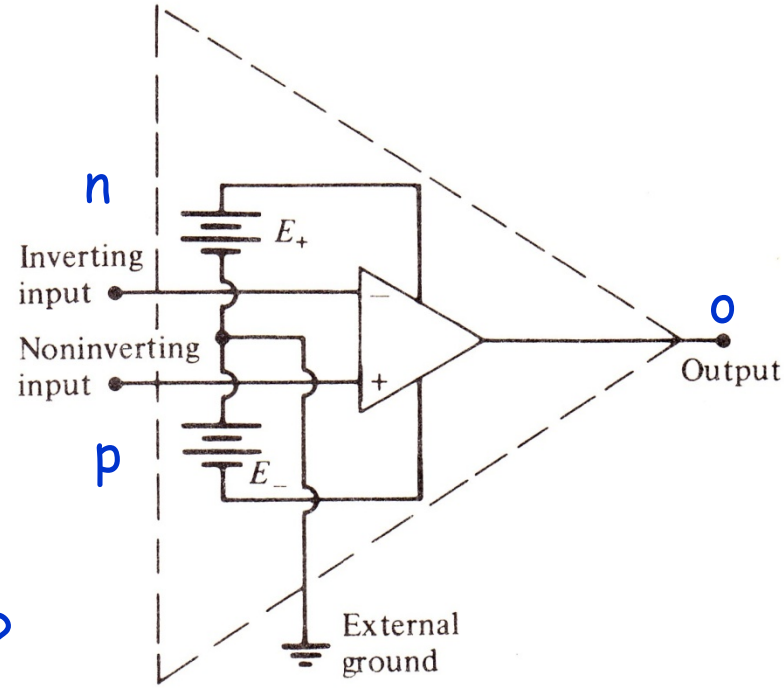




L.O. Chua, C.A. Desoer, S.E. Kuh. "Linear and Nonlinear Circuits", Mc.Graw Hill, 1987, New York



(a)



How many terminals does it have?
How many equations do we need?

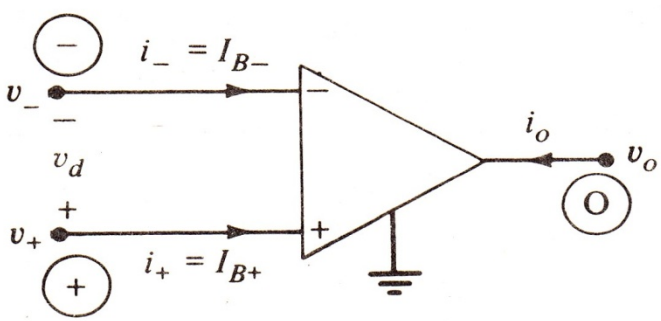
Base currents

Bipolar (µA741)	FET (µA740)
~0,2mA	~0,1nA

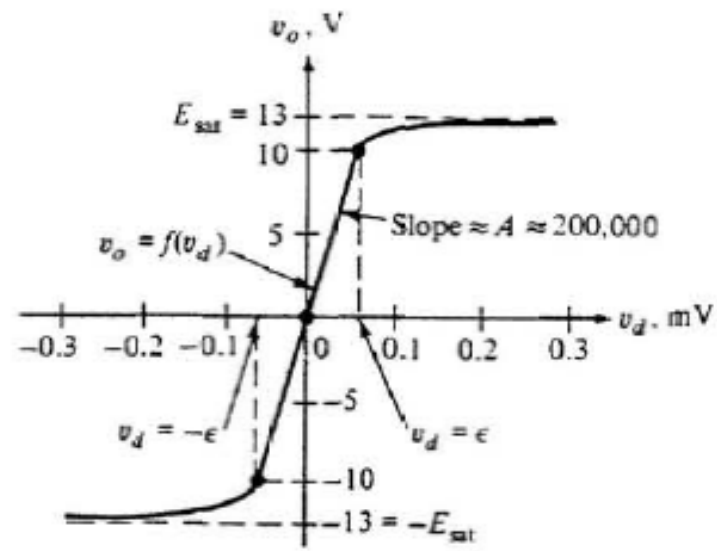
$$i_n = I_{B_n}$$

$$i_p = I_{B_p}$$

$$v_o = f(v_d)$$

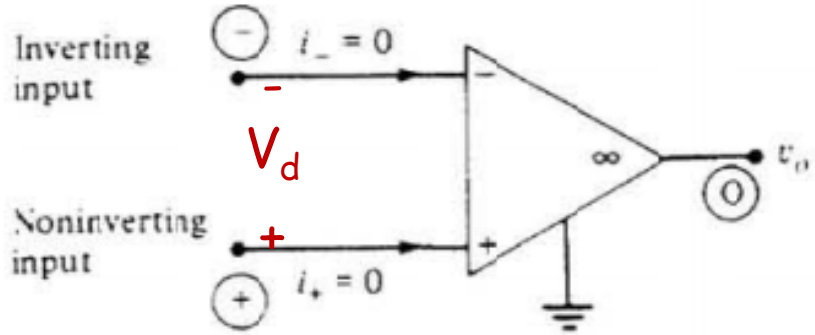


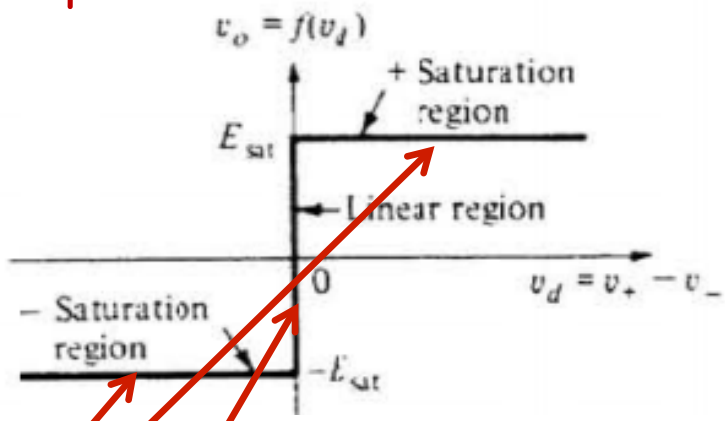
(a)



L.O. Chua, C.A. Desoer, S.E. Kuh. "Linear and Nonlinear Circuits", Mc.Graw Hill, 1987, New York

Ideal Op-Amp





$$i_n = 0$$

$$i_p = 0$$

$$v_d \neq 0 \Rightarrow v_o = E_{sat} \frac{|v_d|}{v_d}$$

$$v_d = 0 \Rightarrow -E_{sat} < v_0 < E_{sat}$$

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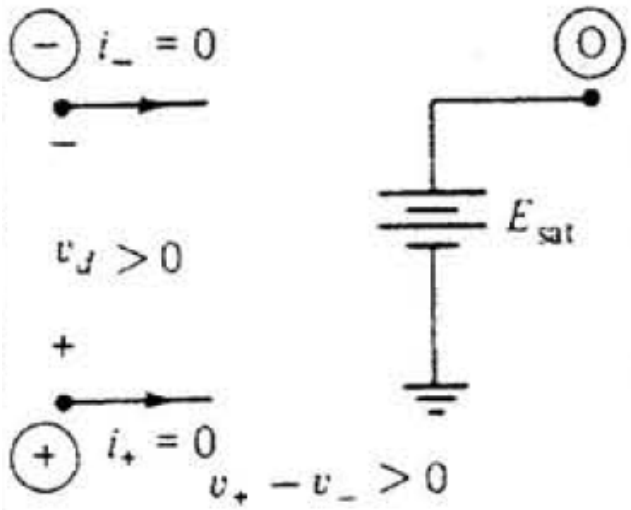
Equivalent model for positive saturation region

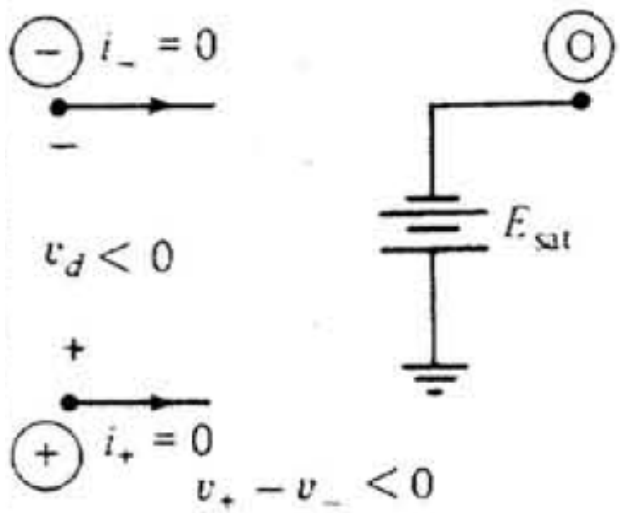
$$v_d = v_p - v_n > 0$$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_p \\ i_n \\ i_o \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_p \\ v_n \\ v_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ E_{sat} \end{bmatrix}$$

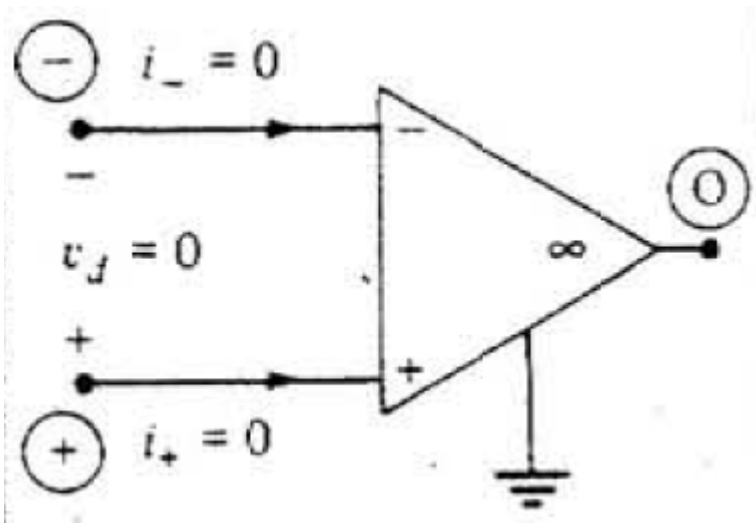
Equivalent model for negative saturation region

$$v_d = v_p - v_n < 0$$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_p \\ i_n \\ i_o \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_p \\ v_n \\ v_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -E_{sat} \end{bmatrix}$$

Equivalent model for linear region



$$-E_{sat} < v_o(t) < E_{sat}, \quad \forall t$$

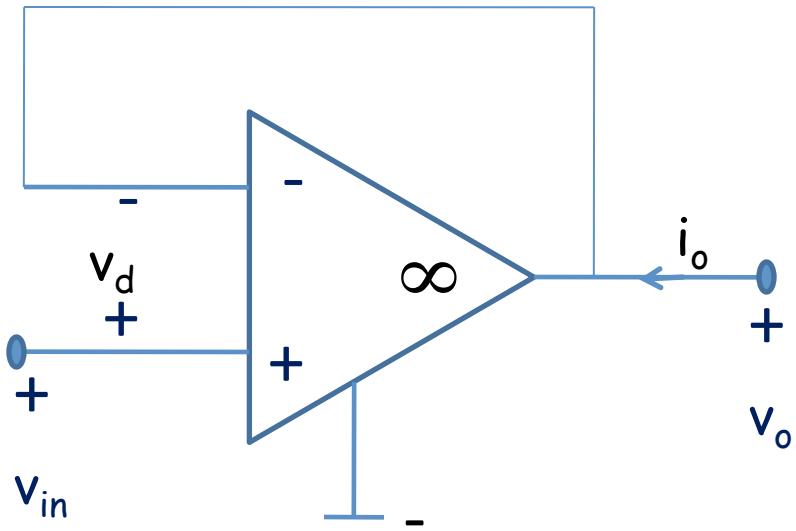
$$i_n = 0$$

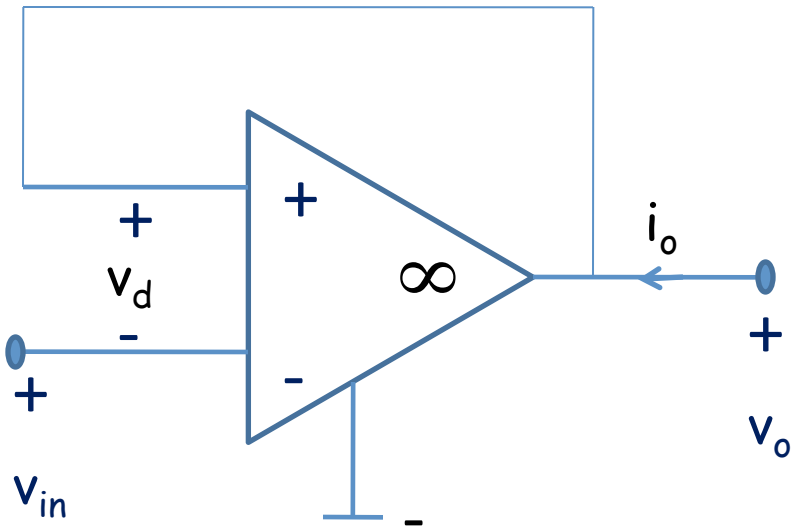
$$i_p = 0 \longrightarrow \text{virtual}$$

$$v_p - v_n = 0 \longrightarrow \text{short-circuit}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_p \\ i_n \\ i_o \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_p \\ v_n \\ v_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Negative-Positive Feedback Circuits





Linear Region: $|v_{in}| < E_{sat}$

Negative Feedboack

$$v_d = v_p - v_n = 0$$

$$v_p = v_{in}$$

$$v_n = v_o$$

$$v_o = v_{in}$$

Positive Feedback

$$v_d = v_p - v_n = 0$$

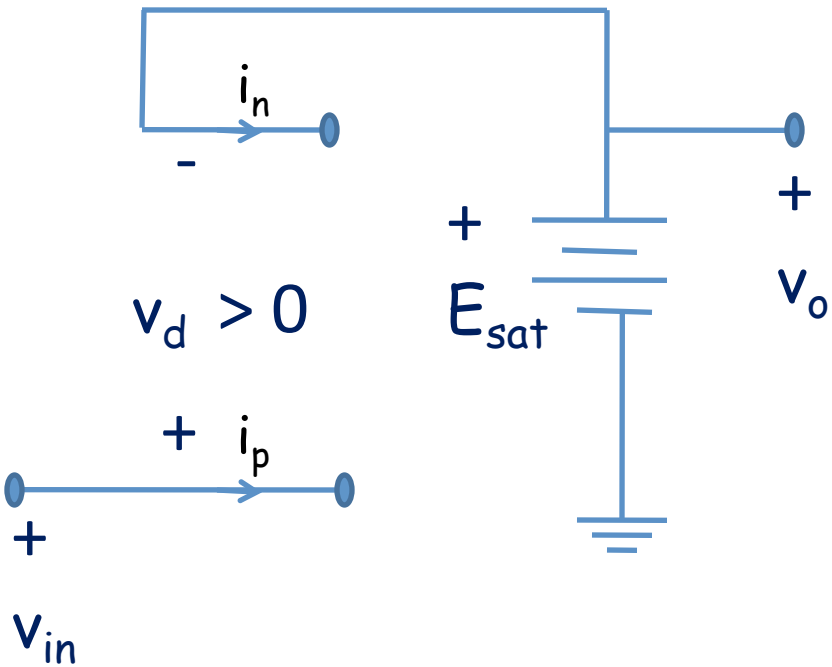
$$v_p = v_o$$

$$v_n = v_{in}$$

$$v_o = v_{in}$$

+ Saturation Region $v_d > 0$

Negative Feedback



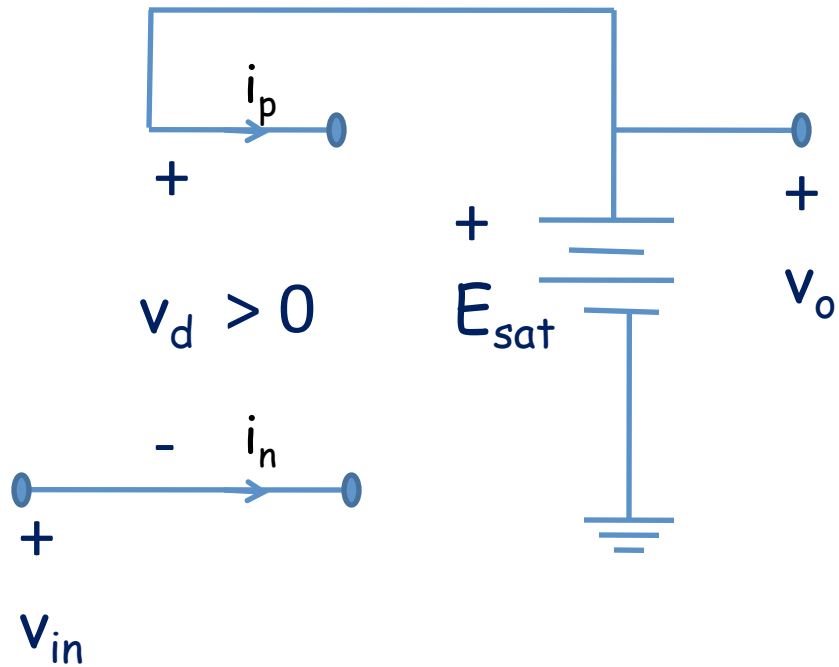
$$v_d = v_p - v_n > 0$$

$$v_p = v_{in}$$

$$v_n = E_{sat}$$

$$v_{in} > E_{sat}$$

Positive Feedback



$$v_d = v_p - v_n > 0$$

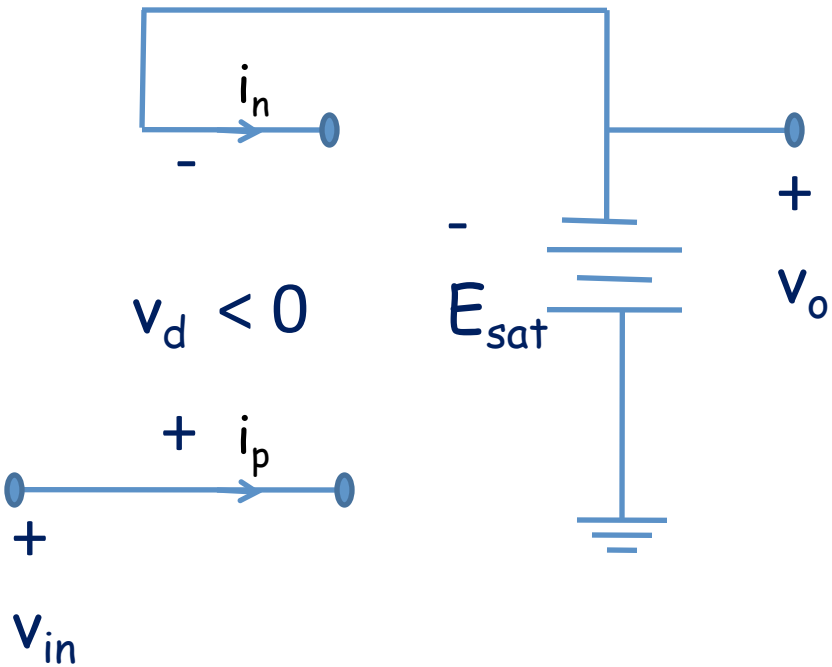
$$v_p = E_{sat}$$

$$v_n = v_{in}$$

$$v_{in} < E_{sat}$$

- Saturation Region $v_d < 0$

Negative Feedback



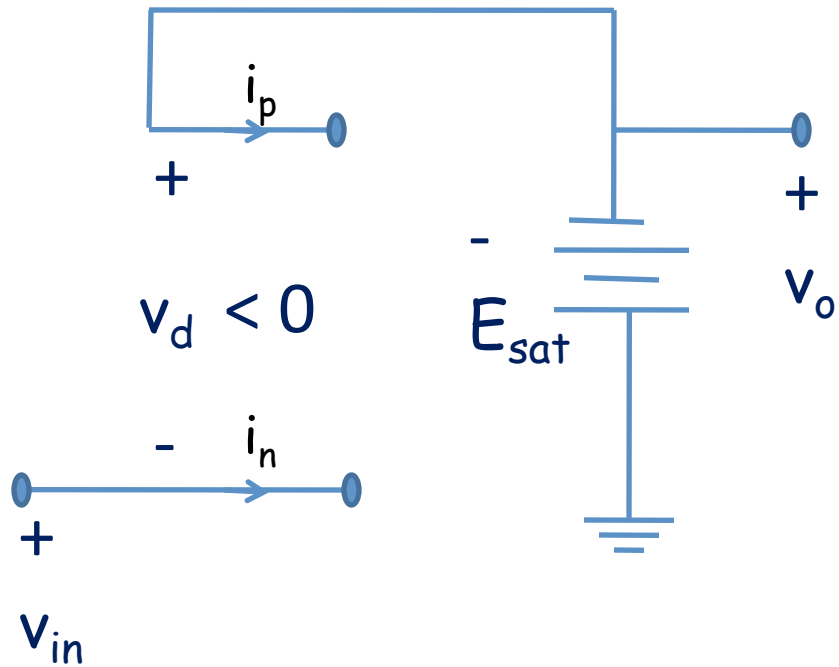
$$v_d = v_p - v_n < 0$$

$$v_p = v_{in}$$

$$v_n = -E_{sat}$$

$$v_{in} < -E_{sat}$$

Positive Feedback



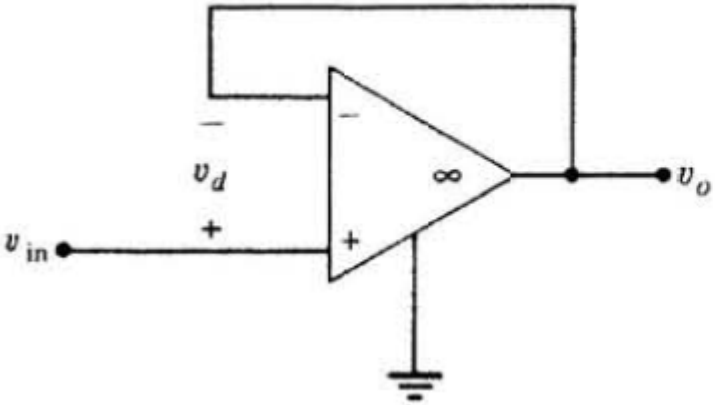
$$v_d = v_p - v_n < 0$$

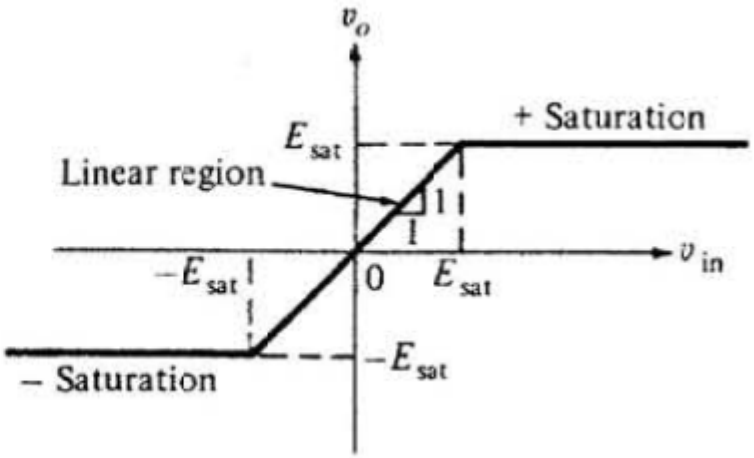
$$v_p = -E_{sat}$$

$$v_n = v_{in}$$

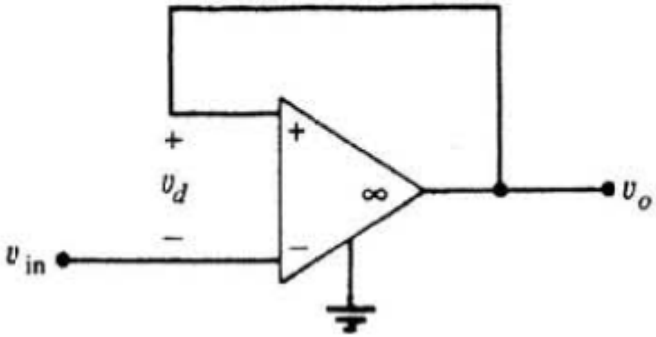
$$v_{in} > -E_{sat}$$

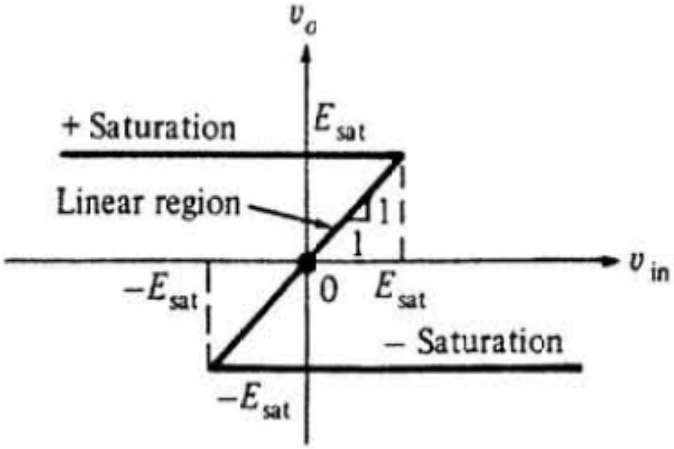
Negative Feedback Circuit





Positive Feedback Circuit

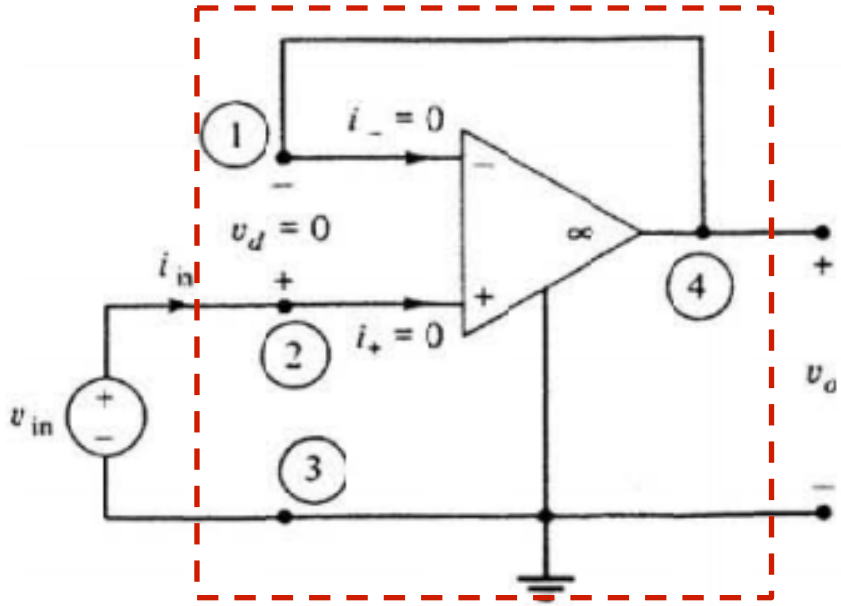




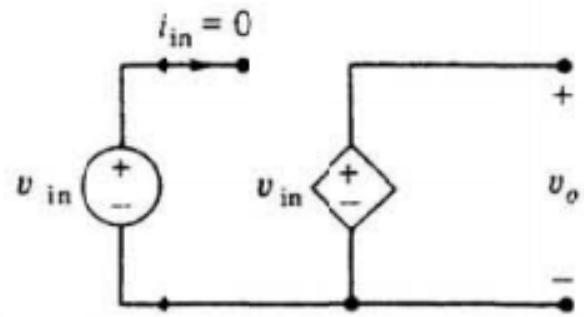
Some Applications

Buffer

Aim: No matter how big is the load at the output, the output voltage is the same as the input voltage.



Voltage-controlled voltage source



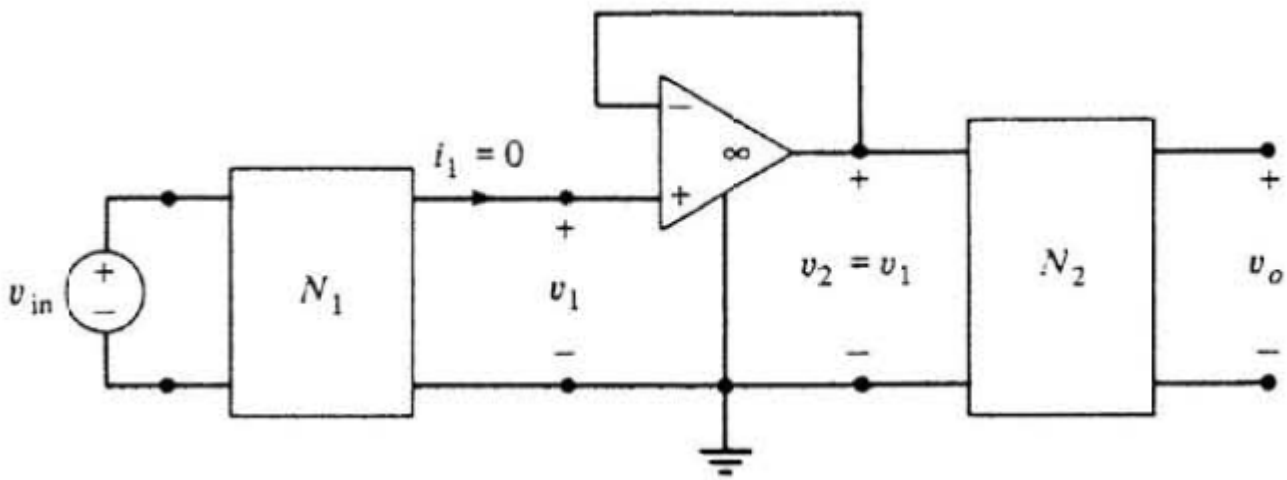
$$v_d = v_{in} - v_o = 0 \Rightarrow v_o = v_{in} \qquad i_{in} = i_+ = 0$$

valid for $\quad -E_{sat} < v_{in} < E_{sat}$

Some Applications

Buffer

Aim: No matter how big is the load at the output, the output voltage is the same as the input voltage.

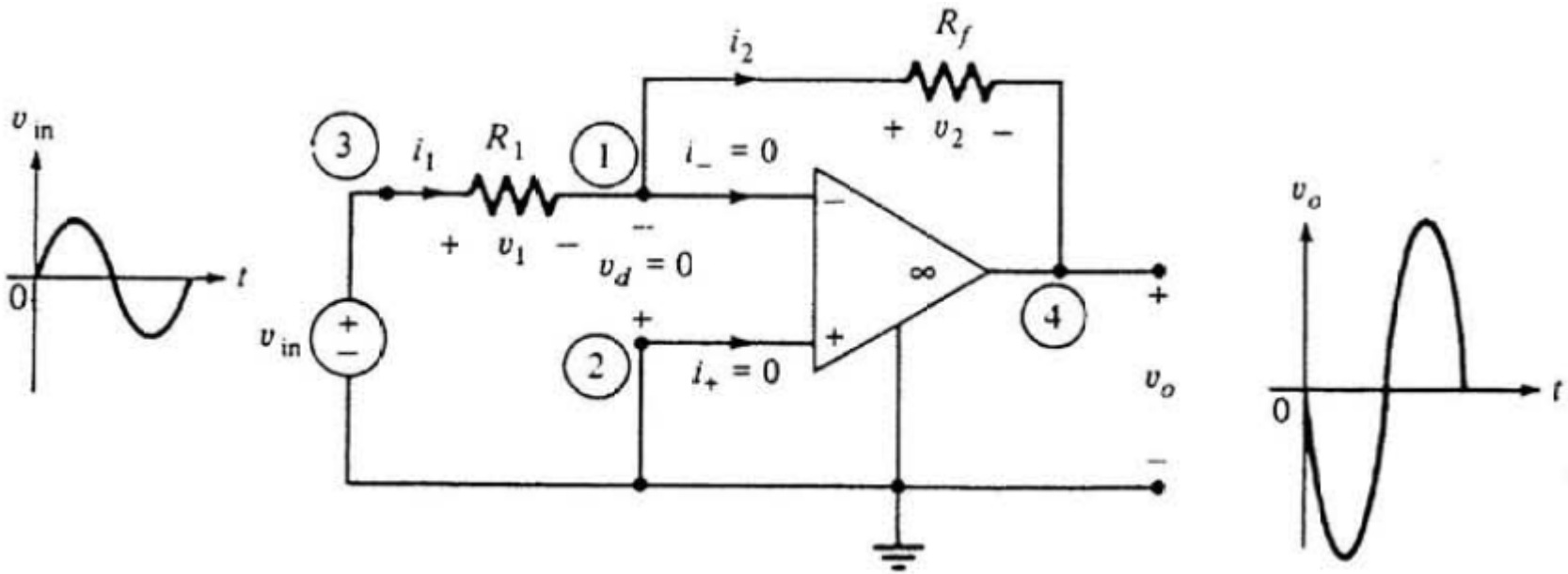


- Internal resistance is large for the input and small for the output.
- This reduces the downloading effect of N_2 onto N_1.
- Simple analysis.

Some Applications

Inverting Amplifier

Aim: A signal with small amplitude at the input is amplified at the output with a 180 degree phase shift.



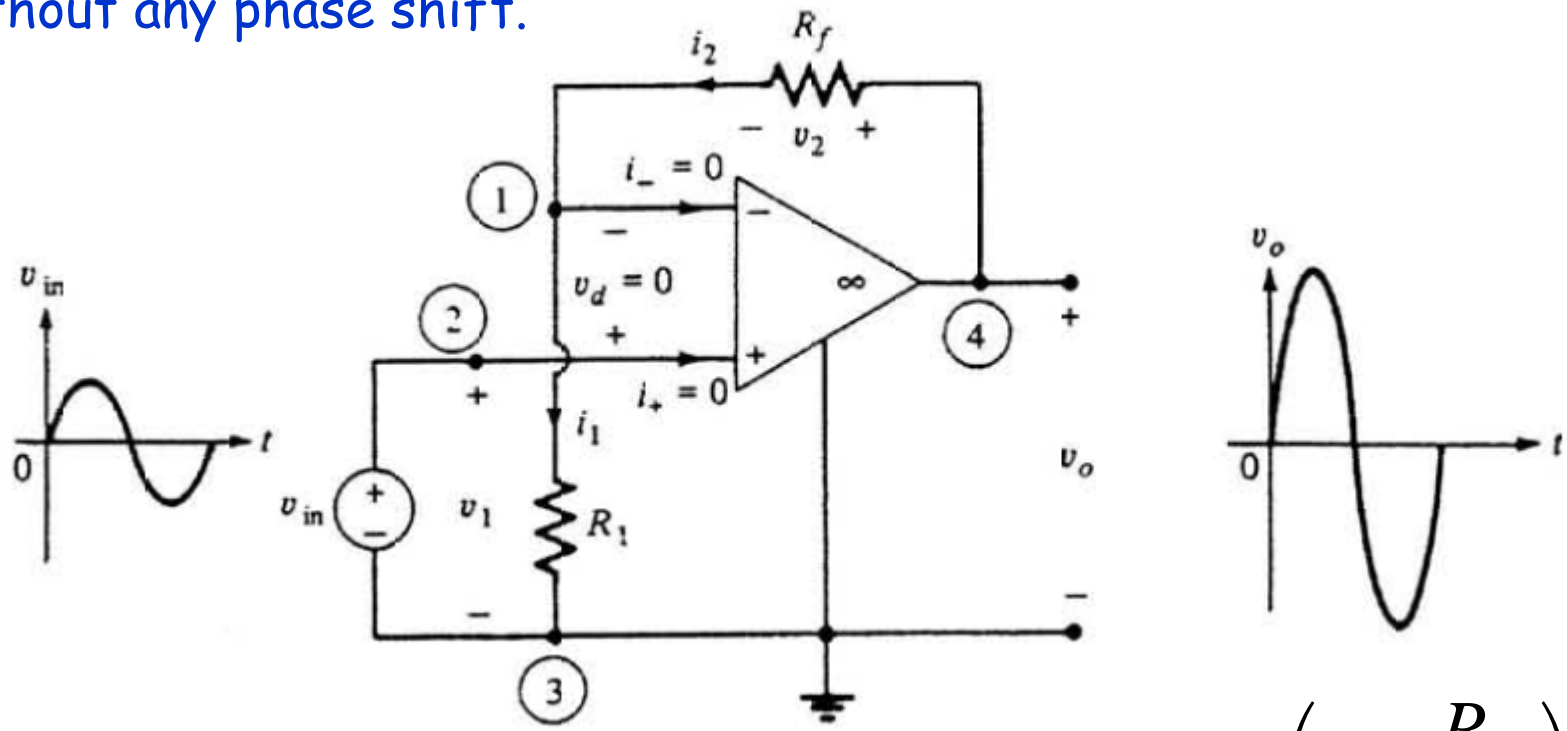
$$v_o = -\left(\frac{R_f}{R_1}\right) v_{in}$$

valid for $$-\left(\frac{R_1}{R_f}\right) E_{sat} < v_{in} < \left(\frac{R_1}{R_f}\right) E_{sat}$$

Some Applications

Non-inverting Amplifier

Aim: A signal with small amplitude at the input is amplified at the output without any phase shift.



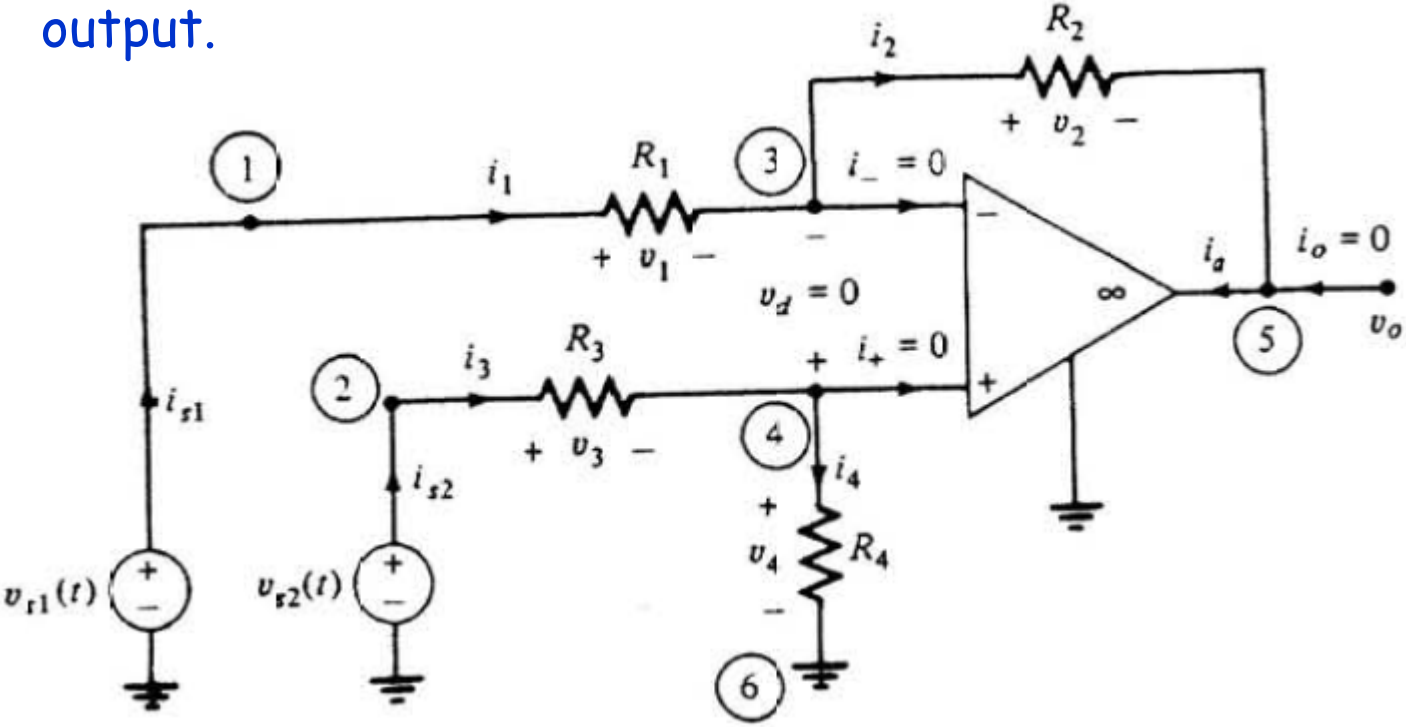
$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_{in}$$

valid for $$-\left(\frac{R_1}{R_1 + R_f}\right) E_{sat} < v_{in} < \left(\frac{R_1}{R_1 + R_f}\right) E_{sat}$$

Some Applications

Differential Amplifier

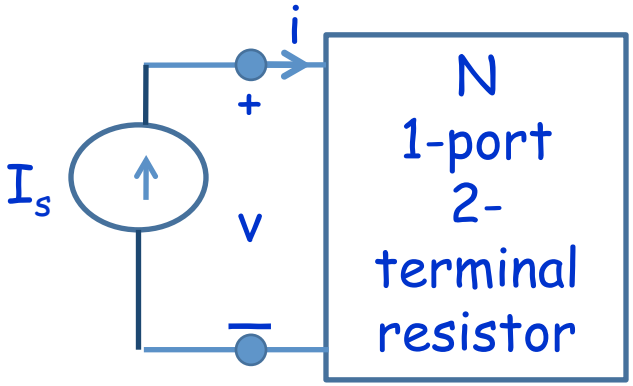
Aim: Difference between two signals at the input is amplified at the output.



$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow v_o = \frac{R_2}{R_1}\left(v_{s2}(t) - v_{s1}(t)\right)$$

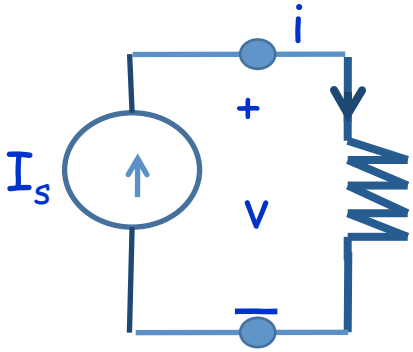
valid for $\quad -\frac{R_1}{R_2}E_{sat} < v_{s2}(t) - v_{s1}(t) < \frac{R_1}{R_2}E_{sat}$

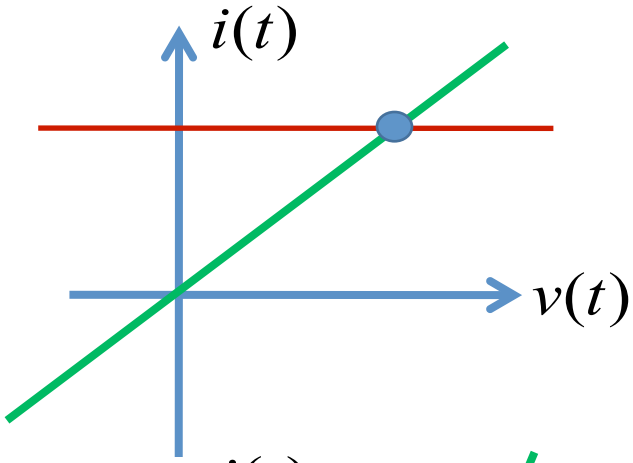
DC Operating Points



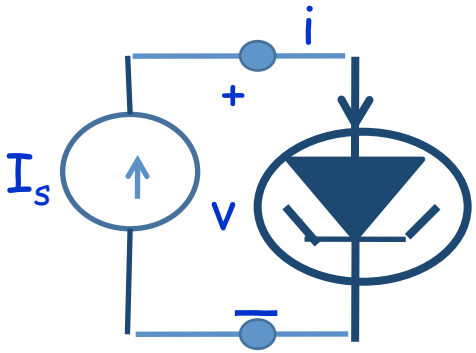
How many solutions:

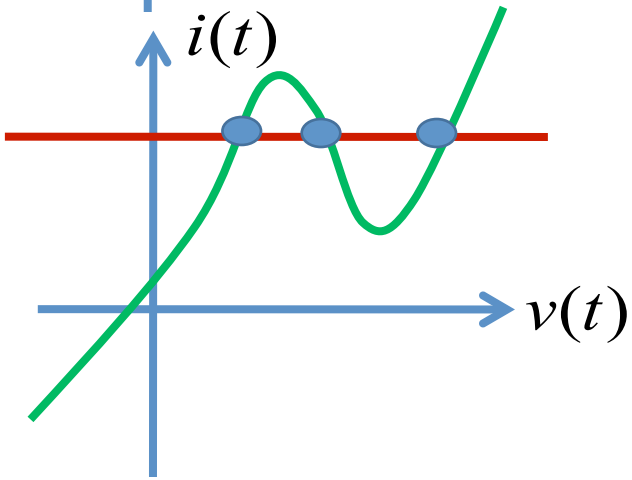
- one
- more than one
- no solution



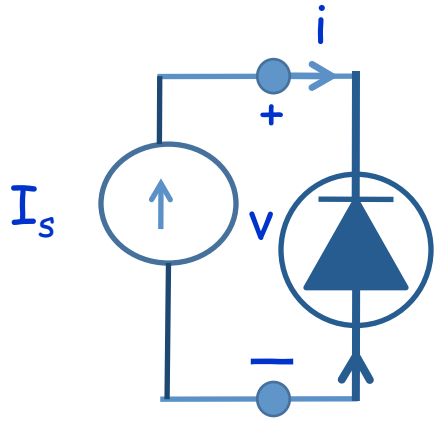


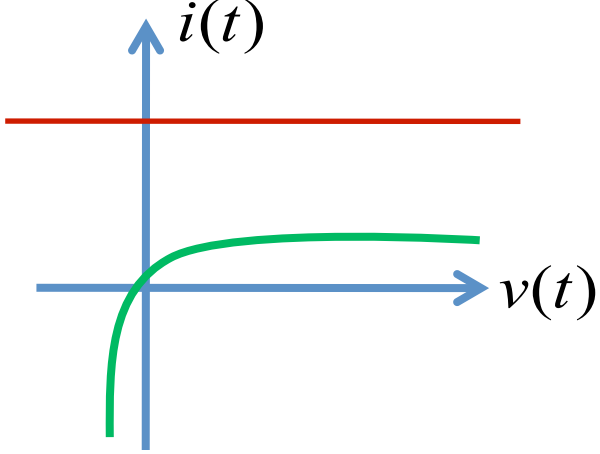
- unique solution





- more than one solution





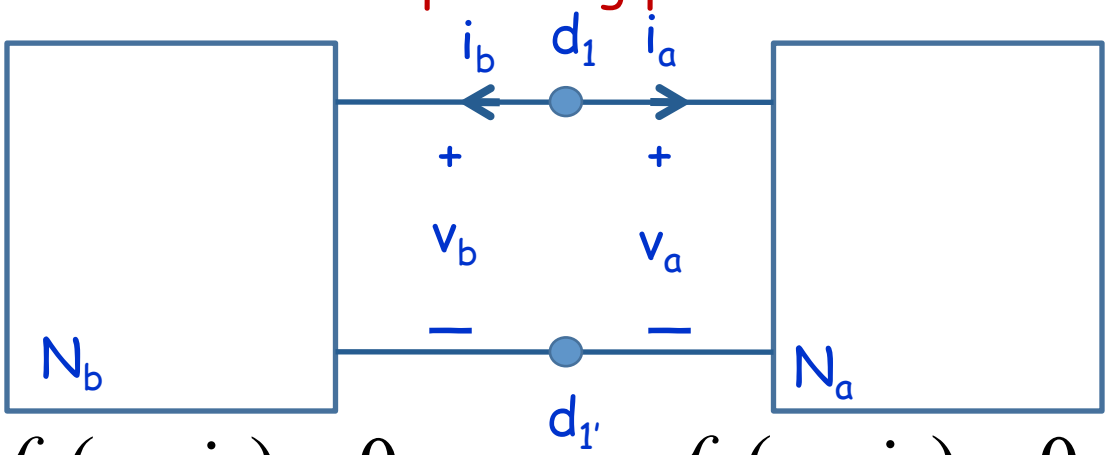
- no solution

Input: value of the independent source

Output: voltage or current value of the element under consideration

- In general, input consists of DC and AC parts. By linearity, one can consider DC and AC solutions seperately.
- DC operating point is the value of the output when only DC input is considered.

How to find DC operating points?



$$f_b(v_b, i_b) = 0 \qquad f_a(v_a, i_a) = 0$$

KCL $i_a = -i_b \quad i \hat{=} i_a = -i_b$

+

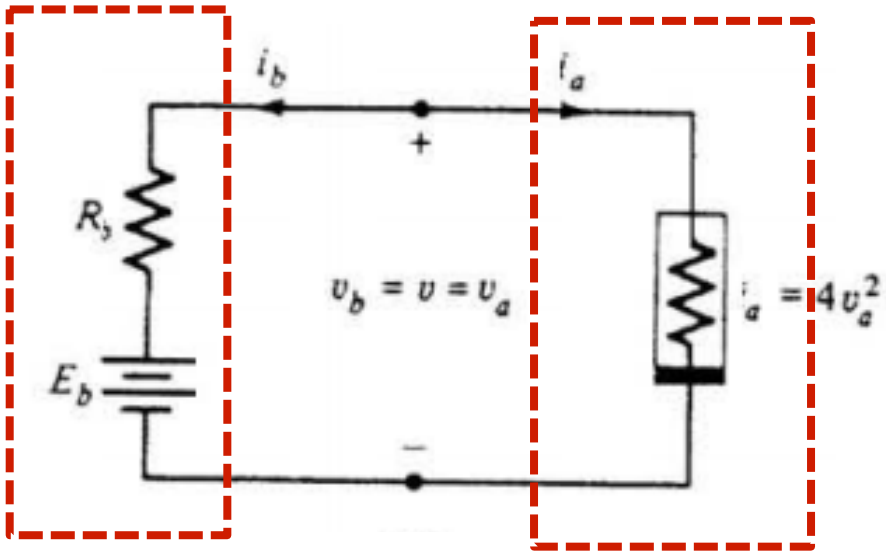
KVL $v_a = v_b \quad v \hat{=} v_a = v_b$

+

EE $f_a(v,i) = 0 \quad f_b(v,-i) = 0$

Solutions of these equations give the DC operating points.

Find the DC operating points (E_b=2,R_b=0.25)



L.O. Chua, C.A. Desoer, S.E. Kuh. "Linear and Nonlinear Circuits", Mc.Graw Hill, 1987, New York

Small Signal Analysis

Assume that the input consists of a DC part and a small AC part:

$$e(t) = E_{DC} + \tilde{e}(t)$$

Then the output is

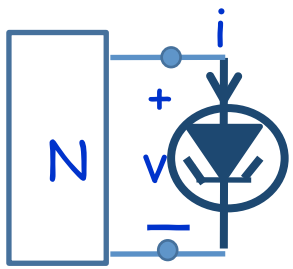
$$v(t) = V_Q + \tilde{v}(t)$$

$$i(t) = I_Q + \tilde{i}(t)$$

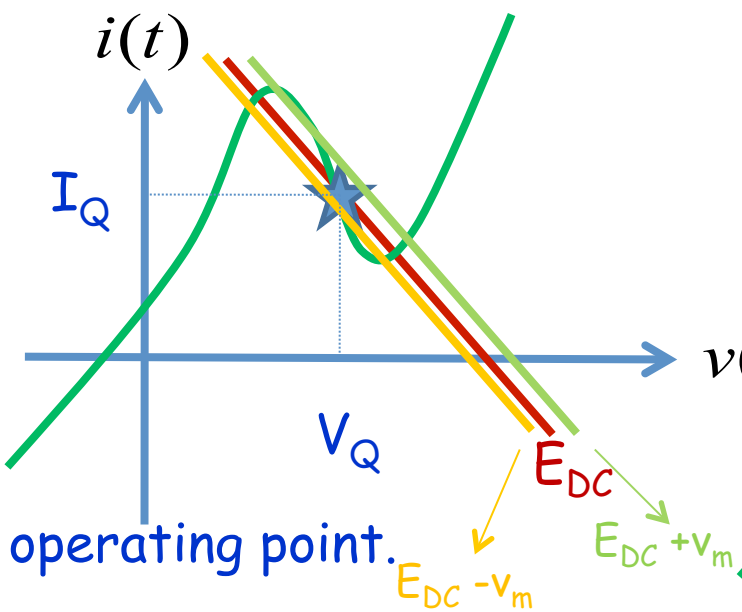
$$(V_Q, I_Q): \text{ Operating point}$$

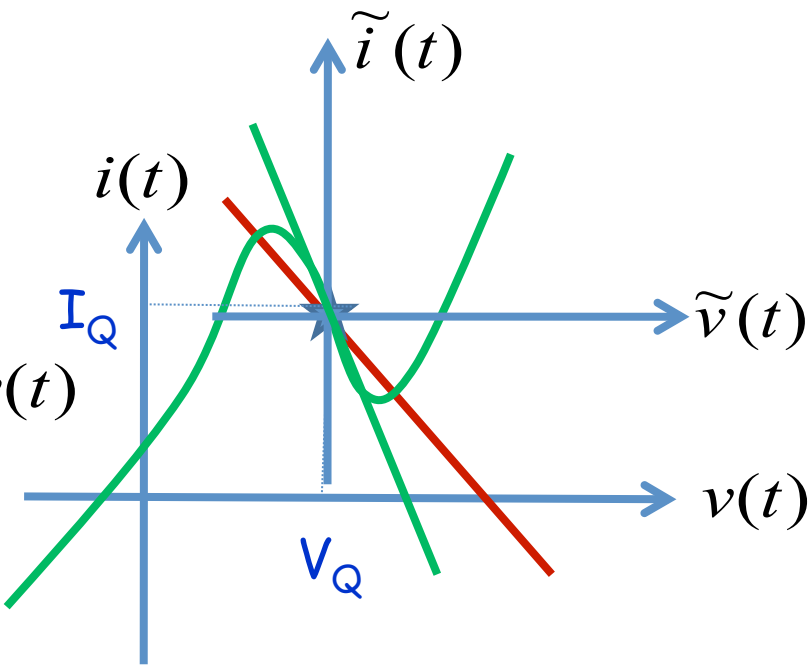
$$(\tilde{v}(t), \tilde{i}(t)): \text{Small-signal}$$

If the AC part is small enough we can apply Taylor expansion and consider the linear term only: Linearization



$$\tilde{e}(t) = v_m \sin \omega t$$





- Find the DC operating point.
- Find the linearization of the nonlinear element.

Linearization

Assumption: $v_m << E_{DC} \Rightarrow \tilde{v}(t) << V_Q$

Taylor expansion:

$$f(x) \cong f(x)\big|_{x=x_a} + f'(x)\big|_{x=x_a}(x-x_a) + \frac{1}{2} f''(x)\big|_{x=x_a}(x-x_a)^2 + \ldots$$

$$i(t) \cong \hat{i}(V_Q) + \left.\frac{d\hat{i}}{dv}\right|_{v=V_Q}(v(t)-V_Q)$$

$$v(t) = V_Q + \tilde{v}(t)$$

$$i(t) - \hat{i}(V_Q) \cong \left.\frac{d\hat{i}}{dv}\right|_{v=V_Q}(v(t)-V_Q)$$

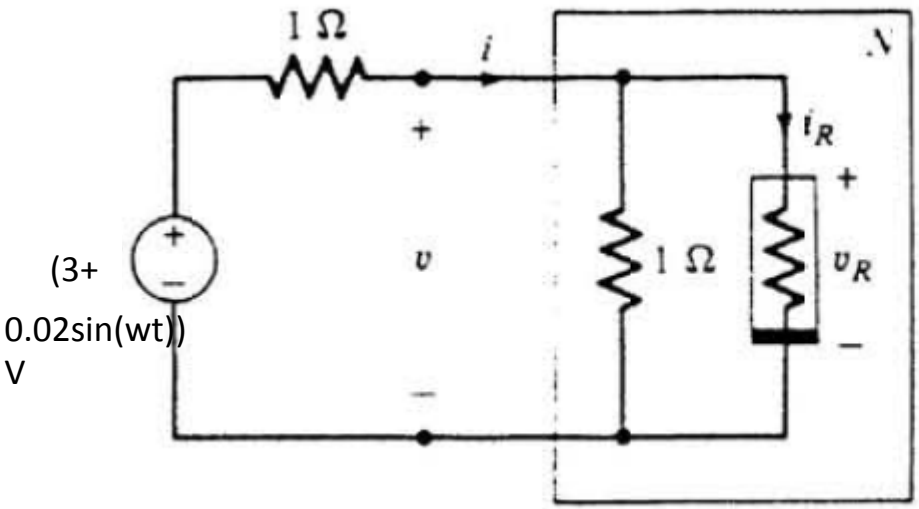
$$i(t) = I_Q + \tilde{i}(t)$$

$$i(t) - I_Q \cong \left.\frac{d\hat{i}}{dv}\right|_{v=V_Q}(v(t)-V_Q)$$

Small-signal conductivity

$$\tilde{i}(t) \cong \left.\frac{d\hat{i}}{dv}\right|_{v=V_Q}\tilde{v}(t) \quad \rightarrow \quad \tilde{i}(t) \cong G\tilde{v}(t)$$

Find the DC operating points for v_R and using the linearization of the nonlinear resistor find an approximate value for v_R.



$$i_R = f(v_R) = v_R^2 - 3\,v_R + 1$$

Small-Signal Analysis for 2-port nonlinear resistors

- Find the operating points.
- Find the linearization of the 2-port.

$$\left.\begin{aligned} v_1 &= \hat{v}_1(i_1, i_2) \\ v_2 &= \hat{v}_2(i_1, i_2) \end{aligned}\right\} \text{Current-conrolled element equations} \qquad \left.\begin{aligned} V_{1_Q} &= \hat{v}_1(I_{1_Q}, I_{2_Q}) \\ V_{2_Q} &= \hat{v}_2(I_{1_Q}, I_{2_Q}) \end{aligned}\right\} \text{Operating point}$$

$$v_1 \cong \hat{v}_1(I_{1_Q}, I_{2_Q}) + \left.\frac{\partial \tilde{v}_1}{\partial i_1}\right|_{(I_{1_Q}, I_{2_Q})} (i_1 - I_{1_Q}) + \left.\frac{\partial \tilde{v}_1}{\partial i_2}\right|_{(I_{1_Q}, I_{2_Q})} (i_2 - I_{2_Q}) + \ldots$$

$$v_2 \cong \hat{v}_2(I_{1_Q}, I_{2_Q}) + \left.\frac{\partial \tilde{v}_2}{\partial i_1}\right|_{(I_{1_Q}, I_{2_Q})} (i_1 - I_{1_Q}) + \left.\frac{\partial \tilde{v}_2}{\partial i_2}\right|_{(I_{1_Q}, I_{2_Q})} (i_2 - I_{2_Q}) + \ldots$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \cong \begin{bmatrix} V_{1_Q} \\ V_{2_Q} \end{bmatrix} + \underbrace{\left.\begin{bmatrix} \dfrac{\partial \tilde{v}_1}{\partial i_1} & \dfrac{\partial \tilde{v}_1}{\partial i_2} \\ \dfrac{\partial \tilde{v}_2}{\partial i_1} & \dfrac{\partial \tilde{v}_2}{\partial i_2} \end{bmatrix}\right|_{(I_{1_Q}, I_{2_Q})}}_{\text{Jakobian matrix}} \begin{bmatrix} i_1 - I_{1_Q} \\ i_2 - I_{2_Q} \end{bmatrix}$$

$$\begin{bmatrix} v_1 - V_{1_Q} \\ v_2 - V_{2_Q} \end{bmatrix} \cong \left.\begin{bmatrix} \dfrac{\partial \tilde{v}_1}{\partial i_1} & \dfrac{\partial \tilde{v}_1}{\partial i_2} \\ \dfrac{\partial \tilde{v}_2}{\partial i_1} & \dfrac{\partial \tilde{v}_2}{\partial i_2} \end{bmatrix}\right|_{(I_{1_Q}, I_{2_Q})} \begin{bmatrix} i_1 - I_{1_Q} \\ i_2 - I_{2_Q} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{v}_1 \\ \tilde{v}_2 \end{bmatrix} \cong \left.\begin{bmatrix} \dfrac{\partial \tilde{v}_1}{\partial i_1} & \dfrac{\partial \tilde{v}_1}{\partial i_2} \\ \dfrac{\partial \tilde{v}_2}{\partial i_1} & \dfrac{\partial \tilde{v}_2}{\partial i_2} \end{bmatrix}\right|_{(I_{1_Q}, I_{2_Q})} \begin{bmatrix} \tilde{i}_1 \\ \tilde{i}_2 \end{bmatrix}$$