

EHB 453, Introduction to Mobile Communications

Lecture 5: OFDMA and MIMO

Prof. Mustafa Ergen





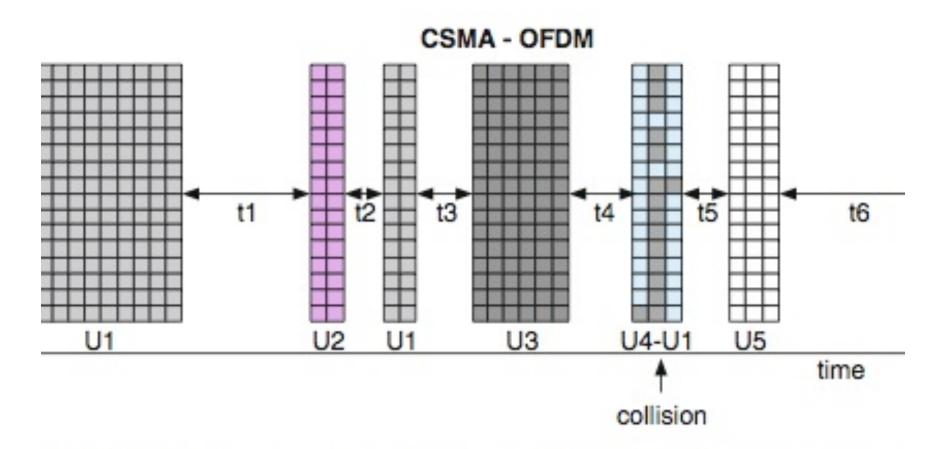


Outline

- What is OFDMA?
- Resource Allocation
- Single Frequency Network
- Flash OFDM



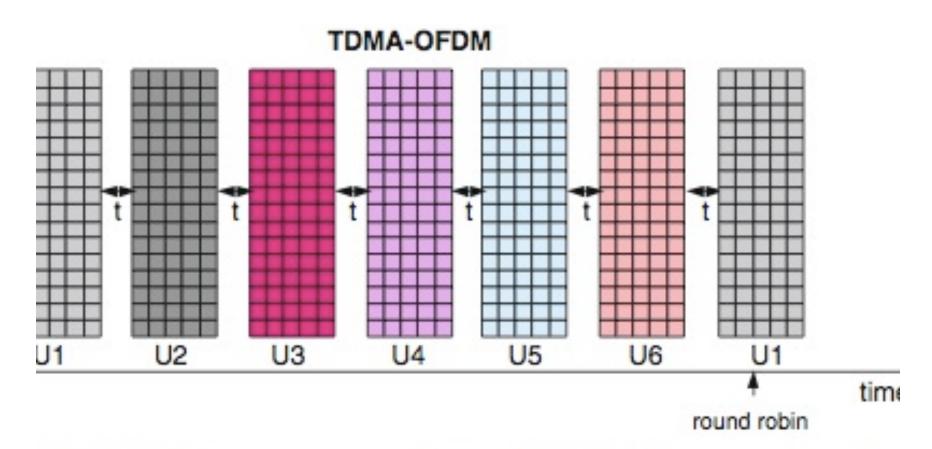
CSMA-OFDM



IA-OFDM: There are 6 users (U), and CSMA scheme has random es (ti) and random packet sizes



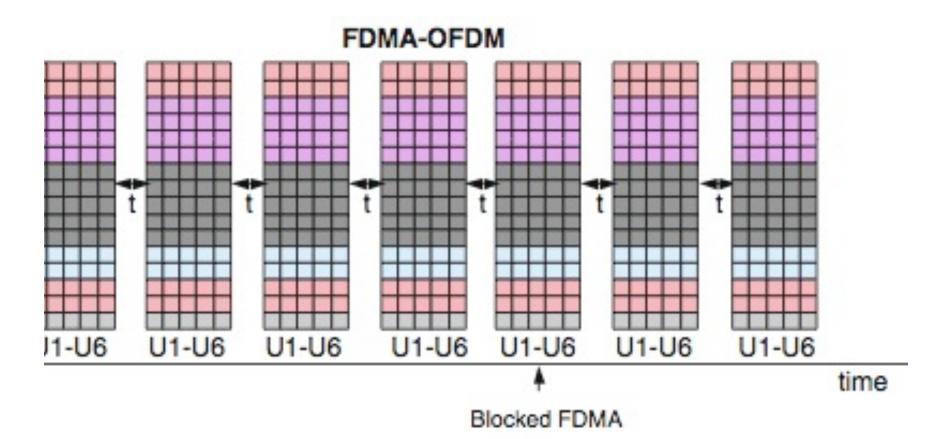
TDMA-OFDM



A-OFDM: There are 6 users (U), and TDMA scheme has fixed tin) and fixed packet sizes



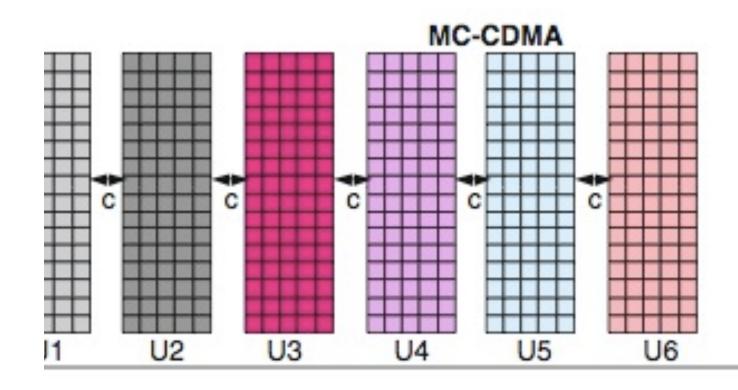
FDMA-OFDM



A-OFDM: There are 6 users (U), and Block-FDMA scheme has fixe s (t) and fixed subcarrier allocation



MC-OFDM

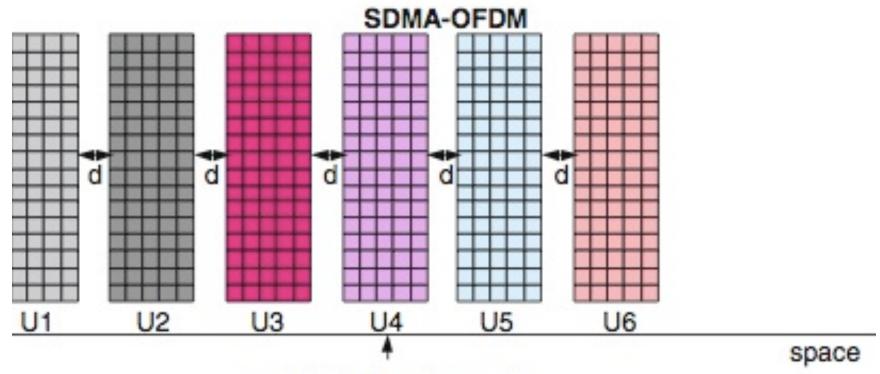


code

MA: There are six users (U), and time-spread MC-CDMA sche
) for orthogonality



SDMA-OFDM

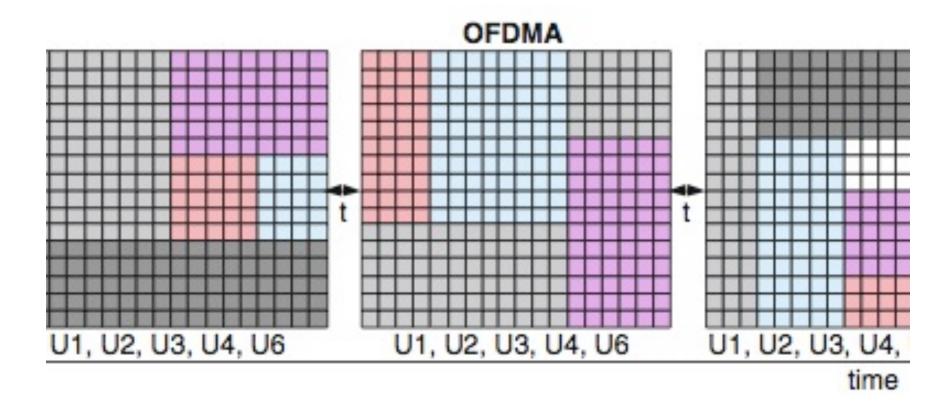


spatial signature for user 4

A-OFDM: There are 6 users (U), and SDMA scheme has physical di r orthogonality



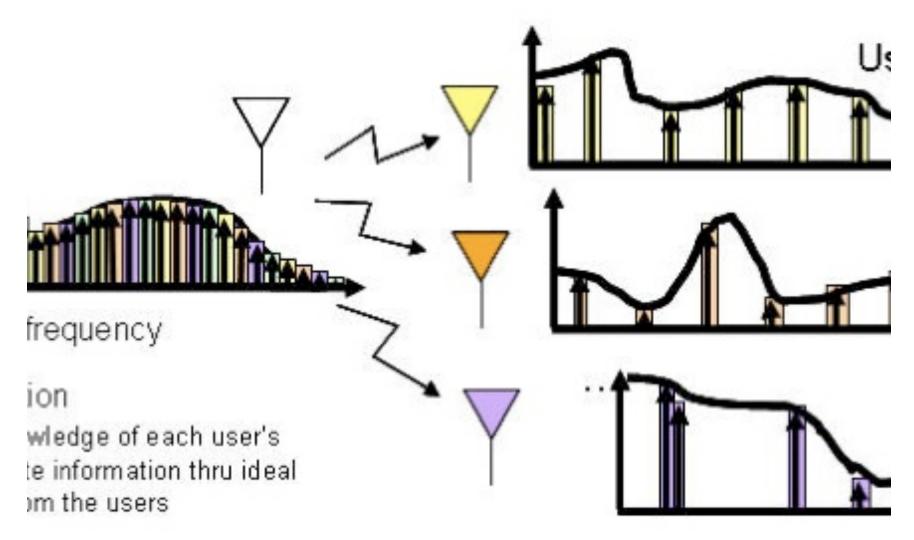
OFDMA



A: There are 6 users (U), and OFDMA scheme has fixed distance lot and subcarrier allocation



OFDMA



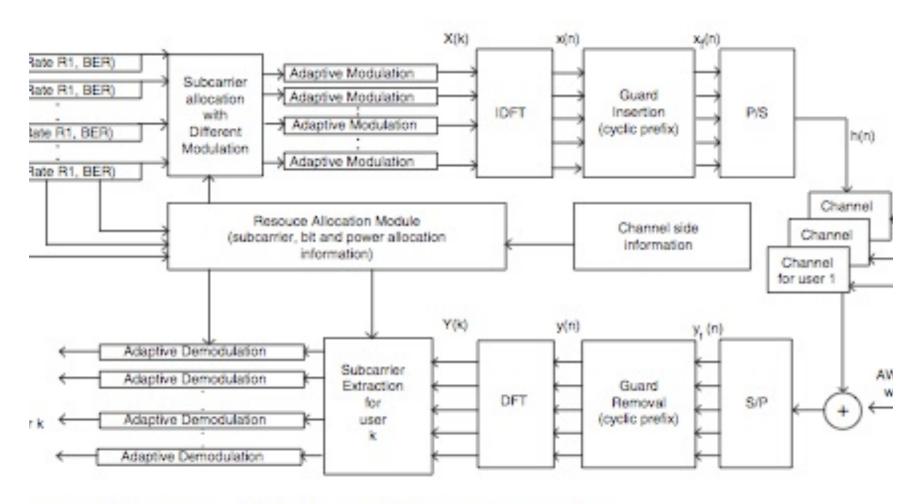


Drawbacks

- Power inefficient compared to OFDM
- OFDMA electronics, including FFTs and Forward Error Correction (FEC) is complex.
- Frequency-selective fading and diversity gain benefits could be at least partly lost if very few sub-carriers are assigned to each user, and if the same carrier is used in every OFDM symbol.
- Dealing with co-channel interference from nearby cells is more complex in OFDM than in CDMA.

Source: http://www.wirelessnetdesignline.com/howto/broadband/198000601;jsessionid=CZFRNRNKHXQGBQE1GHPCKHWATMY32JVN?pgno=2

Scalable OFDM



ogonal frequency division multiple access system



Scalable OFDMA

between 2–6 GHz, considers mobility at a maximum speed of 125 km/h (35 m/s). Doppler shift for operation in 2.5 GHz carrier frequency (f_c) is

$$f_{\rm m} = \frac{v f_{\rm c}}{c} = \frac{35 \,{\rm m/s} \, 2.510^9 \,{\rm Hz}}{3x 10^8 \,{\rm m/s}} = 291 \,{\rm Hz}$$
 (5.1)

and it is 408 Hz when $f_c = 3.5$ GHz and 700 Hz when $f_c = 6$ GHz. When maximum Doppler shift is considered, coherence time of the channel is $T_C = \sqrt{\frac{9}{16.\pi f_m^2}} = 1.1$ ms. This would require an update rate of ≈ 1 KHz for channel estimation and equalization. The delay spread (T_S) for mobile environment is $20 \mu s$ specified by The International Telecommunications Union (ITU-R).² Associated coherence bandwidth (B_C) is

$$B_{\rm C} \approx \frac{1}{5.T_{\rm S}} = \frac{1}{5.20\,\mu\rm s} = 10\,\rm KHz,$$
 (5.2)

where sought frequency correlation is 50%. This means that over a 10-KHz subcarrier width, the fading is considered flat.

SOFDMA subcarrier spacing is independent of channel bandwidth. Scalability ensures that system performance is consistent across different RF channel sizes

Subcarrier Allocation: Fixed QoS Constraints

 With the channel information, the objective of resource allocation problem can be defined as maximizing the throughput subject to a given total power constraint regarding the user's QoS requirements.

C is bits per symbol

Indicator: 1 if that subcarrier is allocated

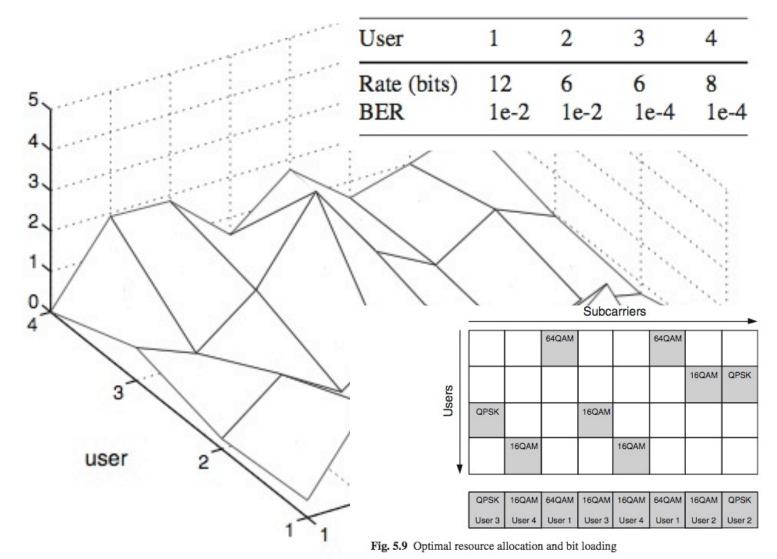
$$\max_{c_{k,n}, \gamma_{k,n}} R_k = \sum_{n=1}^N c_{k,n} \gamma_{k,n}$$
 for all k

subject to
$$P_T = \sum_{k=1}^K \sum_{n=1}^N \frac{f_k(c_{k,n}, BER_k)}{\alpha_{k,n}^2} \gamma_{k,n} \le P_{\max}$$
,

Power allocated to the nth user



Subcarrier Allocation: Fixed QoS

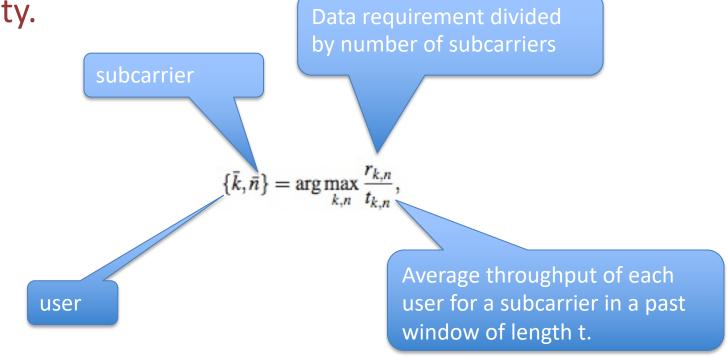




Subcarrier Allocation: Variable QoS

 Another way to approach resource allocation is in terms of capacity. Suppose there is no fixed requirements per symbol and the aim is to maximize capacity.

Data requirement divided

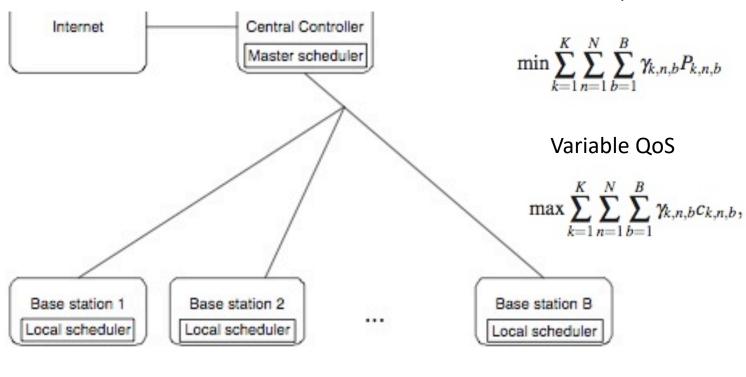


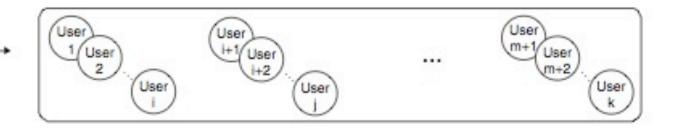


Single Frequency Network

Fixed QoS

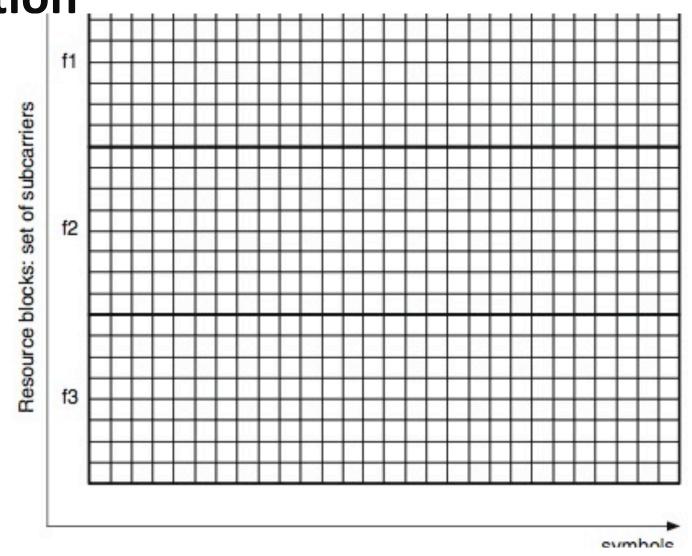
Removes the requirement for cell planning and called single frequency network





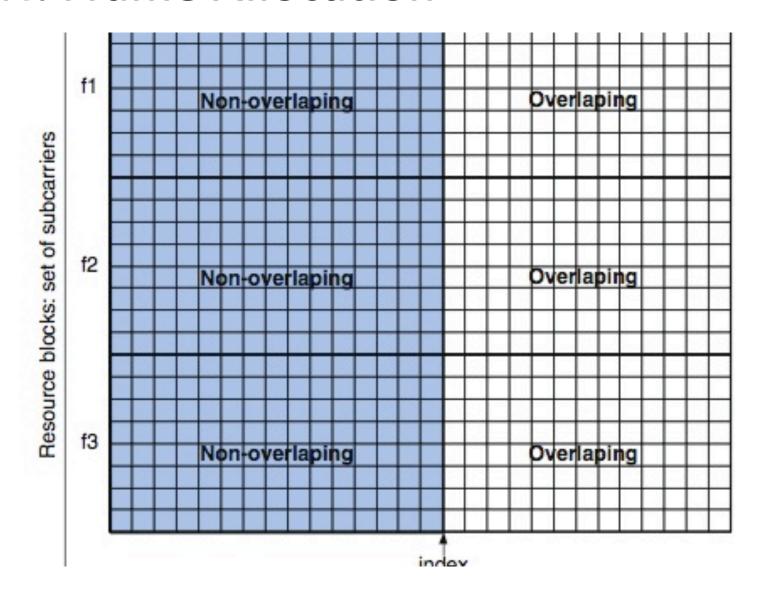


Single Frequency Network: Heuristic Solution



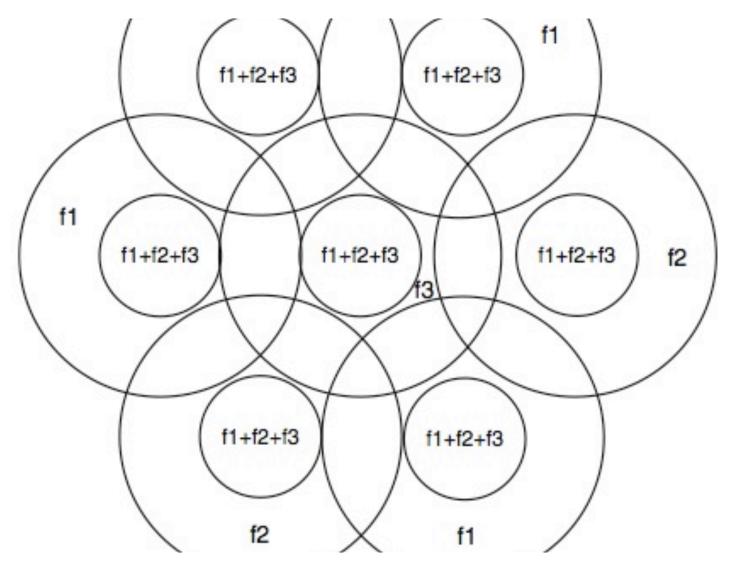


SFN: Frame Allocation



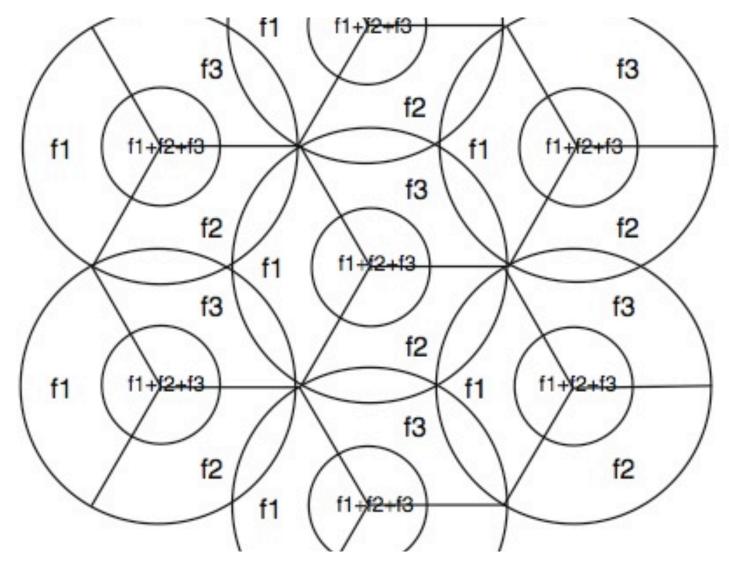


SFN: Omni Directional





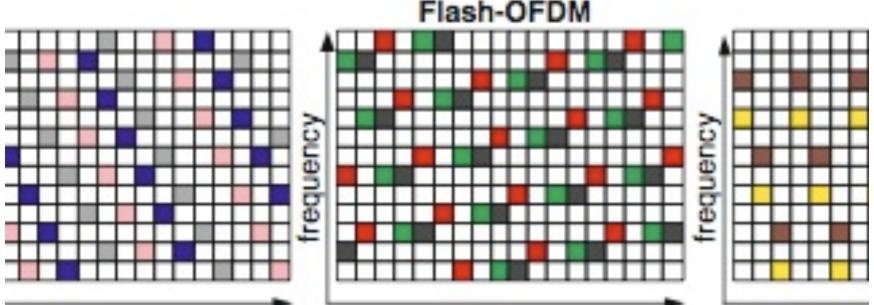
SFN: Sectoral





Flash OFDM

IEEE 802.20: Besides OFDMA, CDMA is used for interference averaging.



Flash-OFDM utilizes an hopping pattern in time and frequency in order to exploit interference diversity as in CDMA as well as frequency diversity. Interference diversity averages out the interference coming from more user, since interference coming solely from one user causes severe interference to each other. A hopping pattern is assigned to a user, which alternates the subcarriers at each symbol.



Latin Squares

Periodic hopping patterns consider frequency diversity and interference diversity. Frequency diversity is exploited by allocating subcarriers as spread as possible and alternate them every symbol time. Interference diversity also considers to allocate hop patterns that are as "apart" as possible from adjacent base stations. If there are N subcarriers then N can be selected as the hopping period and there can be N channels.

Latin Square

No. of Latin squares

N	No. of Latin squares
1	1
2	2
3	12
4	576
5	161280
6	812851200
7	61479419904000
8	108776032459082956800
9	5524751496156892842531225600
10	9982437658213039871725064756920320000
11	776966836171770144107444346734230682311065600000



Subcarrier Sharing: Embedded Modulation

This scheme allows a subcarrier to be used by more than one user. Each carrier This scheme is known as carries 6 bits embedded modulation for 64QAM and exploits the fact that a modulation subcarrier that is high and each user quality to a user maybe uses only 2 out of 6. high quality to multiple users and therefore that subcarrier is utilized to carry bits of multiple users.



References

Mobile Broadband by M. Ergen



Lecture 7

MIMO

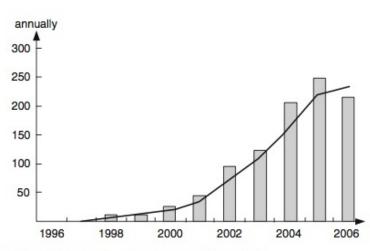


Fig. 6.1 MIMO patent applications per year. (Source: Marvedis)



Outline

- Introduction...why MIMO??
- Shannon capacity of MIMO systems
- The "pipe" interpretation Telatar, AT&T 1995
- To exploit the MIMO channel
 - -BLAST

Foschini, Bell Labs 1996

Space Time Coding

Tarokh, Seshadri & Calderbank 1998

- Beamforming
- Comparisons & hardware issues
- Space time coding in 3G & EDGE^{Release} '99



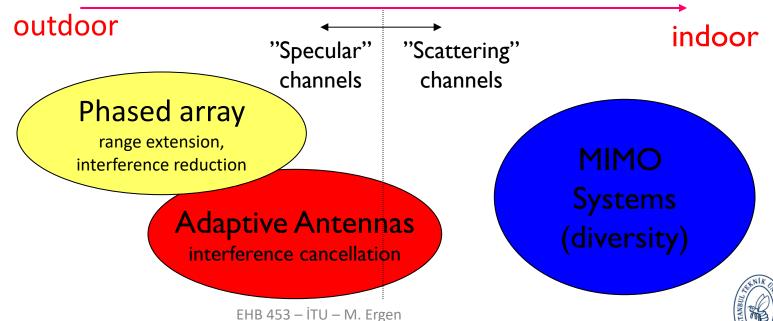
Aspirations (Mathematical) of a System Designer

High data rate

Achieve "Channel Capacity (C)" Quality Minimize Probability of Error (Pe) Minimize complexity/cost of implementation of proposed Real-life Issues System Minimize transmission power required (translates into SNR) Minimize Bandwidth (frequency spectrum) Used

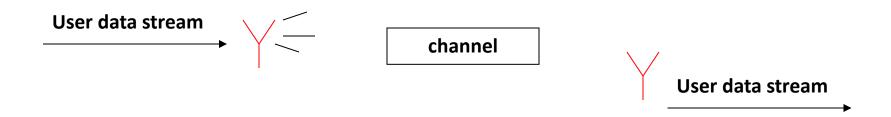
Why MIMO?

- Frequency and time processing are at limits
- Space processing is interesting because it does not increase bandwidth



Antenna Configurations

Single-Input-Single-Output (SISO) antenna system

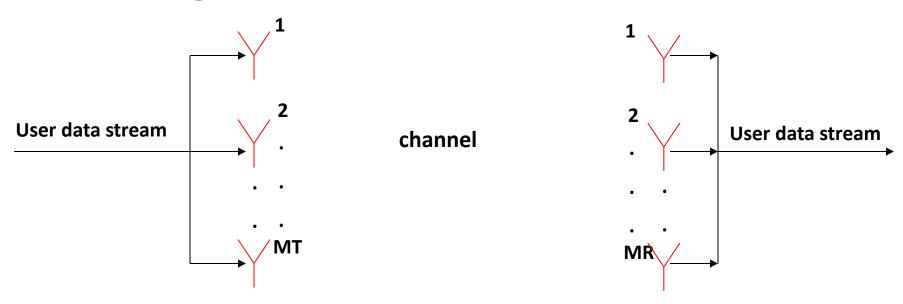


- Theoretically, the 1Gbps barrier can be achieved using this configuration if you are allowed to use much power and as much BW as you so please!
- Extensive research has been done on SISO under power and BW constraints. A combination a smart modulation, coding and multiplexing techniques have yielded good results but far from the 1Gbps barrier



MIMO Antenna Configuration

 Use multiple transmit and multiple receive antennas for a single user



Now this system promises enormous data rates!



Shannon's Capacity (C)

- Given a unit of BW (Hz), the max error-free transmission rate is
- C = log2(1+SNR) bits/s/Hz
- Define
- R: data rate (bits/symbol)
- RS: symbol rate (symbols/second)
- w: allotted BW (Hz)
- Spectral Efficiency is defined as the number of bits transmitted per second per Hz
- R x RS bits/s/Hz
- W
- As a result of filtering/signal reconstruction requirements, RS ≤ W.
 Hence Spectral Efficiency = R if RS = W
- If I transmit data at a rate of R ≤ C, I can achieve an arbitrarily low Pe



Spectral Efficiency

Scheme	b/s/Hz
BPSK	1
QPSK	2
16-QAM	4
64-QAM	6

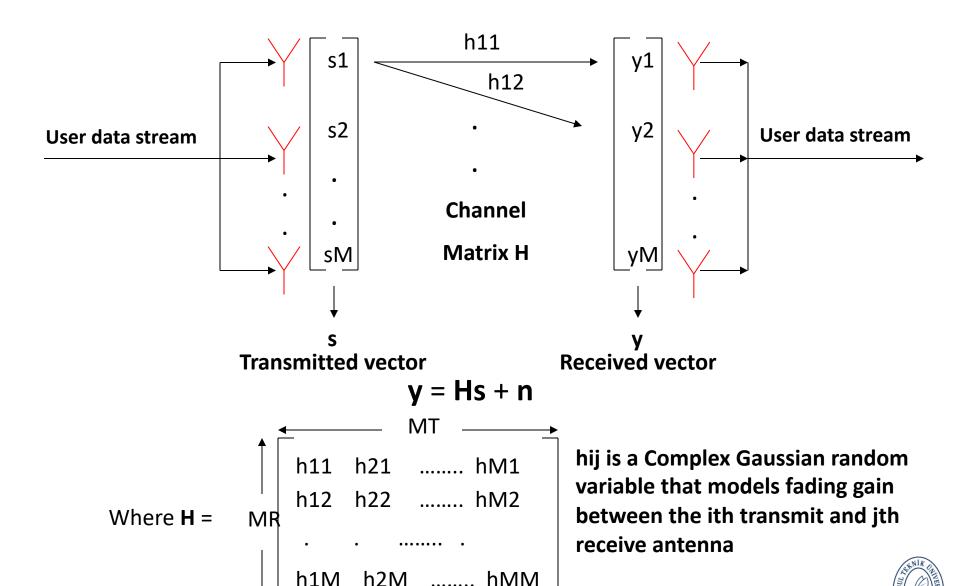
 Spectral efficiencies of some widely used modulation schemes

The Whole point: Given an acceptable Pe, realistic power and BW limits,
 MIMO Systems using smart modulation schemes provide much higher spectral efficiencies than traditional SISO

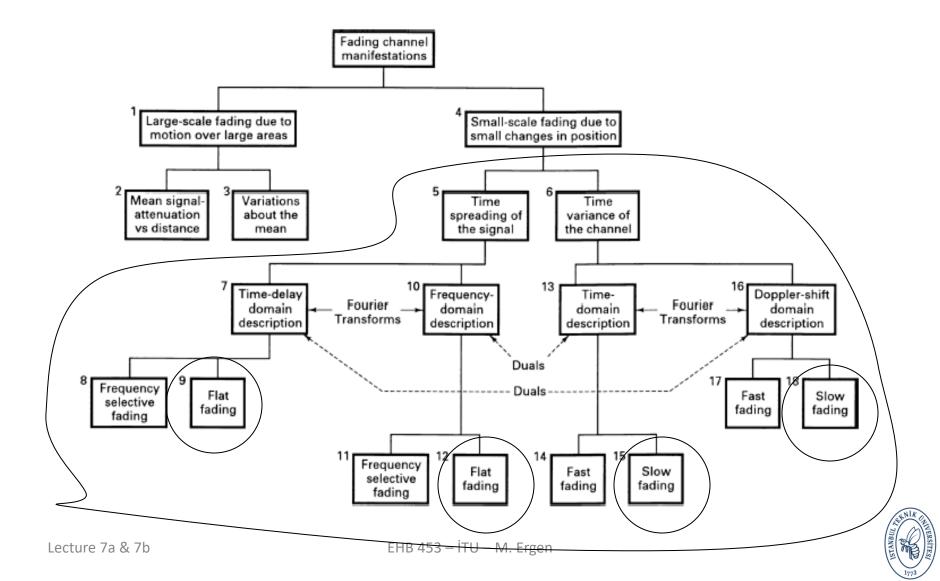


MIMO System Model

Lecture 7a & 7b

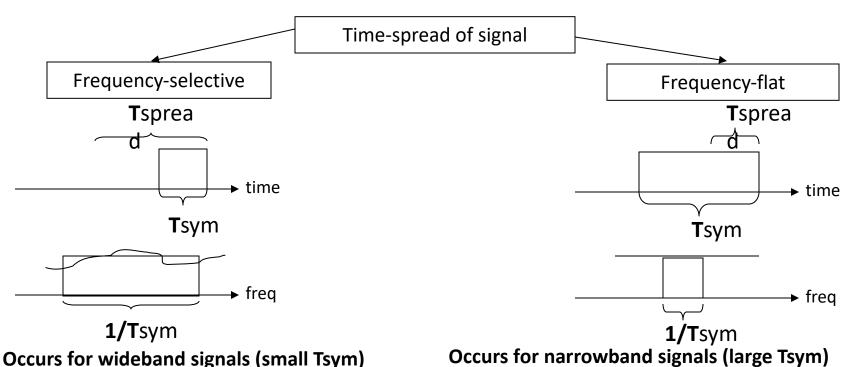


Types of Channels



Fading Channels

- Fading refers to changes in signal amplitude and phase caused by the channel as it makes its way to the receiver
- Define T_{spread} to be the time at which the last reflection arrives and T_{sym} to be the symbol time period



TOUGH TO DEAL IT!

EASIER! Fading gain is complex Gaussian

EHB 453 - İTU - M. Ergen

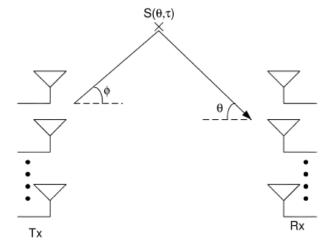




Channel Matrix H

- In addition, assume slow fading
- MIMO Channel Response

$$\mathbf{H}(\tau,t) = \begin{bmatrix} h_{1,1}(\tau,t) & h_{1,2}(\tau,t) & \cdots & h_{1,M_T}(\tau,t) \\ h_{2,1}(\tau,t) & h_{2,2}(\tau,t) & \cdots & h_{2,M_T}(\tau,t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R,1}(\tau,t) & h_{M_R,2}(\tau,t) & \cdots & h_{M_R,M_T}(\tau,t) \end{bmatrix}.$$
 Time-spread



Taking into account slow fading, the MIMO channel impulse response is constructed as,

$$\mathbf{H}(\tau) = \int_{-\pi}^{\pi} \int_{0}^{\tau_{\text{max}}} S(\theta, \tau') \mathbf{a}(\theta) \mathbf{b}^{T}(\phi) g(\tau - \tau') d\tau' d\theta$$

Because of flat fading, it becomes,

$$\mathbf{H}(\tau) \!\!=\!\! \left(\int_{-\pi}^{\pi} \!\! \int_{0}^{\tau_{\mathrm{max}}} \!\! S(\theta, \tau') \mathbf{a}(\theta) \mathbf{b}^{T}\!(\phi) d\tau' d\theta \right) \!\! g(\tau) \!\!=\!\! \mathbf{H} \, g(\tau)$$

a and b are transmit and receive array factor vectors respectively. S is the complex gain that is dependant on direction and delay. g(t) is the transmit and receive pulse shaping impulse response

- With suitable choices of array geometry and antenna element patterns,
 H(↑) = H which is an MR x MT matrix with complex Gaussian i. i. d random variables
- Accurate for NLOS rich-scattering environments, with sufficient antenna spacing at transmitter and receiver with all elements identically polarized



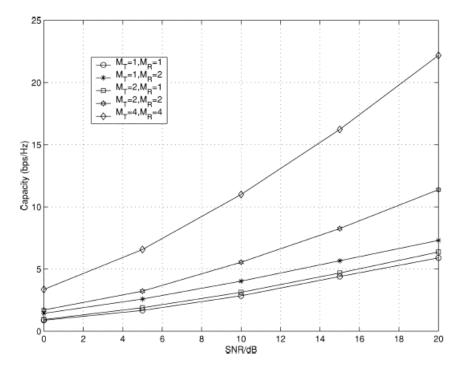
Capacity of MIMO Channels

- y = Hs + n
- Let the transmitted vector s be a random vector to be very general and n is normalized noise. Let the total transmitted power available per symbol period be P. Then,
- C = log 2 (IM + HQHH) b/s/Hz
- where Q = E{ssH} and trace(Q) < P according to our power constraint
- Consider specific case when we have users transmitting at equal power over the channel and the users are uncorrelated (no feedback available). Then,
- CEP = $\log 2 [IM + (P/MT)HHH] b/s/Hz$
- Telatar showed that this is the optimal choice for blind transmission
- Foschini and Telatar both demonstrated that as MT and MR grow,
- CEP = min (MT,MR) log 2 (P/MT) + constant b/s/Hz
- Note: When feedback is available, the Waterfilling solution is yields maximum capacity but converges to equal power capacity at high SNRs



Capacity (contd)

The capacity expression presented was over one realization of the channel. Capacity is a random variable and has to be averaged over infinite realizations to obtain the true ergodic capacity. Outage capacity is another metric that is used to capture this



So MIMO promises enormous rates theoretically! Can we exploit this practically?



MIMO Design Criterion

MIMO Systems can provide two types of gain

Spatial Multiplexing Gain

- Maximize transmission rate (optimistic approach)
- Use rich scattering/fading to your advantage



- Minimize Pe (conservative approach)
- Go for Reliability / QoS etc
- Combat fading
- If only I could have both! As expected, there is a tradeoff
- System designs are based on trying to achieve either goal or a little of both



Diversity

- Each pair of transmit-receive antennas provides a signal path from transmitter to receiver. By sending the SAME information through different paths, multiple independently-faded replicas of the data symbol can be obtained at the receiver end. Hence, more reliable reception is achieved
- A diversity gain d implies that in the high SNR region, my Pe decays at a rate of 1/SNRd as opposed to 1/SNR for a SISO system
- The maximal diversity gain dmax is the total number of independent signal paths that exist between the transmitter and receiver
- For an (MR,MT) system, the total number of signal paths is MRMT
- 1 ≤ d ≤ dmax= MRMT
- The higher my diversity gain, the lower my Pe



Spatial Multiplexing

 $y = Hs + n \rightarrow y' = Ds' + n'$ (through SVD on H) where D is a diagonal matrix that contains the eigenvalues of HH^H

- Viewing the MIMO received vector in a different but equivalent way, $C_{EP} = \log_2 \left[I_M + (P/M_T)DD^H \right] = \log_2 \left[1 + (P/M_T)A_i \right] b/s/Hz$
- Equivalent form tells us that an (M_T, M_R) MIMO channel opens up $m = min (M_T, M_R)$ independent SISO channels between the transmitter and the receiver
- So, intuitively, I can send a maximum of m different information symbols over the channel at any given time

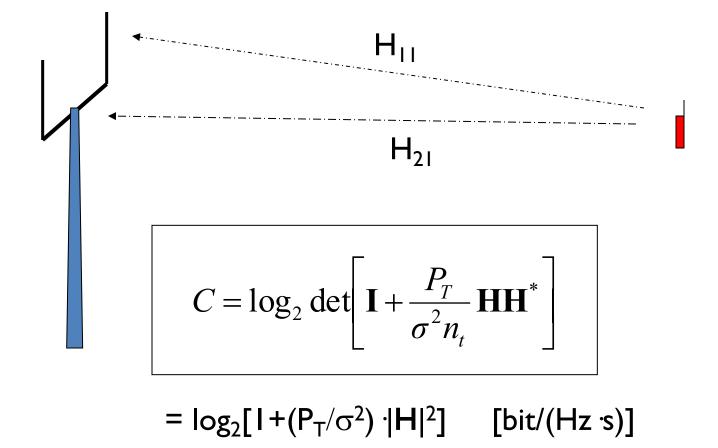


Initial Assumptions

- Flat fading channel (Bcoh>> 1/ Tsymb)
- Slowly fading channel (Tcoh>> Tsymb)
- nr receive and nt transmit antennas
- Noise limited system (no CCI)
- Receiver estimates the channel perfectly
- We consider space diversity only



Receive Diversity

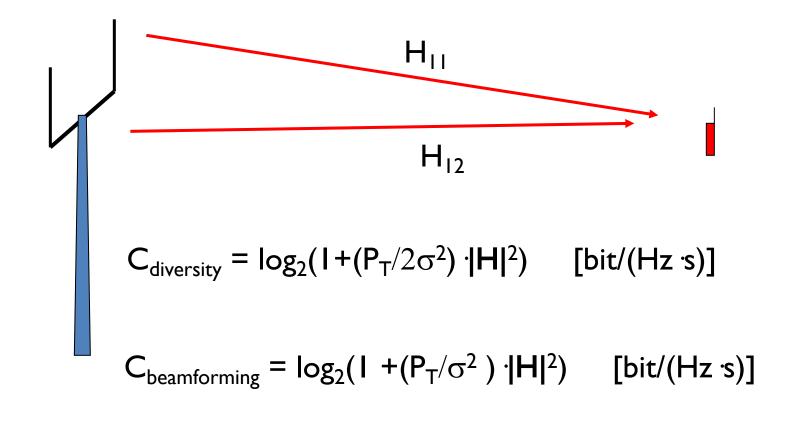


Capacity increases logarithmically with number of receive antennas...

 $H = [H_{11} H_{21}]$



Transmit Diversity

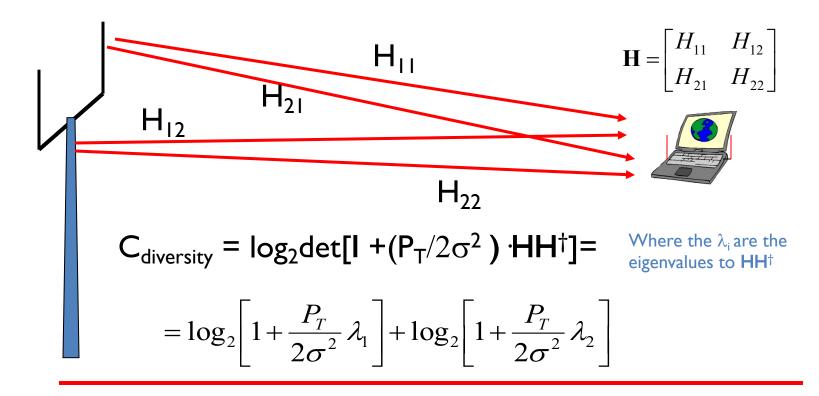




- 3 dB SNR increase if transmitter knows H
- Capacity increases logarithmically with n_t

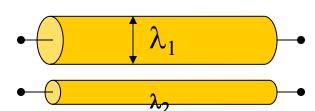


MIMO



Interpretation:

Transmitter



Receiver

m=min(n_r, n_t) parallel channels, equal power allocated to each "pipe"



MIMO capacity in general

H unknown at TX

$$C = \log_2 \det \left[I + \frac{P_T}{\sigma^2 n_t} H H^* \right] =$$

$$= \sum_{i=1}^m \log_2 \left[1 + \frac{P_T}{\sigma^2 n_t} \lambda_i \right]$$

$$m = \min(n_r, n_t)$$

H known at TX

$$C = \sum_{i=1}^{m} \log_2 \left[1 + \frac{p_i \lambda_i}{\sigma^2} \right]$$

Where the power distribution over "pipes" are given by a water filling solution

$$P_{T} = \sum_{i=1}^{m} p_{i} = \sum_{i=1}^{m} \left(v - \frac{1}{\lambda_{i}} \right)^{+}$$

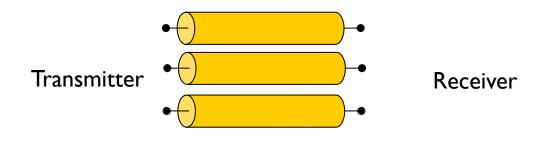


The Channel Eigenvalues

Orthogonal channels HH†=I,
$$\lambda_1 = \lambda_2 = ... = \lambda_m = 1$$

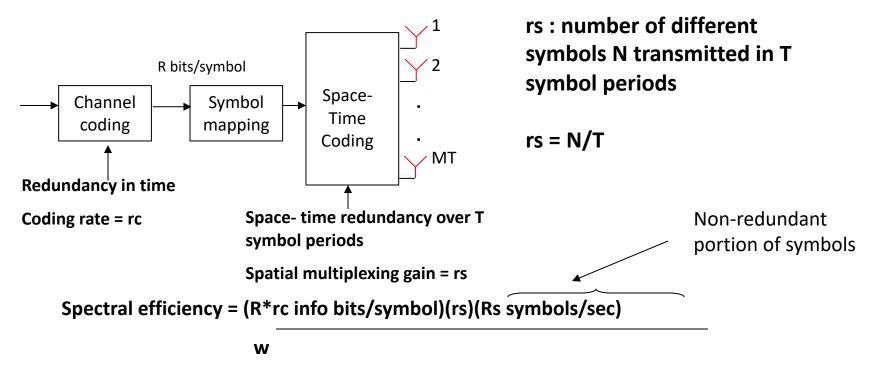
$$C_{\text{diversity}} = \sum_{i=1}^{m} \log_2 \left[1 + \frac{P_T}{\sigma^2 n_t} \lambda_i \right] = \min(n_t, n_r) \cdot \log_2(1 + P_T / \sigma^2 n_t)$$

- \rightarrow Capacity increases linearly with min(n_r , n_t)
 - An equal amount of power P_T/n_t is allocated to each "pipe"





Practical System



= Rrcrs bits/s/Hz assuming Rs = w

rs is the parameter that we are concerned about: $0 \le rs \le MT$

** If rs = MT, we are in spatial multiplexing mode (max transmission rate)

NI TORNALS

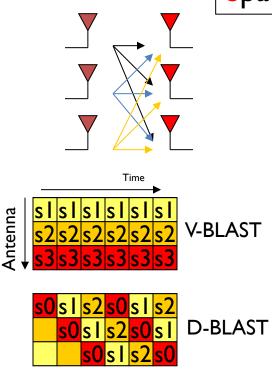
Lecture 7a & 7b

EHB 453 – İTU – M. Ergen

**If rs ≤ 1, we are in diversity mode

BLAST

Bell Labs Layered
Space Time Architecture

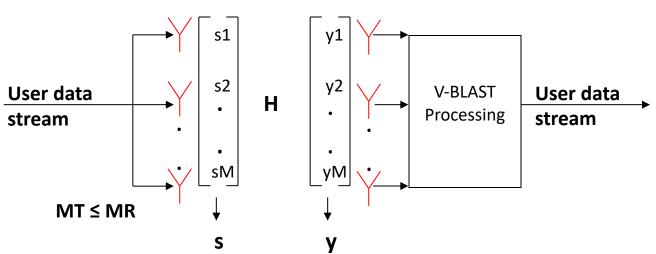


- $n_r \ge n_t$ required
- Symbol by symbol detection.
 Using nulling and symbol cancellation
- V-BLAST implemented -98 by Bell Labs (40 bps/Hz)
- If one "pipe" is bad in BLAST we get errors ...



V-BLAST (Vertical) – Spatial Multiplexing

This is the only architecture that goes all out for maximum rate. Hope the channel helps me out by 'splitting' my info streams!



- Split data into MT streams → maps to symbols → send
- Assume receiver knows H
- Uses old technique of ordered successive cancellation to recover signals $k_{i+1} = \underset{j \notin \{k_1 \cdots k_i\}}{\operatorname{argmin}} \|(\mathbf{G}_{i+1})_j\|^2$
- Sensitive to estimation errors in H
- rs = MT because in one symbol period, you are sending MT different symbols

$$i \leftarrow 1$$

 $\mathbf{G}_1 = \mathbf{H}^+$
 $k_1 = \underset{j}{\operatorname{argmin}} \|(\mathbf{G}_1)_j\|^2$

recursion:

$$\mathbf{w}_{k_i} = (\mathbf{G}_i)_{k_i}$$

$$y_{k_i} = \mathbf{w}_{k_i}^{\mathsf{T}} \mathbf{r}_i$$

$$\hat{a}_{k_i} = \mathcal{Q}(y_{k_i})$$

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \hat{a}_{k_i}(\mathbf{H})_{k_i}$$

$$\mathbf{G}_{i+1} = \mathbf{H}_{\overline{k_i}}^{\mathsf{+}}$$

$$\underset{\mathbf{adS}}{k_{i+1}} = \underset{j \in \{k_1 \cdots k_i\}}{\operatorname{argmin}} \| (\mathbf{G}_{i+1})_j \|^2$$

$$i \leftarrow i + 1$$

D-BLAST – (Diagonal)

- In D-BLAST, the input data stream is divided into sub streams which
- are coded, each of which is transmitted on different antennas time
- slots in a diagonal fashion
- For example, in a (2,2) system

$$\begin{bmatrix} 0 & \pmb{x}_1^{(1)} & \pmb{x}_1^{(2)} & \cdots \\ \pmb{x}_2^{(1)} & \pmb{x}_2^{(2)} & \pmb{x}_2^{(3)} & \cdots \end{bmatrix} \quad \text{interference and nulling it out} \\ \bullet \text{ The estimates of } \pmb{x}^{(1)} \text{ and } \pmb{x}^{(1)} \text{ are fed to a inject decades to decade the first substraces}$$

 receiver first estimates x2⁽¹⁾ and then estimates $x1^{(1)}$ by treating $x2^{(1)}$ as

joint decoder to decode the first substream

- After decoding the first substream, the receiver cancels the contribution of this substream from the received signals and starts to decode the next substream, etc.
- Here, an overhead is required to start the detection process; corresponding to the 0 symbol in the above example
- Receiver complexity high



MT ≤ MR

Alamouti's Scheme - Diversity

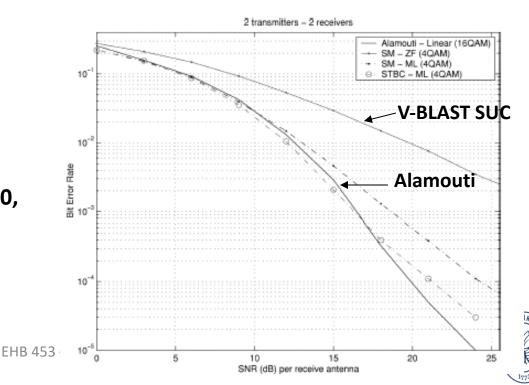
- Transmission/reception scheme easy to implement
- Space diversity because of antenna transmission. Time diversity because of transmission over
 2 symbol periods
- Consider (2, MR) system

$$egin{bmatrix} m{x}_1 & -m{x}_2^\dagger \ m{x}_2 & m{x}_1^\dagger \end{bmatrix}$$

Receiver uses combining and ML detection

• rs = 1

- If you are working with a (2,2) system, stick with Alamouti!
- Widely used scheme: CDMA 2000,
 WCDMA and IEEE 802.16-2004
 OFDM-256



Comparisons

Scheme	Spectral Efficiency	P _e	Implementation Complexity
V-BLAST	HIGH	HIGH	LOW
D-BLAST	MODERATE	MODERATE	HIGH
ALAMOUTI	LOW	LOW	LOW



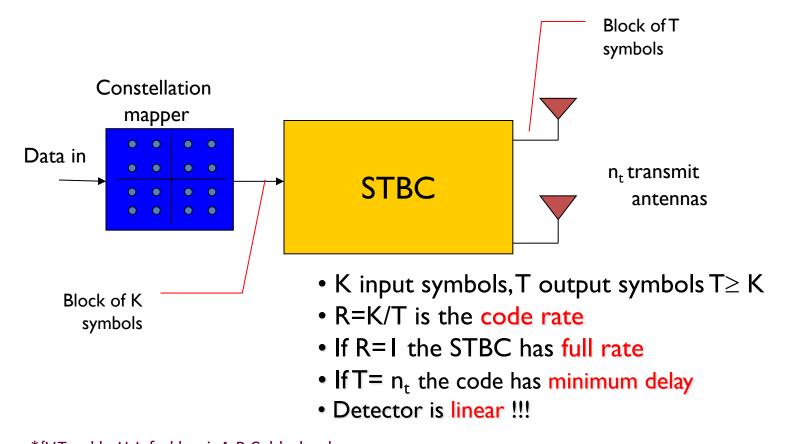
Space Time Coding

- Use parallel channel to obtain diversity not spectral efficiency as in BLAST
- Space-Time trellis codes : coding and diversity gain (require Viterbi detector)
- Space-Time block codes: diversity gain (use outer code to get coding gain)
- $n_r = 1$ is possible
- Properly designed codes acheive diversity of n_r n_t



^{*{}V.Tarokh, N.Seshadri, A.R.Calderbank Space-time codes for high data rate wireless communication: Performance Criterion and Code Construction , IEEE Trans. On Information Theory March 1998 } EHB 453 – ITU – M. Ergen

Orthogonal Space-time Block Codes



^{*{}V.Tarokh, H.Jafarkhani, A.R.Calderbank Space-time block codes from orthogonal designs, IEEE Trans. On Information Theory Junie 1999 Freen



STBC for 2 Transmit Antennas

$$\begin{bmatrix} c_0 c_1 \end{bmatrix} \Rightarrow \begin{bmatrix} c_0 & -c_1^* \\ c_1 & c_0^* \end{bmatrix} \downarrow \text{Antenna}$$

$$\xrightarrow{\text{Time}}$$

Full rate and minimum delay

Assume I RX antenna:

$$r_0 = h_1 c_0 + h_2 c_1 + n_0$$

$$r_1 = -h_1 c_1^* + h_2 c_0^* + n_1$$



$$r = \overline{H}c + n$$

$$\mathbf{r} = \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix}, \qquad \overline{\mathbf{H}} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}, \qquad \mathbf{n} = \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix}, \qquad \mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

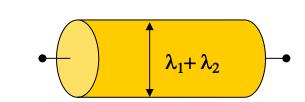
$$\widetilde{\mathbf{r}} = \overline{\mathbf{H}}^* \mathbf{r} = \overline{\mathbf{H}}^* \overline{\mathbf{H}} \mathbf{c} + \overline{\mathbf{H}}^* \mathbf{n} = \|\mathbf{H}\|_F^2 \mathbf{c} + \widetilde{\mathbf{n}}$$

Diagonal matrix due to orthogonality

The MIMO/ MISO system is in fact transformed to an equivalent SISO system with SNR

$$SNR_{eq} = ||H||_{F}^2 SNR/n_t$$

$$| | H | |_{F}^{2} = \lambda_{1} + \lambda_{2}$$





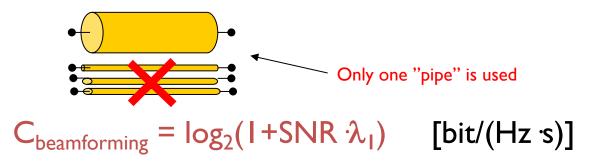
MIMO With Beamforming



Requires that channel H is known at the transmitter Is the capacity-optimal transmission strategy if

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \ge SNR$$

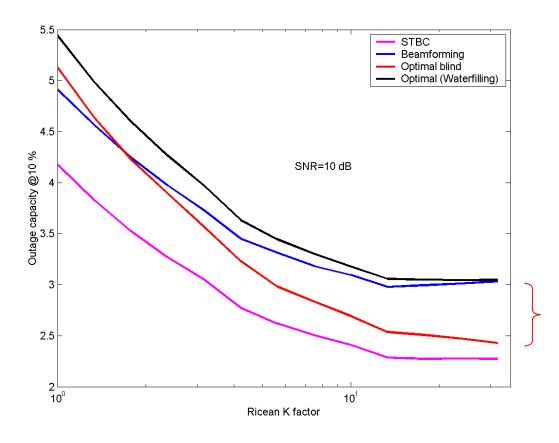
Which is often true for line of sight (LOS) channels





Comparisons...

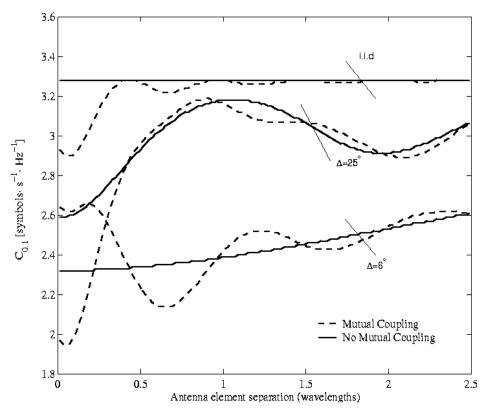
2 * 2 system. With specular component (Ricean fading)



One dominating eigenvalue. BF puts all energy into that "pipe"



Correlated channels/mutual coupling



When angle spread (Δ) is small, we have a dominating eigenvalue. The mutual coupling actually improves the performance of the STBC by making the eigenvalues "more equal" in magnitude.

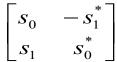


WCDMA Transmit Diversity Concept

(3GPP Release '99 with 2TX antennas)

•2 modes

- Open loop mode is exactly the 2 antenna STBC $\begin{bmatrix} s_2 & -s_1^* \end{bmatrix}$
- Open loop (STTD)



- Closed loop (I bit / slot feedback)
 - Submode I (I phase bit)
 - Submode 2 (3 phase bits / I gain bit)

The feedback bits (1500 Hz) determines the beamformer weights

Submode I Equal power and bit chooses phase between

{0,180} / {90/270}

Submode 2 Bit one chooses power division $\{0.8, 0.2\} / \{0.2, 0.8\}$ and 3 bits chooses phase in an 8-PSK constellation



Message

- Channel capacity increases linearly
 with min(n_r, n_t)
- STBC is in the 3GPPWCDMA proposal



References

- Adapted from notes of HARISH GANAPATHY and Mattias Wennström
- "Layered Space-Time Architecture for Wireless Communication in a Fading
- Environment When using Multi-Element Antennas", G.J.Foschini, Bell Labs Tech Journal, 1996
- "An Overview of MIMO Communications A Key to Gigabit Wireless", A.J Paulraj,
- Gore, Nabar and Bolcskei, IEEE Trans Comm, 2003
- "Improving Fairness and Throughput of Ad Hoc Networks Using Multiple Antennas", Park, Choi and Nettles, submitted Mobicom 2004
- "From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems", Gesbert et al.,IEEE Sel Comm, 2003
- "On Limits of Wireless Communications in a Fading Environment", Foschini and Gans, Wireless Personal Comm, 1998
- "A Simple Transmit Diversity Technique for Wireless Communications", Alamouti, IEEE Sel Comm, 1998
- "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels", Zheng and Tse, IEEE Trans Info Theory, 2003
- "V-BLAST: An Architecture for Realizing Very High Data Rates
- Over the Rich-Scattering Wireless Channel", Wolniansky, Foschini, Golden and Valenzuela, Electronic Letters, 1999
- "MIMO-OFDM Systems for High Data Rate Wireless Networks", Whu

