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Love stories can be unpredictable: *Jules et Jim* in the vortex of life

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Love stories are dynamic processes that begin, develop, and often stay for a relatively long time in a stationary or fluctuating regime, before possibly fading. Although they are, undoubtedly, the most important dynamic process in our life, they have only recently been cast in the formal frame of dynamical systems theory. In particular, why it is so difficult to predict the evolution of sentimental relationships continues to be largely unexplained. A common reason for this is that love stories reflect the turbulence of the surrounding social environment. But we can also imagine that the interplay of the characters involved contributes to make the story unpredictable—that is, chaotic. In other words, we conjecture that sentimental chaos can have a relevant endogenous origin. To support this intriguing conjecture, we mimic a real and well-documented love story with a mathematical model in which the environment is kept constant, and show that the model is chaotic. The case we analyze is the triangle described in *Jules et Jim*, an autobiographic novel by Henri-Pierre Roché that became famous worldwide after the success of the homonymous film directed by François Truffaut. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4882685]

Although hunting for chaos is not as popular as it used to be, we dedicate this paper to the presentation of a new strange attractor. It concerns the most important dynamic process in our life—the evolution of love in interpersonal relationships.[1–4] More precisely, we support the conjecture that romantic relationships can be unpredictable—technically chaotic—on the sole basis of the interplay of the characters involved. This cannot be done without a mathematical model, because love stories are, in general, influenced by the turbulence of the surrounding social environment and far too short to allow the reconstruction of a strange attractor.[5] The first allusion to the conjecture was made by Strogatz,[2] who mentioned the "many-body problem" when presenting his admittedly rudimentary model of Romeo and Juliet. A more technical hint can be found in a paper by Sprott,[6] where a naïve extension of Strogatz's model to the case of a hypothetical triangle is discussed. To give a credible support to the conjecture, we focus on a real and well-documented triangular love story, we identify from it the main psycho-physical traits of the three individuals, and we encapsulate them in a mathematical model with constant environment. We then show that for reasonable values of the individual traits the model compares favorably with the love story and is chaotic.

I. INTRODUCTION

After Strogatz's [1988] pioneering paper,[2] love stories have been modeled with increasing success in terms of differential or difference equations. Many attempts[3,7–10] describe anonymous stories from the state of indifference, in which we are when we first meet, to the establishment of a permanent (stationary or fluctuating) regime, while others[4,11] focus on the phase of marital dissolution. Moreover, mathematical models have also been developed for a few specific (though relatively simple) love stories, described in the literature or in films.[12–15]

By mentioning the analogy with the "many-body problem" of celestial mechanics, Strogatz[2] somehow conjectured that

> *sentimental relationships can be unpredictable—that is, chaotic—on the sole basis of the interplay of the characters involved.*

In other words, the conjecture is that sentimental chaos can have a relevant endogenous component, and not simply reflect the turbulence of the surrounding social environment. A proof of this conjecture can only be based on a mathematical model. In fact, the interactions with other individuals, as well as health, cultural, and economic circumstances, make difficult to identify the origin of the sentimental turbulence. And though nonlinear time series analysis can in principle help in solving the problem, love stories are too short to allow the reconstruction of a strange attractor.[5] Instead, by means of a mathematical model with constant parameters, one can easily cut all the interactions with other individuals and keep the environment constant.

A naïve support to the above conjecture can be found in an extension of Strogatz's model to a hypothetical triangle.[6] However, a credible support can only be given by modeling a real and well-documented love story. This is the aim of this study.

The starting point—the selection of the love story—is rather critical. Indeed, the story must

- be known worldwide, if we want our message to reach the large public;
- contain symptoms of turbulence and unpredictability, to possibly support our conjecture;
- contain a few, at least qualitative, information to allow the validation of the model.

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Under the above constraints, our choice has been the triangular love story described by Roché in his 1953 autobiographic novel *Jules et Jim*.[16] The story begins in Paris a few years before the First World War, where it ends 20 years later. It involves Kathe, her husband Jules, and his best friend Jim (Helen Grund, Franz Hessel, and Henri-Pierre Roché in the real life).

Roché's novel is known because it is considered as one of the main contributions conveying the anti-bourgeois ideology of "free love" that can be condensed in saying that "one should not constrain the people one loves, but leave them free to engage in other relationships." This central idea of Roché's philosophy became very popular in the seventies and was later extensively debated in literary essays. But the love story of *Jules et Jim* became famous worldwide after the success of the 1961 homonymous film—a celebrated masterpiece of the French Nouvelle Vague directed by François Truffaut.

As for the symptoms of turbulence and unpredictability, the reading of the novel gives the impression that Kathe is quite unstable and difficult to predict—she changes partner seven times in the 20 years of concerns, alternating between Jules and Jim. Actually, the uncertainty over the future creates in the triangle (and in the reader) an increasing tension that ceases only when Kathe and Jim commit suicide:

> *Jules would never have again the fear that had been with him since the day he met Kate, first that she would deceive him—and then, quite simply, that she would die, for she had now done that too* (p. 236 in the English translation of Roché's novel[16])

The dramatic end imagined by Roché is hence interpretable as a poetic way of interrupting the torture due to a recurrent shock—the change of partner—that becomes particularly unsustainable because unpredictable. Also the few available data confirm that the partner changes are irregular.

The paper is organized as follows. In Sec. II, we introduce the reader to the basic notions we use to model sentimental relationships.[3] Then in Sec. III, we build our model of *Jules et Jim*. First, we note that the love story between Kathe and any one of her two lovers is scarcely influenced by the presence of the other. This is a direct consequence of the principle of free love that inspires the life of the three characters. If they would rigorously follow this principle, then the triangular relationship would be equivalent to two fully separated pairwise relationships. In turn, the triangle could be modeled with two independent submodels, Kathe-Jules and Kathe-Jim, respectively, described in Secs. III A and III B. However, since the three characters slightly deviate from the pure ideology of free love, the triangle is described more realistically in Sec. III C by weakly coupling the two submodels. Specifically, we introduce two small parameters to take into account that Kathe does not live two independent love stories and that Jules and Jim are slightly complaisant, the first, and jealous, the second. All our modeling choices are supported by literary passages extracted from Roché's novel.

In Sec. IV A, we validate our model against the information available in the novel, which we identify in seven specific features, including the number and the chronology of the partner changes. Since there are no elements in the novel that could suggest reasonable values for the coupling parameters, we perform systematic simulations to check if there is a region in the plane of the two parameters for which all features are satisfactorily reproduced by the model. The result of this validation shows that this region is characterized by small and positive values of the coupling parameters.

Finally, to support our conjecture, we show in Sec. IV B that for the validated values of the coupling parameters, the model trajectory describing the story of *Jules et Jim* asymptotically reaches a chaotic attractor. Moreover, in less than 20 years, the trajectory reaches the attractor and spends close to it a time in which predictions become impracticable according to the computed (largest) Lyapunov exponent.[17]

A broader discussion of our results and a few general conclusions that can be drawn from this study close the paper in Sec. V. Since the results obtained with a model based on subjective interpretations are not as credible as those based on precise physical laws, the reader is invited to check the robustness of our conclusions by interactively simulating our model using an online simulator (see supplementary material[22]), where all model parameters can be significantly changed.

II. MATERIAL AND METHODS

Levinger[1] has been the first to use graphs to represent the time evolution of the feelings of one person for another. Following Levinger's abstraction, and also to minimize the number of equations, we assume that the interest of one person for another can be captured by a single variable, called *feeling*. Low and high positive feelings correspond to friendship and love, while negative feelings indicate antagonism and hate; zero corresponds to indifference. For example, in the pairwise story depicted in the top panels of Fig. 1, she develops from the very beginning a positive feeling for him, while he is initially antagonistic. In contrast, in the other story (bottom panels), she and he are always positively involved, but suffer from remarkable ups and downs. Of course the graphs start from the feelings that they have one for the other at the beginning of the story. Thus, the starting point is the origin of the plane of the feelings if the two individuals are initially indifferent to each other.

Feelings vary over time because of the interplay of consumption and regeneration mechanisms, here considered as time-invariant processes. The basic consumption mechanism is *oblivion*. It explains why a person looses memory of the partner after being abandoned. The regeneration processes typically considered in minimal models[3,7–10,12–15] are the *reaction to love* and the *reaction to appeal*—the mix of beauty, talent, wealth, and other traits that are independent of feelings.

Consider a couple and denote by $x(t)$ and $y(t)$ the feelings that she and he have one for the other at day t. A model is simply a balance of the feelings between any day t and the following day $(t+1)$. In words, her feeling tomorrow is equal to that of today minus the loss of interest between today and tomorrow due to oblivion, plus the recharge of interest, again between today and tomorrow, due to her reactions to his love and appeal.

The loss of interest due to oblivion can be described with a function $F(x)$ increasing with x, to express the fact,

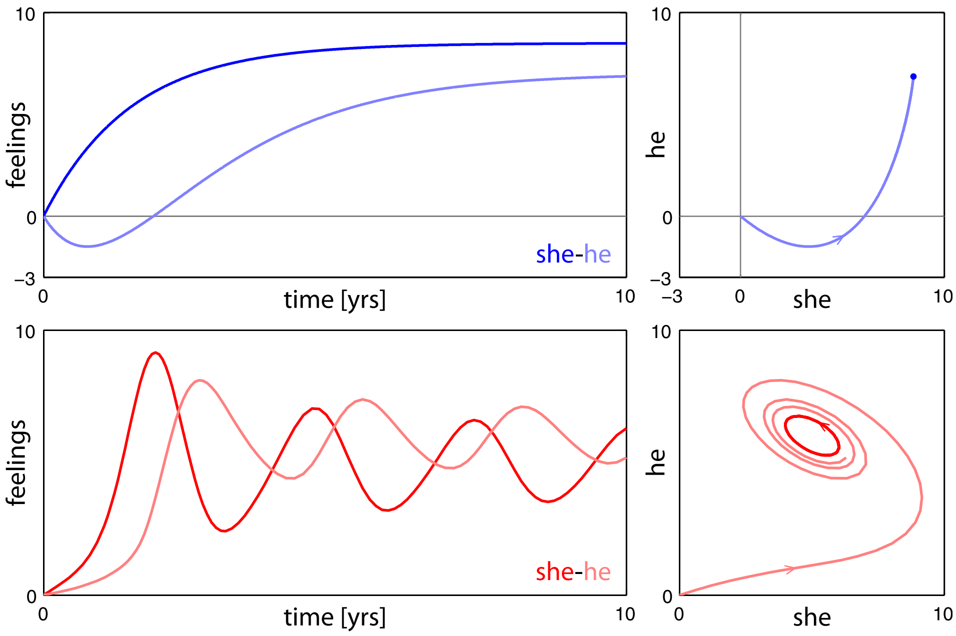

FIG. 1. Graphical representation of two hypothetical love stories. (Left) Feelings' time series. (Right) Trajectories in the plane of the feelings.

common in natural systems, that the rate at which a given property is lost is positively correlated with the abundance of the property. Typically, the loss is assumed to be proportional to x, so the function $F(x)$ is linear and given by the product of a proportionality coefficients f and x. The parameter f, called *forgetting coefficient*, represents the portion of interest lost in one day through oblivion.

As for the recharge of the feeling, we denote by $R_L(y)$ her reaction to the partner's love (where R stands for reaction and L for love) and by $R_A(a_y)$ her reaction to the partner's appeal, here indicated with a_y and assumed to be invariant. Also the reaction $R_A(a_y)$ is assumed to be linear, i.e., $R_A(a_y) = r_A a_y$.

To model the reaction to love, we distinguish between *secure* individuals—who increase their reaction for any increase of the love of the partner—and *insecure* ones—who avoid high involvements by decreasing their reaction (and possibly react negatively) when the love of the partner is above a critical threshold. Secure individuals are therefore characterized by functions $R_L(y)$ increasing with the love y of the partner (see Ref. 3 for a survey). Among these functions, there are linear functions, which however correspond to rather extreme individuals with unbounded capacity of recharge. In contrast, insecure individuals are characterized by functions $R_L(y)$ which are decreasing at sufficiently high values of y (Fig. 2, top).

Another important characteristic of an individual is the propensity to react to the appeal of the partner in a biased way, depending on her/his own state of involvement. For example, parents often see their own children more beautiful than they really are. But the same phenomenon, called *synergism*, has also been observed in a study of perception of physical attractiveness.[18] In this case, the reaction to the partner's appeal can be written in the form $(1 + S(x))\, R_A(a_y)$, where the function S is increasing for positive x (saturating for large x) and is zero for negative x (Fig. 2, bottom). The opposite behavior is also possible, like in *platonic* individuals described by a reaction to appeal of the form $(1 - P(x))$ $R_A(a_y)$, where P is shaped like S and measures the loss of sexual interest for increasing values of the involvement x. Individuals who are neither synergic nor platonic are not biased by their own feelings.

III. THE MODEL OF *JULES ET JIM*

In this section, we propose a mathematical model for the love story of *Jules et Jim* using a didactic style that should make the paper accessible also to non-technically oriented readers. In particular, we present the model as a rule that updates the feelings of Kathe, Jules, and Jim recursively from one day to the next. An equivalent continuous-time formulation of the model is also possible (in terms of ordinary differential equations).

The love story is reduced to a pure triangle in a constant environment. Specifically, we neglect the interactions that

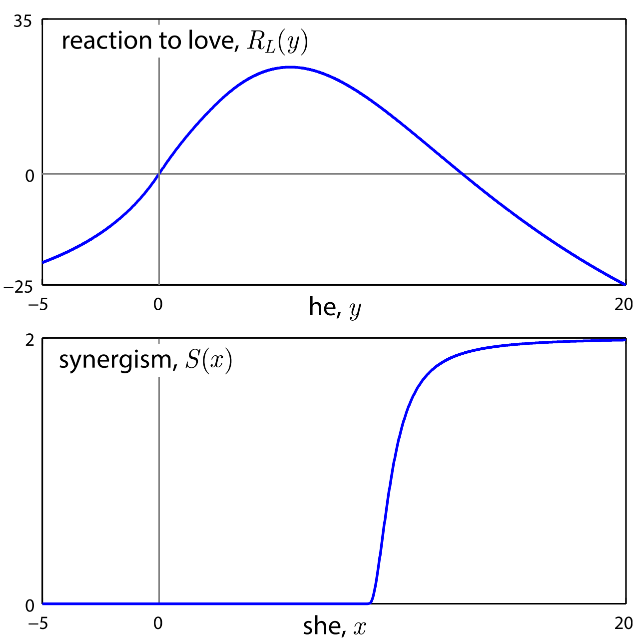

FIG. 2. (Top) Reaction to love typical of an insecure individual. (Bottom) Typical synergism function. See Table I for the analytical expressions.

Kathe, Jules, and Jim have with other minor characters described by Roché, and keep all model parameters constant. Six variables—the feelings of each person for the others—are in principle required in a minimal model. However, Jules and Jim have a deep and permanent friendship:

> *In twenty years Jim and he had never quarrelled. Such disagreements as they did have they noted indulgently* (p. 237)

We therefore consider only the feelings x_1 and x_2 of Kathe for Jules and Jim and the feelings y_1 and y_2 of the two friends for her.

The three follow almost to perfection the ideology of free love:

> *In her mind, each lover was a separate world, and what happened in one world was no concern of the others* (p. 108)

It seems thus reasonable to split Kathe into two independent women, one in love with Jules and one with Jim, and to describe the triangle by means of two independent submodels of pairwise relationship: the Kathe-Jules submodel (Sec. III A) and the Kathe-Jim submodel (Sec. III B).

However, Roché describes specific behaviors on the part of the two friends that violate the rigid principle of free love. Jules is complaisant with Jim—he is pleased when Kathe is with Jim because he realizes this makes her happier. This characteristic, peculiar to Jules, is consistent with his platonic nature (see Sec. III A) and is well described by Roché:

> *'…I'm terrified of losing her, I can't bear to let her go out of my life. Jim—love her, marry her, and let me go on seeing her. What I mean is, if you love her, stop thinking that I'm always in your way'* (p. 27)

Although jealousy is at odds with the ideology of free love, Jim is slightly jealous of Jules:

> *She bestowed her graciousness on each in turn…and Jim was jealous* (p. 97)

The triangle is hence described by coupling the two submodels through suitable parameters that measure the small deviations of the three characters from the principle of free love.

A. The submodel Kathe-Jules

The main peculiarity of Jules is to be platonic:

> *Really, Jules is happy, in his own way, and just wants things to go on. He's seeing you often, in idyllic circumstances, and he's living on hope* (p. 24)

He therefore reduces his reaction to Kathe's appeal when he is more in love with her, i.e., his reaction to appeal is damped by the factor $(1 - P)$, where P is Jules' platonicity (shaped as in Fig. 2, bottom).

In accordance with his platonic nature, Jules is a secure lover, and assuming linear forgetting and reaction functions, the equation regulating his feeling for Kathe is the following:

$$y_1(t+1) = y_1(t) - f_1\, y_1(t) + r_1\, x_1(t) + (1 - P(y_1(t)))\, r_{A1}\, a. \tag{1}$$

Kathe is a passionate woman, and though charmed by Jules, she is at the same time annoyed by his platonic nature:

> *She had been drawn by his mind, his gift of fantasy. But she needed, in addition to Jules, a male of her own sort* (p. 90)

For this reason, her reaction R_L to Jules' love is of the insecure type (Fig. 2, top).

Moreover, Kathe is definitely an enthusiastic person, so her reaction to Jules' appeal is amplified by the factor $(1 + S)$, where S is Kathe's synergism (Fig. 2, bottom).

In conclusion, assuming that Kathe's forgetting and reaction to appeal are linear, her equation is

$$x_1(t+1) = x_1(t) - f\, x_1(t) + R_L(y_1(t)) + (1 + S(x_1(t)))\, r_A\, a_1. \tag{2}$$

The model of the couple Kathe-Jules is therefore composed of Eqs. (1) and (2). The model can be used repeatedly to compute the time evolution of the feelings of Kathe and Jules. For this, we must first assign reasonable values to all parameters, taking into account all possible indications present in the novel. For example, we take Kathe's appeal a greater than Jules' one a_1, because she is, by far, more fascinating than him. Similarly, we assume she forgets faster than him, $f > f_1$, being the more unstable in the couple. Of course the specific values we have selected remain rather arbitrary and based on our subjective interpretations. All the details about the functions R_L, S, and P and the parameter values can be found in Tables I and II.

Now, assuming that the day they meet for the first time, say $t = 0$, Kathe and Jules are completely indifferent one to each other, we can fix $x_1(0) = y_1(0) = 0$ and use the two equations to compute the values of the two feelings the next day, thus obtaining $x_1(1) = r_A\, a_1$ and $y_1(1) = r_{A1}\, a$. It is interesting to note that only appeal matters at the beginning of a love story, since feelings are still latent. To go on to the next day, it is sufficient to increase time of one unit and use the same equations written for $t = 1$ to compute the feelings at day $t = 2$. Note that also the forgetting functions and the reactions to love are now involved. Repeating the same operations for $t = 2$, 3, …, we can compute the feelings of Kathe and Jules at day 3, 4, …, and continue like this for months or years. The results can be easily portrayed to show the evolution of the love story in a time interval of interest. In this way we obtain the graphs in Fig. 3 (top), where the points indicated with 1, 2, and 3 represent the feelings of Kathe and Jules at the end of the first, second, and third year of their relationship.

Kathe and Jules are always positively involved, but their love story does not reach a plateau. Indeed, as time goes on, their feelings tend to oscillate with a period of about 4 years, more precisely 3 years and 10 months. At the beginning of their relationship, Kathe and Jules are increasingly involved, until Kathe has the first inversion in her trend. According to the model, these inversions are recurrent.

B. The submodel Kathe-Jim

The main characteristic of Jim is to be insecure, as all "Don Juan" are to avoid deep involvements:

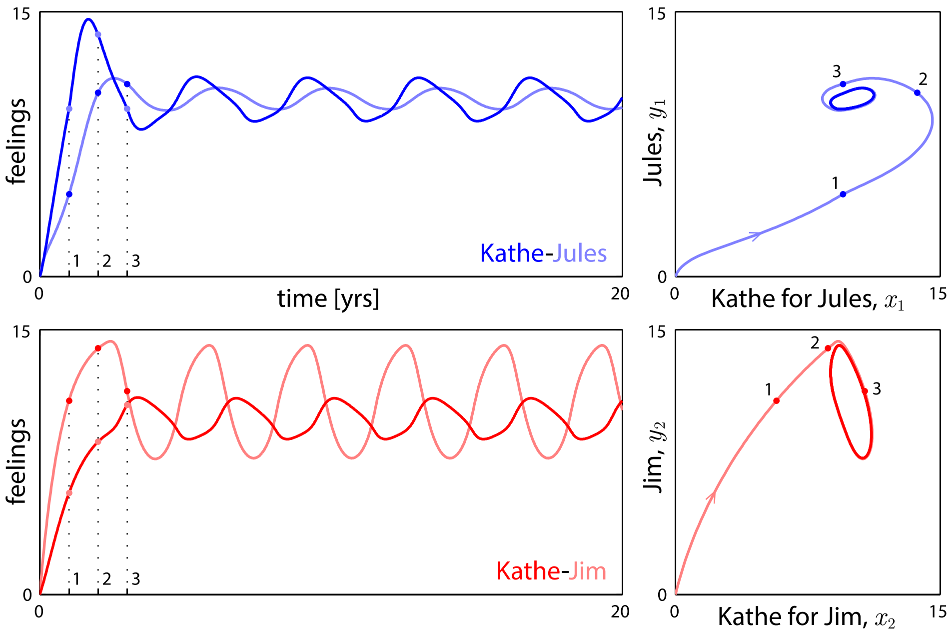

FIG. 3. The hypothetical love stories predicted by the Kathe-Jules (top) and by the Kathe-Jim (bottom) submodels.

'Oh, when,' she said to him one day,—'when are you going to stop giving me bits of yourself and give me everything?' (p. 207)

Thus, his reaction R_{L2} to Kathe's love is nonlinear and shaped as in Fig. 2 (top). Assuming that his forgetting and reaction to appeal are linear, Jim's equation is then

$$y_2(t+1) = y_2(t) - f_2\, y_2(t) + R_{L2}(x_2(t)) + r_{A2}\, a. \quad (3)$$

Kathe is secure in her relationship with Jim (because he is not platonic) and synergic. This is therefore Kathe's equation,

$$x_2(t+1) = x_2(t) - f\, x_2(t) + r_L\, y_1(t) + (1 + S(x_2(t)))\, r_A\, a_2, \quad (4)$$

where S is again Kathe's synergism.

In conclusion, the model of the couple Kathe-Jim is composed of Eqs. (3) and (4). Again parameters must be fixed at reasonable values, e.g., Jim's appeal a_2 smaller than Kathe's one, though larger than Jules' appeal—Jim being a charming "Don Juan;" and Jim's forgetting being faster than Jules' one, $f_2 > f_1$, in agreement with the "Don Juan" nature of Jim (see Table II). Once all parameters are fixed, the model can be repeatedly used to compute the time evolution of the feelings of Kathe and Jim. The result is in Fig. 3 (bottom). In this case too, the involvements of Kathe and Jim increase during the first phase of their relationship and then tend in a few years toward a swinging regime with a period of 3 years and 4 months. This time, the first to invert the positive trend is Jim, who being insecure refuses too deep involvements.

C. The model of the triangle

The model of the triangle is obtained by weakly coupling the two submodels Kathe-Jules and Kathe-Jim. For this we introduce the following extra-characteristics in the behaviors of the three individuals:

– Kathe does not live in fully separated worlds;
– Jules is complaisant with Jim;
– Jim is jealous of Jules.

To implement the first change, we assume that Kathe's forgetting capabilities depend upon her state of involvement. More precisely, we assume that at any given time she forgets less quickly the lover she is more involved with. This is realized by multiplying, in the two equations for Kathe (see below), her forgetting coefficient f by a factor which is greater than 1 in one equation and smaller than 1 in the other. In order to deviate only slightly from the free-love principle, ε must be a small positive parameter.

Jules does not suffer when Kathe is more in love with Jim. Actually, he is pleased because he sees Kathe more happy. As already said, this peculiar characteristic is consistent with the platonic nature of Jules and is well described by Roché. In order to take Jules' complaisance into account, his reaction to Kathe's love is amplified by a factor greater than 1 when she is more in love with Jim, namely, when x_2 is greater than x_1 (see Jules' equation).

In order to take Jim's jealousy into account, his reaction to Kathe's love is dumped by a factor smaller than 1 when she is more in love with Jules, namely, when x_1 is greater than x_2 (see Jim's equation). For simplicity, Jules' complaisance and Jim's jealousy are quantified by the same positive parameter δ, which must also be small if we like to avoid large deviations from the free-love principle.

In conclusion, the model of the triangle is composed of the following four difference equations:

$$\begin{aligned}
x_1(t+1) &= x_1(t) - f\exp(\varepsilon(x_2(t)-x_1(t)))x_1(t)\\
&\quad + R_L(y_1(t)) + (1+S(x_1(t)))r_A a_1, \quad \text{(Kathe for Jules)}\\
x_2(t+1) &= x_2(t) - f\exp(\varepsilon(x_1(t)-x_2(t)))x_2(t)\\
&\quad + r_L y_1(t) + (1+S(x_2(t)))r_A a_2, \quad \text{(Kathe for Jim)}\\
y_1(t+1) &= y_1(t) - f_1 y_1(t) + r_1 x_1(t)\exp(\delta(x_2(t)-x_1(t)))\\
&\quad + (1-P(y_1(t)))r_{A1} a, \quad \text{(Jules)}\\
y_2(t+1) &= y_2(t) - f_2 y_2(t)\\
&\quad + R_{L2}(x_2(t))\exp(\delta(x_2(t)-x_1(t))) + r_{A2} a, \quad \text{(Jim)}
\end{aligned}$$

TABLE I. Nonlinear functions (specified for nonnegative feelings).

Character	Symbol	Expression	Description
Kathe	$R_L(y_1)$	$r_I \dfrac{y_1/y_L}{1+y_1/y_L}\begin{cases}\dfrac{1-((y_1-\tau_I)/y_I)^2}{1+((y_1-\tau_I)/y_I)^2} & \text{if } y_1 \geq \tau_I \\ 1 & \text{if } y_1 < \tau_I\end{cases}$	Kathe's reaction to Jules' love
	$S(x)$	$\begin{cases}s\dfrac{((x-\tau_S)/x_S)^2}{1+((x-\tau_S)/x_S)^2} & \text{if } x \geq \tau_S \\ 0 & \text{if } x < \tau_S\end{cases}$	Kathe's synergism
Jules	$P(y_1)$	$\begin{cases}p\,\dfrac{((y_1-\tau_P)/y_P)^2}{1+((y_1-\tau_P)/y_P)^2} & \text{if } y_1 \geq \tau_P \\ 0 & \text{if } y_1 < \tau_P\end{cases}$	Jules' platonicity
Jim	$R_{L2}(x_2)$	$r_{I2} \dfrac{x_2/x_L}{1+x_2/x_L}\begin{cases}\dfrac{1-((x_2-\tau_{I2})/x_I)^2}{1+((x_2-\tau_{I2})/x_I)^2} & \text{if } x_2 \geq \tau_{I2} \\ 1 & \text{if } x_2 < \tau_{I2}\end{cases}$	Jim's reaction to Kathe's love

TABLE II. Model parameters.

Character	Context	Symbol	Value	Description
Kathe	Forgetting	f	2/365	Kathe's forgetting coefficient
	Reaction to love	r_L	1/365	Kathe's reaction coefficient to Jim's love
	$R_L(y_1)$	r_I	80/365	Kathe's-to-Jules maximum insecureness
		y_L	10	Sensitivity of Kathe's reaction to Jules' love
		τ_I	2.5	Kathe's-to-Jules insecureness threshold
		y_I	10.5	Sensitivity of Kathe's-to-Jules insecureness
	Reaction to appeal	r_A	1/365	Kathe's reaction coefficient to appeal
	$S(x)$	s	2	Kathe's maximum synergism
		τ_S	9	Kathe's synergism threshold
		x_S	1	Sensitivity of Kathe's synergism
	Appeal	a	20	Kathe's appeal
Jules	Forgetting	f_1	1/365	Jules' forgetting coefficient
	Reaction to love	r_{L1}	1/365	Jules' reaction coefficient to love
	Reaction to appeal	r_{A1}	0.5/365	Jules' reaction coefficient to appeal
	$P(y_1)$	p	1	Jules' maximum platonicity
		τ_P	0	Jules' platonicity threshold
		y_P	1	Sensitivity of Jules' platonicity
	Appeal	a_1	4	Jules' appeal
Jim	Forgetting	f_2	2/365	Jim's forgetting coefficient
	$R_{L2}(x_2)$	r_{I2}	20/365	Jim's maximum insecureness
		x_L	10	Sensitivity of Jim's reaction to love
		τ_{I2}	9	Jim's insecureness threshold
		x_I	1	Sensitivity of Jim's insecureness
	Reaction to appeal	r_{A2}	1/365	Jim's reaction coefficient to appeal
	Appeal	a_2	5	Jim's appeal

and differs from the ensemble of the two independent submodels for the presence of the two small coupling parameters ε and δ (see Tables I and II for the analytical expressions and the reference values of the other parameters).

IV. RESULTS

A. Validation of the model

We now validate our model of *Jules et Jim* against the following quantitative/qualitative features we have identified in the novel:

(i) In the twenty years of concern, Kathe changes partner seven times, alternating between Jules and Jim;
(ii) the chronology of the partner changes is well documented by Roché;
(iii) during the first years Kathe is more attracted by Jules (she marries him);
(iv) at the very beginning of the story, Kathe is more attracted by Jim, who misses a strategic date:

> *If Kate and Jim had met at the café, things might have turned out very differently* (p. 80)

(v) Jim's ups and downs are more relevant than those of Jules:

"Jim was easy for her to take, but hard to keep. Jim's love drops to zero when Kate's does, and shoots up to a hundred with hers. I never reached their zero or their hundred" (p. 231)

(vi) The drops in interest of Kathe for Jules anticipate those of Jules for Kathe:

The danger was that Kate would leave. She had done it once already…and it had looked as if she didn't mean to return…She was full of stress again, Jules could feel that she was working up for something (p. 89)

(vii) The drops in interest of Jim for Kathe anticipate those of Kathe for Jim:

He himself was incapable of living for months at a time in close contact with Kate, it always brought him into a state of exhaustion and involuntary recoil which was the cause of their disasters (p. 189)

We keep all parameters (except ε and δ) at the values of Table II and we first look for pairs (ε, δ) for which feature (i) is reproduced by the model. For this we fix a dense grid in the (ε, δ) plane and we systematically simulate our model for each point of the grid, always starting from the state of indifference—since Jules and Jim are together when they are introduced to Kathe—and stopping the simulation after 20 years. The pairs (ε, δ) in the overshaded region in Fig. 4 are those for which the model predicts seven changes of partner—seven changes of sign of Kathe's unbalance x_1–x_2 after she marries Jules.

And for the particular values of ε and δ corresponding to the white dot in the figure, the predicted chronology of the partner changes is in best agreement with (ii). Kathe's unbalance is graphed in Fig. 5 (bottom-left) and the correlation between the seven instants suggested by the model and those indicated by Roché is 0.97! (Fig. 5, bottom-right).

We then compare the model predictions of Fig. 5 with features (iii)–(vii). Feature (iii) is well predicted because $x_1 > x_2$ in the first years of the story (see Kathe's unbalance). Feature (iv) is also predicted, even if not visible at the scale of the figure. In fact, Jules' appeal is lower than that of Jim ($a_1 < a_2$), and this implies that during the very first days of the story the feeling of Kathe for Jules is lower than that for Jim ($x_1(1) = r_A a_1$ and $x_2(1) = r_A a_2$). But after a couple of weeks, according to the model, Kathe's preference is in favor of Jules and she marries him soon after. Fig. 5 is also in agreement with feature (v), y_2-oscillations being larger than y_1-oscillations, and with features (vi) and (vii), as evident from the rotation directions in the projections of the model trajectory in the planes Kathe-Jules (x_1, y_1) and Kathe-Jim (x_2, y_2).

Finally, to fully validate our model, we have ascertained the robustness of our results with respect to perturbations of all parameters. This is mandatory in a context where most parameters describe qualitative, rather than quantitative, characterial aspects. For this we have first checked that features (i)–(vii) are satisfactorily reproduced for all pairs (ε, δ) in the overshaded region of Fig. 4. Then, we have systematically simulated our model by perturbing (up to 10%) the parameters of the Kathe-Jules and Kathe-Jim submodels, and we have checked that it was always possible to fit features (i)–(vii) with small values of the coupling parameters ε and δ. The reader can verify the robustness of our results by using the online simulator (see supplementary material[22]), where all model parameters can be significantly changed.

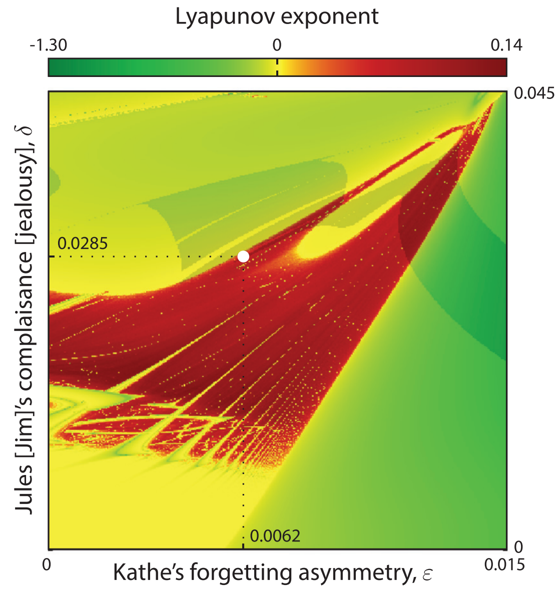

FIG. 4. The largest Lyapunov exponent of the model trajectory starting from $x_1(0) = x_2(0) = y_1(0) = y_2(0) = 0$ (computation based on the discrete QR standard algorithm[19] and checked to be consistent with other nonnegative initial conditions). The exponent is positive (red) for chaotic attractors; zero (yellow) for quasi-periodic attractors and bifurcating cycles; negative (green) for stable cycles. For (ε, δ) in the overshaded region the model predicts that Kathe changes partner seven times in 20 years (see Fig. 5, bottom).

B. Support of the conjecture

To support our conjecture, we need to show that, for reasonable parameter settings, the trajectory of our model originating at the state of indifference converges to a chaotic attractor and that the associated unpredictability is at work in the first 20 years.

Focusing on our validated parameter setting (Table II and (ε, δ) at the white dot in Fig. 4), we obtain the chaotic attractor depicted in Fig. 6 for which we estimate a Lyapunov exponent of 0.07 yr^{-1}. The characteristic time of divergence of nearby trajectories after which predictions become impracticable (the inverse of the Lyapunov exponent[17]) is hence about 15 yr. Moreover, from Fig. 5, we see that the attractor is reached only a few years after the beginning of the love story, so that we can conclude that unpredictability can be felt before the end of the story.

The Lyapunov exponent has been computed for all pairs (ε, δ) considered in Fig. 4 (see the color-code), and the result is the typical bifurcation diagram expected for weakly coupled oscillators. For extremely weak coupling, the model attractor is a torus (see the yellow region close to $\varepsilon = \delta = 0$). Then, for larger coupling, the two oscillators can synchronize on a cycle on torus, and this occurs in the well-known

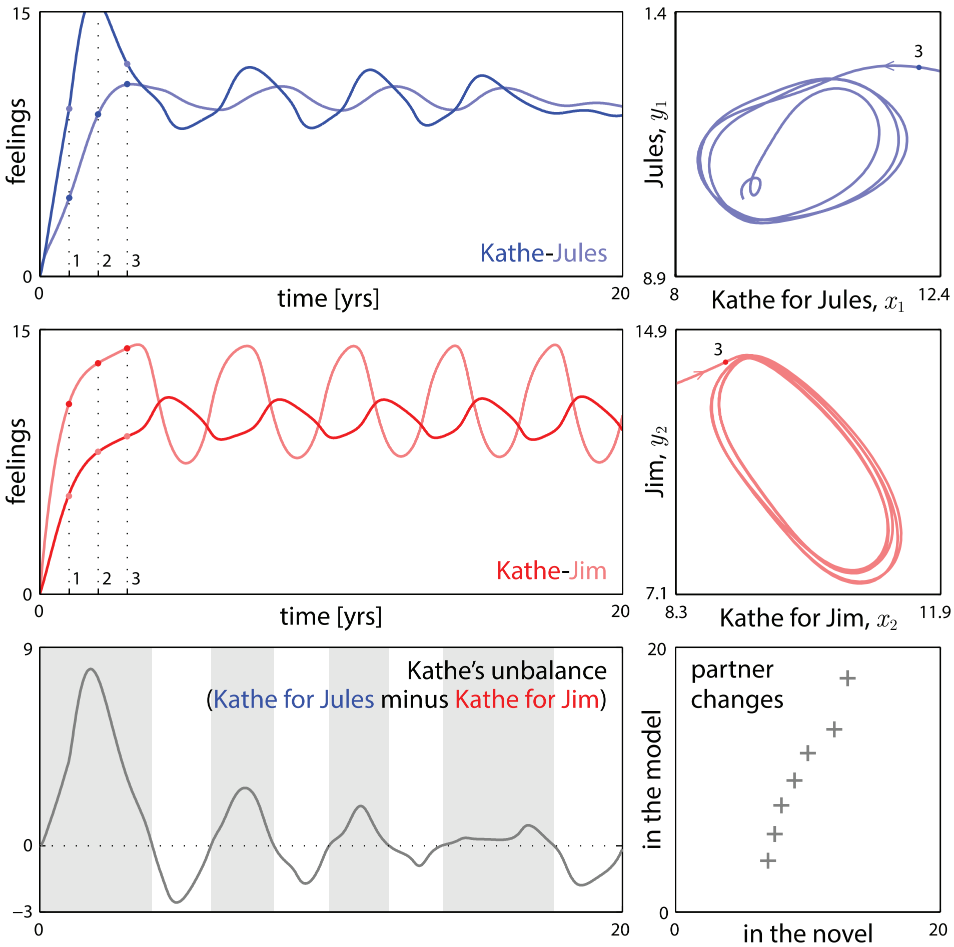

FIG. 5. The love story predicted by the validated model (ε and δ at the white dot in Fig. 4, other parameters as in Table II). (Top panels) Time series of the feelings and trajectory projections in the planes of the Kathe-Jules and Kathe-Jim submodels. (Bottom panels) Kathe's preference and comparison with the novel.

Arnold tongues (the very thin greenish regions). Increasing the coupling, the attractor undergoes a complex structure of bifurcations—not discussed in detail—that describe the classical torus-destruction route to chaos. The genericity of Fig. 4 confirms once more the robustness of our results.

Note that only a weak coupling allows to support the conjecture, since the model attractor is periodic if the coupling is too strong, whereas the uncoupled ensemble of the Kathe-Jules and Kathe-Jim submodels describes a periodic or quasi-periodic love story. Interestingly, chaos can be found for $\varepsilon = 0$, but not for $\delta = 0$, suggesting that the complaisance of Jules and the jealousy of Jim are the key elements triggering the complexity of their story.

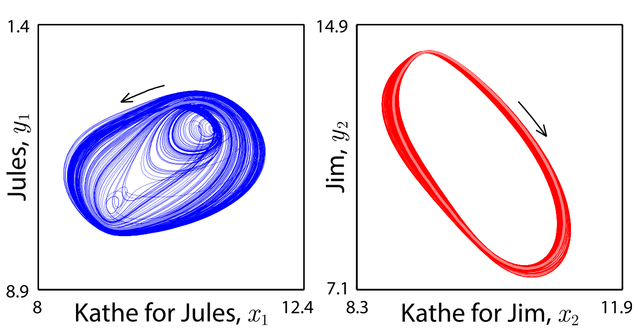

FIG. 6. Projections of the chaotic attractor reached by the validated model (the limit cycles of Fig. 3 are superimposed for comparison with the uncoupled ensemble of the Kathe-Jules and Kathe-Jim submodels).

V. DISCUSSION AND CONCLUSIONS

As is well known, even from personal experience, sentimental relationships are influenced by the social environment in which individuals live. It is therefore not surprising if the feelings characterizing romantic relationships in turbulent environments cannot be predicted. More subtle and interesting is the idea discussed in this paper: love stories can be unpredictable even in constant environments, i.e., on the sole basis of the characters involved. This idea has been conjectured in Ref. 2 and then supported in Ref. 6 through a naïve mathematical model of a hypothetical love story.

Here we have proved the conjecture by making reference to the real and well documented triangular love story, involving Kathe, Jules, and Jim, described by Roché in his 1953 novel *Jules et Jim*.[16]

There are five aspects of our study that are worth to be mentioned because of general interest. The first concerns the method of analysis, which is general and consistent with standard psychoanalysis. First, the main psycho-physical traits of the individuals involved are identified, in this case from a careful reading of the novel. Then, these characteristics are encapsulated in a mathematical model—the formal analogue of the verbal descriptions more traditionally used in psychology—which is validated by comparing the model predictions with the most relevant features of the love story. The result is very satisfactory: for suitable values of the parameters, the matching between the story predicted by the model and the story described by Roché goes beyond what is typically expected in the context of social dynamics. This allowed us to prove the conjecture by simply checking that the validated model has a positive Lyapunov exponent. Specifically, we show that both the time needed to reach the strange attractor from the initial state of indifference and the characteristic time of divergence of nearby trajectories in the attractor are significantly smaller than the length of the love story (20 years), so that it is possible to infer that Kathe, Jules, and Jim had high chances to feel the unpredictability of their story.

As a second interesting aspect, we found that unpredictability is triggered by minor and almost hidden traits of the three characters: Kathe forgets slightly more quickly the lover she is less involved with, while Jules and Jim are slightly complaisant and jealous. In the general context of systems theory, this confirms that small parameters can play strategic roles in promoting complex dynamics, while in the specific context of interpersonal relationships this justifies the interest that psychoanalysts have in apparently minor details.

A third aspect we like to mention is concerned with the structure of our model. It is composed of two oscillators (the submodels Kathe-Jules and Kathe-Jim), which interfere through weak coupling mechanisms. This is common in several fields of science, where systems can be viewed as interconnected oscillating units. For example, in ecology each consumer population has a favorite resource but can also feed on a secondary species, which, in turn, can be the favorite resource for another consumer. Thus, complex food webs are naturally described as consumer-resource units interconnected through the feeding preferences. A model of two consumers competing for two resources has therefore the same structure than the model considered in this paper. This is of great potential interest, because some of the general results obtained in mathematical ecology,[20] and/or results in the theory of coupled oscillators,[21] could guide the modeling of complex interpersonal relationships.

The fourth aspect to be remarked is that a love story can be chaotic without necessarily involving three individuals, as in the case studied in this paper. Indeed, sentimental chaos can be present in the more standard situation involving two individuals, provided at least one is characterized, in addition to the romantic sphere, by a second important emotional compartment. This is typical of individuals involved in creative professions, where inspiration, satisfaction, and self-esteem can interfere with the romantic sphere. And since a model of this situation would be at least three-dimensional, instabilities (chaos) can easily arise. For example, the destabilizing effect of inspiration has been pointed out in the romantic relationship between Petrarch, the famous poet of the 13th century, and his mistress.[12]

Finally, the last general message we like to extract from our study is the fact that a mathematical study can be used to highlight the genius of an artist—in this case François Truffaut, one of the prominent directors of the "Nouvelle Vague"—who featured Roché's novel in his most important film, *Jules et Jim*, made in 1961 after discussing the idea with Roché. Jeanne Moreau and Oskar Werner, already well known, played Kathe and Jules, while Henri Serre, selected because of a certain resemblance to Roché, played Jim. Truffaut omits many minor characters of the novel, thus considering an almost steady social environment, but successfully reproduces the feelings between Helen Grund and the two friends. Indeed Helen Grund, the only one of the three who could watch the film after Hessel and Roché passed away, wrote to Truffaut:

> *But what disposition in you, what affinity could have enlightened you to the point of recreating—in spite of the odd inevitable deviation and compromise—the essential quality of our intimate emotions?*

Truffaut magistrally adds, here and there, explicit elements pointing to the fact that love stories can be turbulent because of attracting and repelling forces. Since the discussion of these original elements would bring us too far, we only mention here the most explicit reference to attraction and repulsion, *Le tourbillon de la vie* (the vortex of life), the soundtrack sung by Jeanne Moreau. This song is undoubtedly a beautiful hymn to chaos, characterized by recurrent phases of convergence and divergence. Further details on the genius of François Truffaut in using the metaphor of stretching and folding will be published elsewhere.

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