Plotting and Transforming a Circle in 3D

$x_c := 2$ $r := 5$ $x(t) := r\cdot\cos(t) + x_c$

$y_c := 3$ $y(t) := r\cdot\sin(t) + y_c$

$z(t) := 0$

$s := 4$ $n := 10$ $j := 0..n$ $t_j := \dfrac{j\cdot 2\cdot\pi}{n}$

$$T := \begin{pmatrix} 1 & 0 & 0 & x_c \\ 0 & 1 & 0 & y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & 0 & -x_c \\ 0 & 1 & 0 & -y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$P_{0,j} := x(t_j)$ $P_{0,n+j+1} := x(t_j)$

$P_{1,j} := y(t_j)$ $P_{1,n+j+1} := y(t_j)$

$P_{2,j} := z(t_j)$ $P_{2,n+j+1} := z(t_j) + 5$

$P_{3,j} := 1$ $P_{3,n+j+1} := 1$

$$x = r\cdot\cos(\alpha)$$
$$y = r\cdot\sin(\alpha)$$

$$x' = r\cdot\cos(\alpha + \theta) = r\cdot\cos(\alpha)\cdot\cos(\theta) - r\cdot\sin(\alpha)\cdot\sin(\theta)$$
$$y' = r\cdot\sin(\alpha + \theta) = r\cdot\sin(\alpha)\cdot\cos(\theta) + r\cdot\cos(\alpha)\cdot\sin(\theta)$$

$$x' = x\cdot\cos(\theta) - y\cdot\sin(\theta)$$
$$y' = x\cdot\sin(\theta) + y\cdot\cos(\theta)$$

$$(x' \;\; y') = (x \;\; y)\cdot\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$Rx(\theta) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Mirr := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Rz(\theta) := \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Ry(\theta) := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$i := 0..3$ $k := 0..\dfrac{cols(P)}{4}$

$PP_{i,2\cdot j} := P_{i,j}$ $PP_{i,2\cdot j+1} := P_{i,n+1+j}$ $PPP := PP$

$PPP_{i,4\cdot k} := PP_{i,4\cdot k+1}$ $PPP_{i,4\cdot k+1} := PP_{i,4\cdot k}$

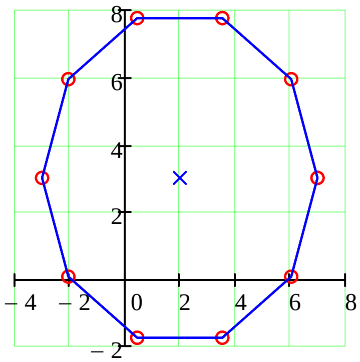
$P := augment(P, PPP)$

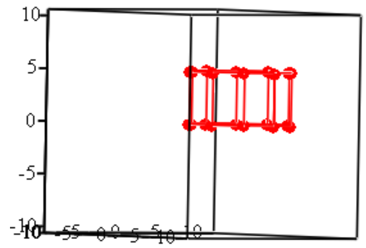
$j := 0..cols(P) - 1$

$p := Rz\left(\dfrac{2\cdot\pi\cdot 2}{10}\right)\cdot P$ $\theta := \dfrac{\pi\cdot FRAME}{20}$

$pss_{0,j} := p_{0,j}$ $pss_{1,j} := p_{1,j}$ $pss_{2,j} := p_{2,j}$

$p := T\cdot Rz(\theta)\cdot T1\cdot P$ $p1 := T\cdot P$

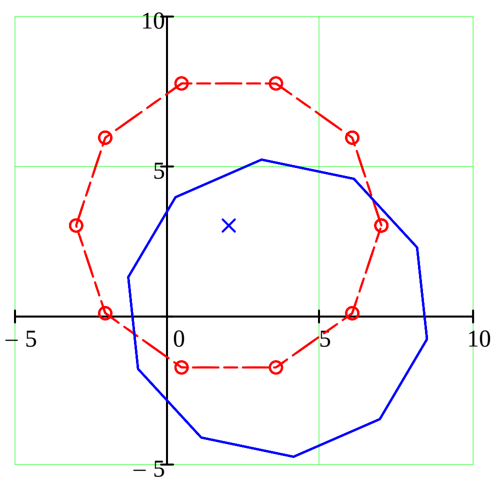




$$\left[\left(p^{T}\right)^{\langle 0\rangle},\left(p^{T}\right)^{\langle 1\rangle},\left(p^{T}\right)^{\langle 2\rangle}\right]$$

Mirror Transformation :

$\theta := 30\mathrm{deg}$ $\qquad$ $p := Rz(\theta)\cdot Mirr\cdot Rz(-\theta)\cdot P$



$$Ry := \begin{pmatrix} 0.7 & 0 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \\ -0.7 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad P0 := \begin{pmatrix} 5.5 \\ 6.5 \\ 5 \\ 1 \end{pmatrix}$$

$$p := T\cdot Ry\cdot T1\cdot P0 \qquad p = \begin{pmatrix} 7.95 \\ 6.5 \\ 1.05 \\ 1 \end{pmatrix}$$

$x_c := 3 \qquad y_c := 3$

Perspective Transformation : $\qquad d_x := 0 \quad d_y := 0 \quad d_z := 10 \qquad j := 0..\,\mathrm{cols}(P) - 1$

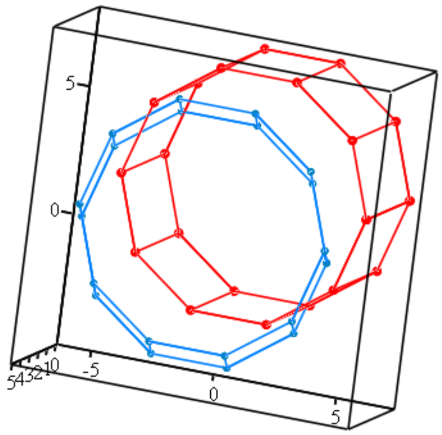
$$T_p := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d_z} & 0 \end{pmatrix}$$

$$p_{x_j} := \left[\left(p^{T}\right)^{\langle 0\rangle}\right]_j \qquad p_{y_j} := \left[\left(p^{T}\right)^{\langle 1\rangle}\right]_j \qquad p_{z_j} := \left[\left(p^{T}\right)^{\langle 2\rangle}\right]_j$$

$$p_{0,j} := \left(p_x{}^{T}\right)_{0,j} \qquad p_{1,j} := \left(p_y{}^{T}\right)_{0,j} \qquad p_{2,j} := \left(p_z{}^{T}\right)_{0,j}$$

$$T := \begin{pmatrix} 1 & 0 & 0 & x_c \\ 0 & 1 & 0 & y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & 0 & -x_c \\ 0 & 1 & 0 & -y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$p := T\cdot T_p\cdot T1\cdot P$

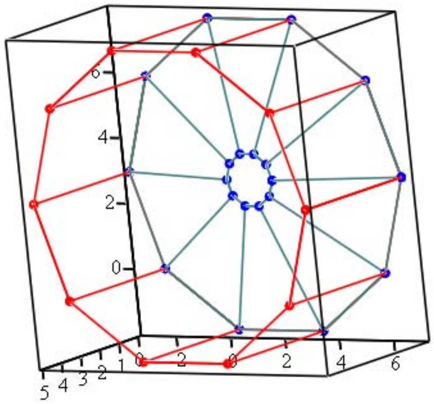


$$\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right],\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right]$$

$$x_c := 2 \quad y_c := 3 \quad T := \begin{pmatrix} 1 & 0 & 0 & x_c \\ 0 & 1 & 0 & y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad T1 := \begin{pmatrix} 1 & 0 & 0 & -x_c \\ 0 & 1 & 0 & -y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad d_x := 0 \quad d_y := 0 \quad d_z := 1$$

$$p := T1\cdot P \qquad p_{x_j} := \frac{\left[\left(p^T\right)^{\langle 0\rangle}\right]_j}{d_z\cdot\left[\left(p^T\right)^{\langle 2\rangle}\right]_j + 1} \quad p_{y_j} := \frac{\left[\left(p^T\right)^{\langle 1\rangle}\right]_j}{d_z\cdot\left[\left(p^T\right)^{\langle 2\rangle}\right]_j + 1} \quad p_{z_j} := \frac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{d_z\cdot\left[\left(p^T\right)^{\langle 2\rangle}\right]_j + 1}$$

$$p_{0,j} := \left(p_x{}^T\right)_{0,j} \qquad p_{1,j} := \left(p_y{}^T\right)_{0,j} \qquad p_{2,j} := \left(p_z{}^T\right)_{0,j} \qquad p := T\cdot p$$



$$\left[\left(P^T\right)^{\langle 0\rangle},\left(P^T\right)^{\langle 1\rangle},\left(P^T\right)^{\langle 2\rangle}\right],\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right]$$

$$x' = x\cdot\cos(\theta) - y\cdot\sin(\theta) \qquad (x' \;\; y') = (x \;\; y)\cdot\begin{pmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{pmatrix}$$

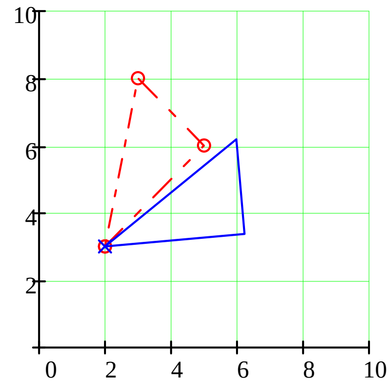
$$y' = x\cdot\sin(\theta) + y\cdot\cos(\theta)$$

Plotting and Transforming a Triangle in 2D

$$Rz(\theta) := \begin{pmatrix}\cos(\theta) & \sin(\theta) & 0\\ -\sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{pmatrix} \qquad T := \begin{pmatrix}1 & 0 & 2\\ 0 & 1 & 3\\ 0 & 0 & 1\end{pmatrix} \qquad T1 := \begin{pmatrix}1 & 0 & -2\\ 0 & 1 & -3\\ 0 & 0 & 1\end{pmatrix} \qquad P := \begin{pmatrix}2 & 5 & 3 & 2\\ 3 & 6 & 8 & 3\\ 1 & 1 & 1 & 1\end{pmatrix}$$

$\theta := 40\text{deg}$ $\qquad p := T\cdot Rz(\theta)\cdot T1\cdot P$ $\qquad p := T\cdot Rz(\theta)\cdot T1\cdot P$

$$p = \begin{pmatrix} 2 & 6.226 & 5.98 & 2 \\ 3 & 3.37 & 6.187 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



$\theta := -30\text{deg}$ $\qquad p := T1 \cdot P$

$$p = \begin{pmatrix} 0 & 3 & 1 & 0 \\ 0 & 3 & 5 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

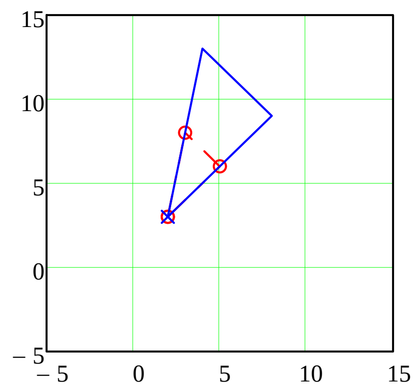
Scaling a Triangle

$$P := \begin{pmatrix} 2 & 5 & 3 & 2 \\ 3 & 6 & 8 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad T := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$Sx := 2$

$Sy := 2$

$$S := \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad p := T \cdot S \cdot T1 \cdot P$$



3D Mirror Transformation :

$$P = a + u \cdot b + v \cdot c$$

1. Plane Drawing :

$m := 2 \qquad i := 0..m \qquad u_i := \frac{i}{m} \qquad n := 2 \qquad j := 0..n \qquad v_j := \frac{j}{n}$

$$P_0 := \begin{pmatrix} 2 \\ 0 \\ 19 \end{pmatrix} \quad P_1 := \begin{pmatrix} 1 \\ 10 \\ 15 \end{pmatrix} \quad P_2 := \begin{pmatrix} 8 \\ 6 \\ 20 \end{pmatrix} \qquad n := \frac{(P_1 - P_0) \times (P_2 - P_0)}{\left|(P_1 - P_0) \times (P_2 - P_0)\right|} \qquad n = \begin{pmatrix} 0.437 \\ -0.296 \\ -0.849 \end{pmatrix} \qquad ns := n$$

$$P_2 := P_0 + n \times (P_1 - P_0) \qquad P_3 := P_2 + (P_1 - P_0)$$

$$P_{i,j} := P_0 + u_i \cdot (P_1 - P_0) + v_j \cdot (P_2 - P_0) \qquad x_{i,j} := (P_{i,j})_0 \quad y_{i,j} := (P_{i,j})_1 \quad z_{i,j} := (P_{i,j})_2$$

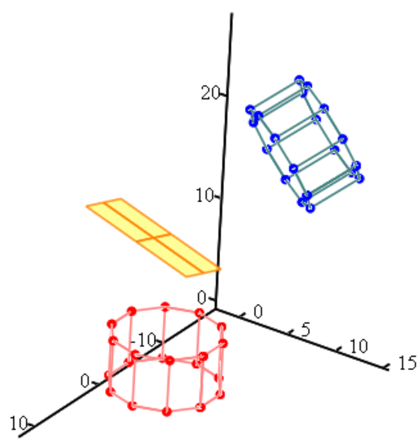
2. 3D Part Drawing :

$$pr := \begin{pmatrix} 7 & 2 & -3 & 2 & 7 & 2 & 2 & -3 & 2 & 2 \\ 3 & 8 & 3 & -2 & 3 & 3 & 8 & 3 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 5 & 0 \end{pmatrix} \quad pr := pss \qquad i := 0 .. \text{cols}(pr) - 1 \qquad j := 0 .. \text{rows}(pr) - 2$$

$$prs_{j,i} := pr_{j,i}$$

3. Mirror of part with respect to plane :

$$p^{\langle i \rangle} := pr^{\langle i \rangle} + 2 \cdot n \cdot \left[\left(P_0 - pr^{\langle i \rangle} \right) \cdot n \right]$$



$$\left[\left(pr^T\right)^{\langle 0 \rangle}, \left(pr^T\right)^{\langle 1 \rangle}, \left(pr^T\right)^{\langle 2 \rangle} \right], \left[\left(p^T\right)^{\langle 0 \rangle}, \left(p^T\right)^{\langle 1 \rangle}, \left(p^T\right)^{\langle 2 \rangle} \right], (x, y, z)$$

3. 3D Mirror by using mirror transformation matrix :

$$n = \begin{pmatrix} 0.437 \\ -0.296 \\ -0.849 \end{pmatrix}$$

$$\theta_x := \text{atan}\left[\frac{n_1}{\sqrt{(n_0)^2 + (n_2)^2}} \right] \qquad \theta_x = -17.213 \cdot \text{deg} \qquad \theta_y := \text{atan}\left(\frac{n_0}{n_2} \right) \qquad \theta_y = -27.255 \cdot \text{deg}$$

$$\text{Mirr} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Rx(\theta) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Rz(\theta) := \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T := \begin{pmatrix} 1 & 0 & 0 & P0_0 \\ 0 & 1 & 0 & P0_1 \\ 0 & 0 & 1 & P0_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad T1 := \begin{pmatrix} 1 & 0 & 0 & -P0_0 \\ 0 & 1 & 0 & -P0_1 \\ 0 & 0 & 1 & -P0_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad Ry(\theta) := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$n := \frac{P_1 - P_0}{\left|P_1 - P_0\right|} \quad n = \begin{pmatrix} -0.092 \\ 0.925 \\ -0.37 \end{pmatrix} \quad \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} := n \quad |n| = 1$$

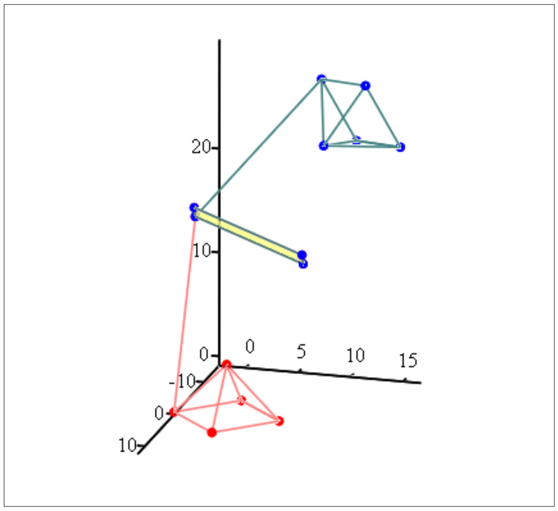
$$pr := \begin{pmatrix} 7 & 2 & -3 & 2 & 7 & 2 & 2 & -3 & 2 & 2 & P0_0 & P1_0 & P3_0 & P2_0 & P0_0 \\ 3 & 8 & 3 & -2 & 3 & 3 & 8 & 3 & 3 & -2 & P0_1 & P1_1 & P3_1 & P2_1 & P0_1 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 5 & 0 & P0_2 & P1_2 & P3_2 & P2_2 & P0_2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$c(\theta) \equiv \cos(\theta)$$

$$s(\theta) \equiv \sin(\theta)$$

$$R(\theta) := \begin{bmatrix} n_x{}^2\cdot(1 - c(\theta)) + c(\theta) & n_x\cdot n_y\cdot(1 - c(\theta)) - n_z\cdot s(\theta) & n_x\cdot n_z\cdot(1 - c(\theta)) + n_y\cdot s(\theta) & 0 \\ n_x\cdot n_y\cdot(1 - c(\theta)) + n_z\cdot s(\theta) & n_y{}^2\cdot(1 - c(\theta)) + c(\theta) & n_y\cdot n_z\cdot(1 - c(\theta)) - n_x\cdot s(\theta) & 0 \\ n_x\cdot n_z\cdot(1 - c(\theta)) - n_y\cdot s(\theta) & n_y\cdot n_z\cdot(1 - c(\theta)) + n_x\cdot s(\theta) & n_z{}^2\cdot(1 - c(\theta)) + c(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

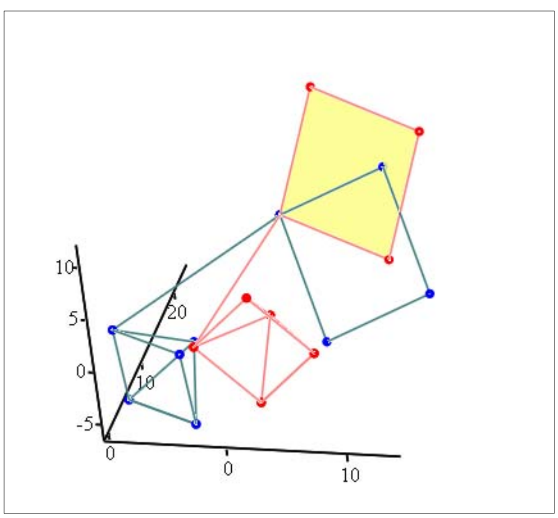
$$R(\theta) = \begin{pmatrix} 0.867 & -0.196 & -0.458 & 0 \\ 0.173 & 0.981 & -0.092 & 0 \\ 0.467 & 4.217 \times 10^{-4} & 0.884 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad p := T\cdot\left(Ry\left(\theta_y\right)\cdot Rx\left(\theta_x\right)\right)\cdot Mirr\cdot Rx\left(-\theta_x\right)\cdot Ry\left(-\theta_y\right)\cdot T1\cdot pr$$



$$\left[\left({pr}^T\right)^{\langle 0\rangle},\left({pr}^T\right)^{\langle 1\rangle},\left({pr}^T\right)^{\langle 2\rangle}\right],\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right],(x,y,z)$$

$\theta_z := 50deg$

$p := T\cdot\left(Ry\left(\theta_y\right)\cdot Rx\left(\theta_x\right)\right)\cdot Rz\left(\theta_z\right)\cdot Rx\left(-\theta_x\right)\cdot Ry\left(-\theta_y\right)\cdot T1\cdot pr$



$$\left[\left({pr}^T\right)^{\langle 0\rangle},\left({pr}^T\right)^{\langle 1\rangle},\left({pr}^T\right)^{\langle 2\rangle}\right],\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right],(x,y,z)$$