Plotting and Transforming a Circle in 3D

$x_c := 2$ $\quad r := 5$ $\quad x(t) := r\cdot\cos(t) + x_c$ $\quad s := 4$ $\quad n := 4$ $\quad j := 0..n$ $\quad t_j := \dfrac{j\cdot 2\cdot\pi}{n}$

$y_c := 3$ $\quad y(t) := r\cdot\sin(t) + y_c$

$z(t) := 0$

$$T := \begin{pmatrix} 1 & 0 & 0 & x_c \\ 0 & 1 & 0 & y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & 0 & -x_c \\ 0 & 1 & 0 & -y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$P_{0,j} := x(t_j)$ $\quad P_{0,n+j+1} := x(t_j)$

$P_{1,j} := y(t_j)$ $\quad P_{1,n+j+1} := y(t_j)$

$P_{2,j} := z(t_j)$ $\quad P_{2,n+j+1} := z(t_j) + 5$

$P_{3,j} := 1$ $\quad P_{3,n+j+1} := 1$

$$x = r\cdot\cos(\theta)$$

$$y = r\cdot\sin(\theta)$$

$$x' = r\cdot\cos(\theta+\alpha) = r\cdot\cos(\alpha)\cdot\cos(\theta) - r\cdot\sin(\alpha)\cdot\sin(\theta)$$

$$y' = r\cdot\sin(\theta+\alpha) = r\cdot\sin(\alpha)\cdot\cos(\theta) + r\cdot\cos(\alpha)\cdot\sin(\theta)$$

$$x' = x\cdot\cos(\theta) - y\cdot\sin(\theta)$$

$$y' = x\cdot\sin(\theta) + y\cdot\cos(\theta)$$

$$(x' \;\; y') = (x \;\; y)\cdot\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$Rx(\theta) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Mirr := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Rz(\theta) := \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$Ry(\theta) := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$i := 0..3 \qquad k := 0..\frac{cols(P)}{4}$

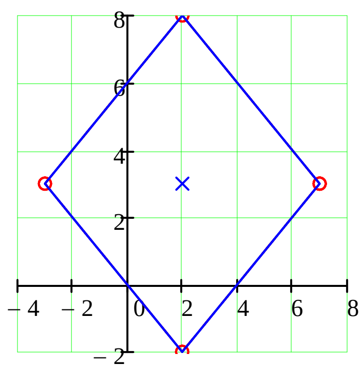
$PP_{i,2\cdot j} := P_{i,j} \qquad PP_{i,2\cdot j+1} := P_{i,n+1+j} \qquad PPP := PP$

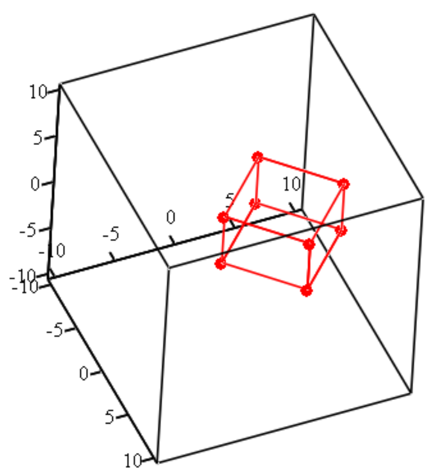
$PPP_{i,4\cdot k} := PP_{i,4\cdot k+1} \qquad PPP_{i,4\cdot k+1} := PP_{i,4\cdot k}$

$P := augment(P, PPP)$

$p := Rz\left(\frac{2\cdot\pi\cdot 2}{10}\right)\cdot P \qquad \theta := \frac{\pi\cdot FRAME}{20}$

$p := T\cdot Rz(\theta)\cdot T1\cdot P \qquad p1 := T\cdot P$





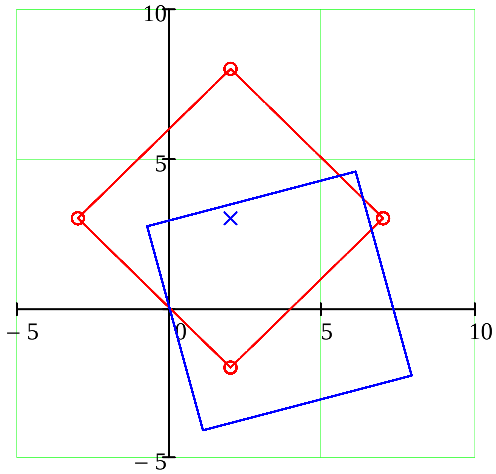
$\left[\left(p^T\right)^{\langle 0\rangle}, \left(p^T\right)^{\langle 1\rangle}, \left(p^T\right)^{\langle 2\rangle}\right]$

$$Ry := \begin{pmatrix} 0.7 & 0 & 0.7 & 0 \\ 0 & 1 & 0 & 0 \\ -0.7 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P0 := \begin{pmatrix} 5.5 \\ 6.5 \\ 5 \\ 1 \end{pmatrix} \qquad p := T\cdot Ry\cdot T1\cdot P0 \qquad p = \begin{pmatrix} 7.95 \\ 6.5 \\ 1.05 \\ 1 \end{pmatrix}$$

Mirror Transformation :

$\theta := 30deg$ $\quad\quad p := Rz(\theta)\cdot Mirr\cdot Rz(-\theta)\cdot P$



$x_c := 2 \quad y_c := 3$

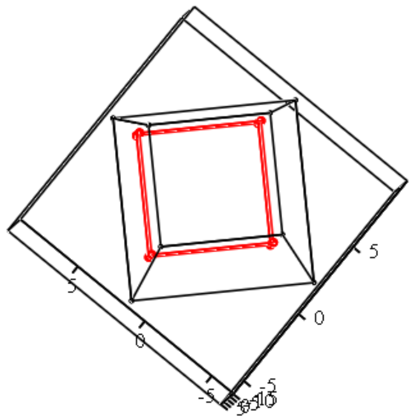
Perspective Transformation : $\quad d_x := 0 \quad d_y := 0 \quad d_z := -15 \quad j := 0 .. cols(P) - 1$

$$T_p := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{d_z} & 0 \end{pmatrix} \quad T1 := \begin{pmatrix} 1 & 0 & 0 & -x_c \\ 0 & 1 & 0 & -y_c \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad T := \begin{pmatrix} 1 & 0 & 0 & x_c \\ 0 & 1 & 0 & y_c \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p := T_p\cdot T1\cdot P \quad p_{x_j} := \frac{\left[\left(p^T\right)^{\langle 0\rangle}\right]_j}{\frac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{d_z}} \quad p_{y_j} := \frac{\left[\left(p^T\right)^{\langle 1\rangle}\right]_j}{\frac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{d_z}} \quad p_{z_j} := \frac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{\frac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{d_z}}$$

$$p_{0,j} := \left(p_x{}^T\right)_{0,j} \quad p_{1,j} := \left(p_y{}^T\right)_{0,j} \quad p := T\cdot p \quad p_{2,j} := \left[\left(P^T\right)^{\langle 2\rangle}\right]_j$$

$$p_{2,j} := \left(p_z{}^T\right)_{0,j}$$



$$\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right],\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right]$$

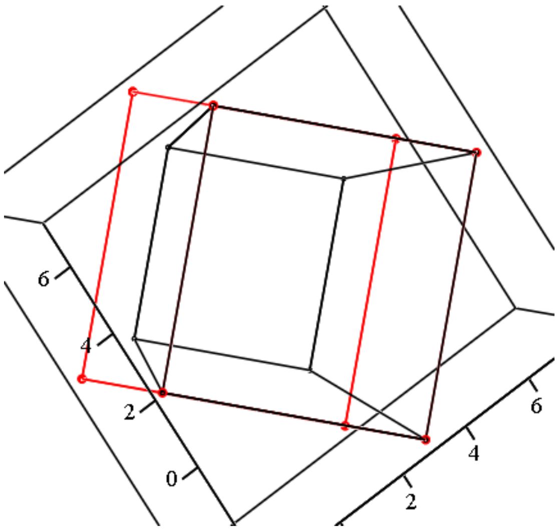
$x_c := 2$ $\quad y_c := 3$ $\quad T := \begin{pmatrix} 1 & 0 & 0 & x_c \\ 0 & 1 & 0 & y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\quad T1 := \begin{pmatrix} 1 & 0 & 0 & -x_c \\ 0 & 1 & 0 & -y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$d_x := 0$ $\quad d_y := 0$ $\quad d_z := 10$

$p := T1\cdot P$ $\quad p_{x_j} := \dfrac{\left[\left(p^T\right)^{\langle 0\rangle}\right]_j}{\dfrac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{d_z}+1}$ $\quad p_{y_j} := \dfrac{\left[\left(p^T\right)^{\langle 1\rangle}\right]_j}{\dfrac{\left[\left(p^T\right)^{\langle 2\rangle}\right]_j}{d_z}+1}$

$p_{0,j} := \left(p_x{}^T\right)_{0,j}$

$p_{1,j} := \left(p_y{}^T\right)_{0,j}$ $\quad p := T\cdot p$



$$\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right],\left[\left(p^T\right)^{\langle 0\rangle},\left(p^T\right)^{\langle 1\rangle},\left(p^T\right)^{\langle 2\rangle}\right]$$

$$x' = x\cdot\cos(\theta) - y\cdot\sin(\theta)$$

$$y' = x\cdot\sin(\theta) + y\cdot\cos(\theta)$$

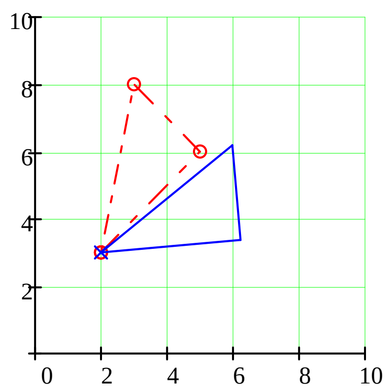
$$(x' \quad y') = (x \quad y)\cdot\begin{pmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{pmatrix}$$

Plotting and Transforming a Triangle in 2D

$$Rz(\theta) := \begin{pmatrix}\cos(\theta) & \sin(\theta) & 0\\ -\sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{pmatrix} \qquad T := \begin{pmatrix}1 & 0 & 2\\ 0 & 1 & 3\\ 0 & 0 & 1\end{pmatrix} \qquad T1 := \begin{pmatrix}1 & 0 & -2\\ 0 & 1 & -3\\ 0 & 0 & 1\end{pmatrix} \qquad P := \begin{pmatrix}2 & 5 & 3 & 2\\ 3 & 6 & 8 & 3\\ 1 & 1 & 1 & 1\end{pmatrix}$$

$\theta := 40\text{deg}$ $\qquad p := T\cdot Rz(\theta)\cdot T1\cdot P$ $\qquad p := T\cdot Rz(\theta)\cdot T1\cdot P$

$$p = \begin{pmatrix}2 & 6.226 & 5.98 & 2\\ 3 & 3.37 & 6.187 & 3\\ 1 & 1 & 1 & 1\end{pmatrix}$$



$\theta := -30\text{deg}$ $\qquad p := T1\cdot P$

$$p = \begin{pmatrix}0 & 3 & 1 & 0\\ 0 & 3 & 5 & 0\\ 1 & 1 & 1 & 1\end{pmatrix}$$

Scaling a Triangle

$$P := \begin{pmatrix} 2 & 5 & 3 & 2 \\ 3 & 6 & 8 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \qquad T := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$Sx := 2$

$Sy := 2$

$$S := \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad p := T \cdot S \cdot T1 \cdot P$$