BLG 540E TEXT RETRIEVAL SYSTEMS

Term Weighting, Scoring and the Vector Space Model

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Ranked retrieval

- > Thus far, our queries have all been Boolean.
 - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
 - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
 - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
 - Most users don't want to wade through 1000s of results.
 - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query I: "flights from istanbul" \rightarrow 900,000 hits
- Query 2: "flights from istanbul to narita": I hit
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
 - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval models, the system returns an ordering over the (top) documents in the collection with respect to a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval models have normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
 - Indeed, the size of the result set is not an issue
 - We just show the top $k \ (\approx 10)$ results
 - We don't overwhelm the user
 - Premise: the ranking algorithm works

Importance of ranking:

- Viewing abstracts: Users are a lot more likely to read the abstracts of the top-ranked pages (1, 2, 3, 4) than the abstracts of the lower ranked pages (7, 8, 9, 10).
- Clicking: Distribution is even more skewed for clicking
- In I out of 2 cases, users click on the top-ranked page.
- Even if the top-ranked page is not relevant, 30% of users will click on it.
- Getting the ranking right is very important.
- Getting the top-ranked page right is most important.

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score say in [0, 1] to each document
- This score measures how well document and query "match".

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this.

Jaccard coefficient

- Recall from Lecture 3: A commonly used measure of overlap of two sets A and B
- ▶ jaccard(A,B) = $|A \cap B| / |A \cup B|$
- jaccard(A,A) = I
- ▶ jaccard(A,B) = 0 if $A \cap B = 0$
- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: ides of march
- Document I: caesar died in march
- Document 2: the long march

Issues with Jaccard for scoring

- It doesn't consider term frequency (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms. Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length

Recall (Lecture 1): Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
 - Each document is a count vector in \mathbb{N}^{v} : a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- John is quicker than Mary and Mary is quicker than John have the same vectors
- This is called the <u>bag of words</u> model.

Term frequency tf

- The term frequency tf_{t,d} of term t in document d is defined as the number of times that t occurs in d.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
 - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
 - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

Log-frequency weighting

The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} tf_{t,d}, & \text{if } tf_{t,d} > 0\\ 0, & \text{otherwise} \end{cases}$$

- ▶ 0 → 0, I → I, 2 → I.3, I0 → 2, I000 → 4, etc.
- Score for a document-query pair: sum over terms t in both q and d:

score =
$$\sum_{t \in q \cap d} (1 + \log tf_{t,d})$$

• The score is 0 if none of the query terms is present in the document.

Document frequency

Rare terms are more informative than frequent terms

- Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- \rightarrow We want a high weight for rare terms like *arachnocentric*.

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., high, increase, line)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
- ► → For frequent terms, we want high positive weights for words like high, increase, and line
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

idf weight

- df_t is the <u>document</u> frequency of t: the number of documents that contain t
 - df_t is an inverse measure of the informativeness of t
 - $df_t \leq N$
- We define the idf (inverse document frequency) of t by

$$\operatorname{idf}_{t} = \log_{10} \left(\frac{N}{df}_{t} \right)$$

We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

Will turn out the base of the log is immaterial.

idf example, suppose N = 1 million

term	df _t	idf _t
calpurnia	1	6
animal	100	4
sunday	1,000	4
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\operatorname{idf}_{t} = \log_{10} \left(\frac{N}{df}_{t} \right)$$

There is one idf value for each term *t* in a collection.

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
 - iPhone
- idf has no effect on ranking one term queries
 - idf affects the ranking of documents for queries with at least two terms
 - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

Collection vs. Document frequency

- The collection frequency of t is the number of occurrences of t in the collection, counting multiple occurrences.
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

Which word is a better search term (and should get a higher weight)?

Sec. 6.2.2

tf-idf weighting

The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$\mathbf{w}_{t,d} = (1 + \log tf_{t,d}) \times \log_{10}(N/df_t)$$

- Best known weighting scheme in information retrieval
 - Note: the "-" in tf-idf is a hyphen, not a minus sign!
 - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Sec. 6.2.2

Final ranking of documents for a query

Score(q,d) = $\sum_{t \in q \cap d} \text{tf.idf}_{t,d}$

Binary \rightarrow count \rightarrow weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Sec. 6.3

Documents as vectors

- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors most entries are zero.

The Vector-space model

D



Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- ▶ proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

Formalizing vector space proximity

- First cut: distance between two points
 - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea ...
- ... because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea



 \vec{q} and \vec{d}_2 is large although the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.

Use angle instead of distance

- Thought experiment: take a document d and append it to itself. Call this document d'.
- "Semantically" d and d' have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.
- Key idea: Rank documents according to angle with query.

From angles to cosines

- > The following two notions are equivalent.
 - Rank documents in <u>decreasing</u> order of the angle between query and document
 - Rank documents in <u>increasing</u> order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0°, 180°]

From angles to cosines



But how – and why – should we be computing cosines?

Length normalization

A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L₂ norm:

$$\left\|\vec{x}\right\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L₂ norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
 - Long and short documents now have comparable weights

cosine(query,document)



 q_i is the tf-idf weight of term *i* in the query d_i is the tf-idf weight of term *i* in the document

 $\cos(\vec{q}, \vec{d})$ is the cosine similarity of \vec{q} and \vec{d} ... or, equivalently, the cosine of the angle between \vec{q} and \vec{d} .

Cosine for length-normalized vectors

For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q}, \vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized
Cosine similarity illustrated



Cosine similarity amongst 3 documents

- How similar are
- the novels
- SaS: Sense and
- Sensibility (Jane Austen)
- PaP: Pride and
- Prejudice (Jane Austen), and
- WH: Wuthering
- Heights? (Emily Bronte)

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

After length normalization

term	SaS	PaP	WH	term	SaS	PaP	WH
affection	3.06	2.76	2.30	affection	0.789	0.832	0.524
jealous	2.00	1.85	2.04	jealous	0.515	0.555	0.465
gossip	1.30	0	1.78	gossip	0.335	0	0.405
wuthering	0	0	2.58	wuthering	0	0	0.588

 $cos(SaS,PaP) \approx$ 0.789 × 0.832 + 0.515 × 0.555 + 0.335 × 0.0 + 0.0 × 0.0 \approx 0.94 $cos(SaS,WH) \approx$ 0.79 $cos(PaP,WH) \approx$ 0.69 Computing cosine scores

COSINESCORE(q)

- 1 float Scores[N] = 0
- 2 float Length[N]
- 3 for each query term t
- 4 **do** calculate $w_{t,q}$ and fetch postings list for t
- 5 **for each** $pair(d, tf_{t,d})$ in postings list
- 6 **do** Scores[d]+ = $w_{t,d} \times w_{t,q}$
- 7 Read the array Length
- 8 for each d
- 9 **do** Scores[d] = Scores[d]/Length[d]
- 10 return Top K components of Scores[]

tf-idf weighting has many variants

Term frequency		Docum	ent frequency	Normalization			
n (natural)	tf _{t,d}	n (no)	1	n (none)	1		
l (logarithm)	$1 + \log(tf_{t,d})$	t (idf)	$\log \frac{N}{df_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$		
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	1/u		
b (boolean)	$egin{cases} 1 & ext{if } \operatorname{tf}_{t,d} > 0 \ 0 & ext{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^lpha$, $lpha < 1$		
L (log ave)	$\frac{1 + \log(\operatorname{tf}_{t,d})}{1 + \log(\operatorname{ave}_{t \in d}(\operatorname{tf}_{t,d}))}$						

Columns headed 'n' are acronyms for weight schemes.

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation ddd.qqq, using the acronyms from the previous table
- A very standard weighting scheme is: Inc.Itc
- Document: logarithmic tf (I as first character), no idf and cosine normalization
- Query: logarithmic tf (l in leftmost column), idf (t in second column), cosine normalization ...

tf-idf example: lnc.ltc

Document: *car insurance auto insurance* Query: *best car insurance*

Term	Query					Document				Pro d	
	tf- raw	tf-wt	df	idf	wt	n'liz e	tf-raw	tf-wt	wt	n'liz e	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is *N*, the number of docs?

Doc length = $\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., K = 10) to the user

Computing Scores in a Complete Search System

Outline

- Speeding up vector space ranking
- Putting together a complete search system
 - Will require learning about a number of miscellaneous topics and heuristics

Efficient cosine ranking

- Find the K docs in the collection "nearest" to the query \Rightarrow K largest query-doc cosines.
- Efficient ranking:
 - Computing a single cosine efficiently.
 - Choosing the K largest cosine values efficiently.
 - Can we do this without computing all N cosines?

Efficient cosine ranking

- What we're doing in effect: solving the K-nearest neighbor problem for a query vector
- In general, we do not know how to do this efficiently for high-dimensional spaces
- But it is solvable for short queries, and standard indexes support this well

Special case – unweighted queries

- No weighting on query terms
 - Assume each query term occurs only once
- Then for ranking, don't need to normalize query vector

Faster cosine: unweighted query

FASTCOSINESCORE(q)

- 1 float Scores[N] = 0
- 2 for each *d*
- 3 **do** Initialize *Length*[*d*] to the length of doc *d*
- 4 for each query term t
- 5 **do** calculate $w_{t,q}$ and fetch postings list for *t*
- 6 **for each** $pair(d, tf_{t,d})$ in postings list
- 7 **do** add $wf_{t,d}$ to *Scores*[*d*]
- 8 Read the array *Length*[*d*]
- 9 **for each** *d*

- 10 **do** Divide *Scores*[*d*] by *Length*[*d*]
- 11 **return** Top *K* components of *Scores*[]

Figure 7.1 A faster algorithm for vector space scores.

Computing the *K* largest cosines: selection vs. sorting

- Typically we want to retrieve the top K docs (in the cosine ranking for the query)
 - not to totally order all docs in the collection
- Can we pick off docs with K highest cosines?
- Let J = number of docs with nonzero cosines
 - We seek the K best of these J

Use heap for selecting top *K*

- Binary tree in which each node's value > the values of children
- Takes 2J operations to construct, then each of K "winners" read off in 2log J steps.
- ▶ For J=1M, K=100, this is about 10% of the cost of sorting.



Bottlenecks

- Primary computational bottleneck in scoring: <u>cosine</u> <u>computation</u>
- Can we avoid all this computation?
- Yes, but may sometimes get it wrong
 - a doc not in the top K may creep into the list of K output docs
 - Is this such a bad thing?

Cosine similarity is only a proxy

- User has a task and a query formulation
- Cosine matches docs to query
- Thus cosine is anyway a proxy for user happiness
- If we get a list of K docs "close" to the top K by cosine measure, should be ok

Generic approach

- Find a set A of contenders, with K < |A| << N
 - A does not necessarily contain the top K, but has many docs from among the top K
 - Return the top *K* docs in *A*
- Think of A as <u>pruning</u> non-contenders
- The same approach is also used for other (non-cosine) scoring functions
- Will look at several schemes following this approach

Index elimination

- Basic algorithm FastCosineScore only considers docs containing at least one query term
- Take this further:
 - Only consider high-idf query terms
 - Only consider docs containing many query terms

High-idf query terms only

- For a query such as catcher in the rye
- Only accumulate scores from catcher and rye
- Intuition: in and the contribute little to the scores and so don't alter rank-ordering much
- Benefit:
 - ▶ Postings of low-idf terms have many docs → these (many) docs get eliminated from set A of contenders

Docs containing many query terms

- Any doc with at least one query term is a candidate for the top K output list
- For multi-term queries, only compute scores for docs containing several of the query terms
 - Say, at least 3 out of 4
- Easy to implement in postings traversal

3 of 4 query terms



Scores only computed for docs 8, 16 and 32.

Champion lists

- Precompute for each dictionary term t, the r docs of highest weight in t's postings
 - Call this the <u>champion list</u> for t
 - (aka <u>fancy list</u> or <u>top docs</u> for *t*)
- Note that r has to be chosen at index build time
 - Thus, it's possible that r < K
- At query time, only compute scores for docs in the champion list of some query term
 - Pick the K top-scoring docs from amongst these

Static quality scores

- We want top-ranking documents to be both relevant and authoritative
- Relevance is being modeled by cosine scores
- Authority is typically a query-independent property of a document
- Examples of authority signals
 - Wikipedia among websites
 - Articles in certain newspapers
 - A paper with many citations
 - (Pagerank)

Modeling authority

- Assign to each document a query-independent <u>quality</u> <u>score</u> in [0,1] to each document d
 - Denote this by g(d)
- Thus, a quantity like the number of citations is scaled into [0,1]

Net score

- Consider a simple total score combining cosine relevance and authority
- net-score(q,d) = g(d) + cosine(q,d)
 - Can use some other linear combination than an equal weighting
 - Indeed, any function of the two "signals" of user happiness more later
- Now we seek the top K docs by <u>net score</u>

Top *K* by net score – fast methods

- First idea: Order all postings by g(d)
- Key: this is a common ordering for all postings
- Thus, can concurrently traverse query terms' postings for
 - Postings intersection
 - Cosine score computation

Why order postings by *g*(*d*)?

- Under g(d)-ordering, top-scoring docs likely to appear early in postings traversal
- In time-bound applications (say, we have to return whatever search results we can in 50 ms), this allows us to stop postings traversal early
 - Short of computing scores for all docs in postings

High and low lists

- For each term, we maintain two postings lists called high and low
 - Think of high as the champion list
- When traversing postings on a query, only traverse high lists first
 - If we get more than K docs, select the top K and stop
 - Else proceed to get docs from the *low* lists
- Can be used even for simple cosine scores, without global quality g(d)
- A means for segmenting index into two <u>tiers</u>

Impact-ordered postings

- We only want to compute scores for docs for which wf_{t,d} is high enough
- We sort each postings list by wf_{td}
- Now: not all postings in a common order!
- How do we compute scores in order to pick off top *K*?
 - Two ideas follow

1. Early termination

- When traversing t's postings, stop early after either
 - a fixed number of *r* docs
 - $wf_{t,d}$ drops below some threshold
- Take the union of the resulting sets of docs
 - One from the postings of each query term
- Compute only the scores for docs in this union

2. idf-ordered terms

- When considering the postings of query terms
- Look at them in order of decreasing idf
 - High idf terms likely to contribute most to score
- As we update score contribution from each query term
 - Stop if doc scores relatively unchanged
- Can apply to cosine or some other net scores

Cluster pruning: preprocessing

- Pick \sqrt{N} docs at random: call these *leaders*
- For every other doc, pre-compute nearest leader
 - Docs attached to a leader: its followers;
 - Likely: each leader has ~ \sqrt{N} followers.

Cluster pruning: query processing

Process a query as follows:

- Given query Q, find its nearest leader L.
- Seek K nearest docs from among L's followers.

Sec. 7.1.6

Visualization

D


Why use random sampling

Fast

Leaders reflect data distribution

General variants

- Have each follower attached to b1=3 (say) nearest leaders.
- From query, find b2=4 (say) nearest leaders and their followers.

Parametric and zone indexes

- Thus far, a doc has been a sequence of terms
- In fact documents have multiple parts, some with special semantics:
 - Author
 - Title
 - Date of publication
 - Language
 - Format
 - etc.

These constitute the <u>metadata</u> about a document

Fields

We sometimes wish to search by these metadata

- E.g., find docs authored by William Shakespeare in the year 1601, containing *alas poor Yorick*
- Year = 1601 is an example of a <u>field</u>
- Also, author last name = shakespeare, etc
- Field or parametric index: postings for each field value
 - Sometimes build range trees (e.g., for dates)
- Field query typically treated as conjunction
 - (doc must be authored by shakespeare)

Zone

- A <u>zone</u> is a region of the doc that can contain an arbitrary amount of text e.g.,
 - Title
 - Abstract
 - References ...
- Build inverted indexes on zones as well to permit querying
- E.g., "find docs with *merchant* in the title zone and matching the query gentle rain"

Example zone indexes



Tiered indexes

Break postings up into a hierarchy of lists

- Most important
- •
- Least important
- Can be done by g(d) or another measure
- Inverted index thus broken up into <u>tiers</u> of decreasing importance
- > At query time use top tier unless it fails to yield K docs
 - If so drop to lower tiers

Example tiered index



Query term proximity

- Free text queries: just a set of terms typed into the query box – common on the web
- Users prefer docs in which query terms occur within close proximity of each other
- Let w be the smallest window in a doc containing all query terms, e.g.,
- For the query strained mercy the smallest window in the doc The quality of mercy is not strained is <u>4</u> (words)
- Would like scoring function to take this into.

Query parsers

- Free text query from user may in fact spawn one or more queries to the indexes, e.g. query rising interest rates
 - Run the query as a phrase query
 - If <K docs contain the phrase rising interest rates, run the two phrase queries rising interest and interest rates
 - If we still have <K docs, run the vector space query rising interest rates
 - Rank matching docs by vector space scoring
- This sequence is issued by a <u>query parser</u>

Aggregate scores

- We've seen that score functions can combine cosine, static quality, proximity, etc.
- How do we know the best combination?
- Some applications expert-tuned
- Increasingly common: machine-learned

Putting it all together



References

- Introduction to Information Retrieval, chapters 6 & 7.
- The slides were adapted from
 - the book's companion website:
 - http://nlp.stanford.edu/IR-book/information-retrieval-book.html