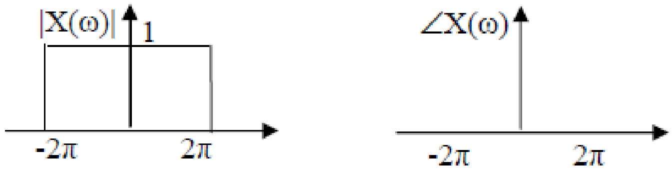
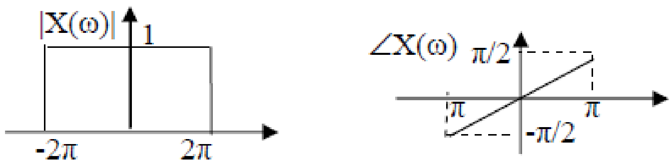
MKM 501 E – HW3

1) Find and graph the inverse transforms of the following four pairs of magnitude and phase responses:

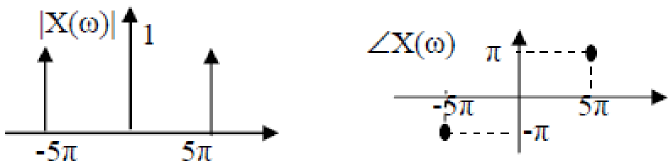
a.



b.



c.



2) Using the multiplication-convolution duality of the Fourier Transform, find an expression for y(t) that does not use the convolution operator *:

a. $y(t) = \mathrm{rect}(t) * \cos(\pi t)$

b. $\mathcal{F}[y(t)] = \mathrm{sinc}^2(\omega/2)$

3. Look at the graphs of the signals in Figure 1. Can you say whether or not they are low-pass, high-pass or band-pass signals? Reason your answer.

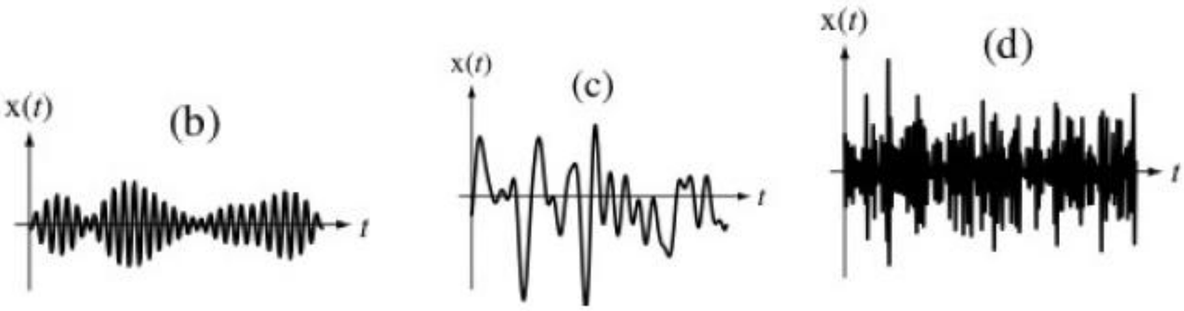


Figure 1:

4. Find the Fourier transform of the following signals, using the tables and FT properties:

(a) For $x(t) = \left[2e^{(-1+j2\pi)t} + 2e^{(-1-j2\pi)t}\right] u(t)$, find $X(\omega)$.

(b) For $x(t) = 1.6\mathrm{sinc}^2(-t)\sin(4\pi t)$, find $X(f)$

(c) For $x(t) = e^{-t/4}u(t) * \sin(2\pi t)$, find $X(f)$

(d) For $x(t) = \frac{d}{dt}(\mathrm{sinc}(t))$, find $X(\omega)$

(e) For $x(t) = \int_{-\infty}^{t} \mathrm{rect}(\lambda)d\lambda$, find $X(\omega)$.

5. Plot the magnitude and phase spectrum of the following signals (you have to use FT tables and the FT properties in order to compute $X(\omega)$):

   (a) $x(t) = \text{sgn}(t) - \text{sgn}(-t)$, where $\text{sgn}(t)$ is the sign function; that is, $\text{sgn}(t) = 1$, if $t > 0$, $\text{sgn}(t) = -1$, for $t < 0$, and $\text{sgn}(0) = 0$.

   (b) $x(t) = \int_{-\infty}^{t} \sin(2\pi\lambda) d\lambda$. *Hint:* Recall that $\delta(s)$ is a real signal that is zero for $s \neq 0$.

6. Compute the Impulse Response of each of the systems with the Frequency Response given next. Classify each Impulse Response as a low-pass, high-pass or band-pass signal.

   (a) $H(j\omega) = 10\left[\delta(\omega - \frac{1}{2}) + \delta(\omega + \frac{1}{2})\right]$

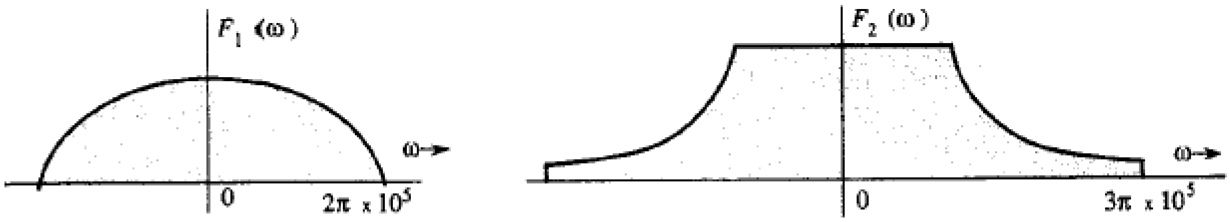
   (b) $H(j\omega) = -2\pi\delta(\omega) * j\pi\left(\delta(\omega - 300) - \delta(\omega + 300)\right)$

   (c) $H(j\omega) = 3\text{sinc}(f)\text{e}^{-j2\pi f}$

7. (a) Suppose a system Impulse Response is given by $h(t) = 7(u(t) - u(t - 20))$. What is the FT of the system response to the signal $x(t) = \text{e}^{-20t}u(t)$?

   (b) Suppose a system Impulse Response is given by $h(t) = \text{e}^{-20t}u(t)$. What would be the input $x(t)$ we need to apply in order to obtain the output $y(t) = 5(\text{e}^{10t} - \text{e}^{-20t})u(t)$?

8) Figure P5.1-1 shows Fourier spectra of signals $f_1(t)$ and $f_2(t)$. Determine the Nyquist sampling rates for signals $f_1(t), f_2(t), f_1^2(t), f_2^3(t)$, and $f_1(t)f_2(t)$.



9) Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals **(a)** $\text{sinc}^2(100\pi t)$ **(b)** $0.01\,\text{sinc}^2(100\pi t)$ **(c)** $\text{sinc}(100\pi t) + 3\,\text{sinc}^2(60\pi t)$ **(d)** $\text{sinc}(50\pi t)\text{sinc}(100\pi t)$.

10) A signal $f(t) = \text{sinc}(200\pi t)$ is sampled (using uniformly spaced impulses) at a rate of (a) 150 Hz (b) 200 Hz (c) 300 Hz. For each of the three cases (i) sketch the spectrum of the sampled signal, (ii) explain if you can recover the signal $f(t)$ from the sampled signal, (iii) if the sampled signal is passed through an ideal lowpass filter of bandwidth 100 Hz, sketch the spectrum of the output signal.