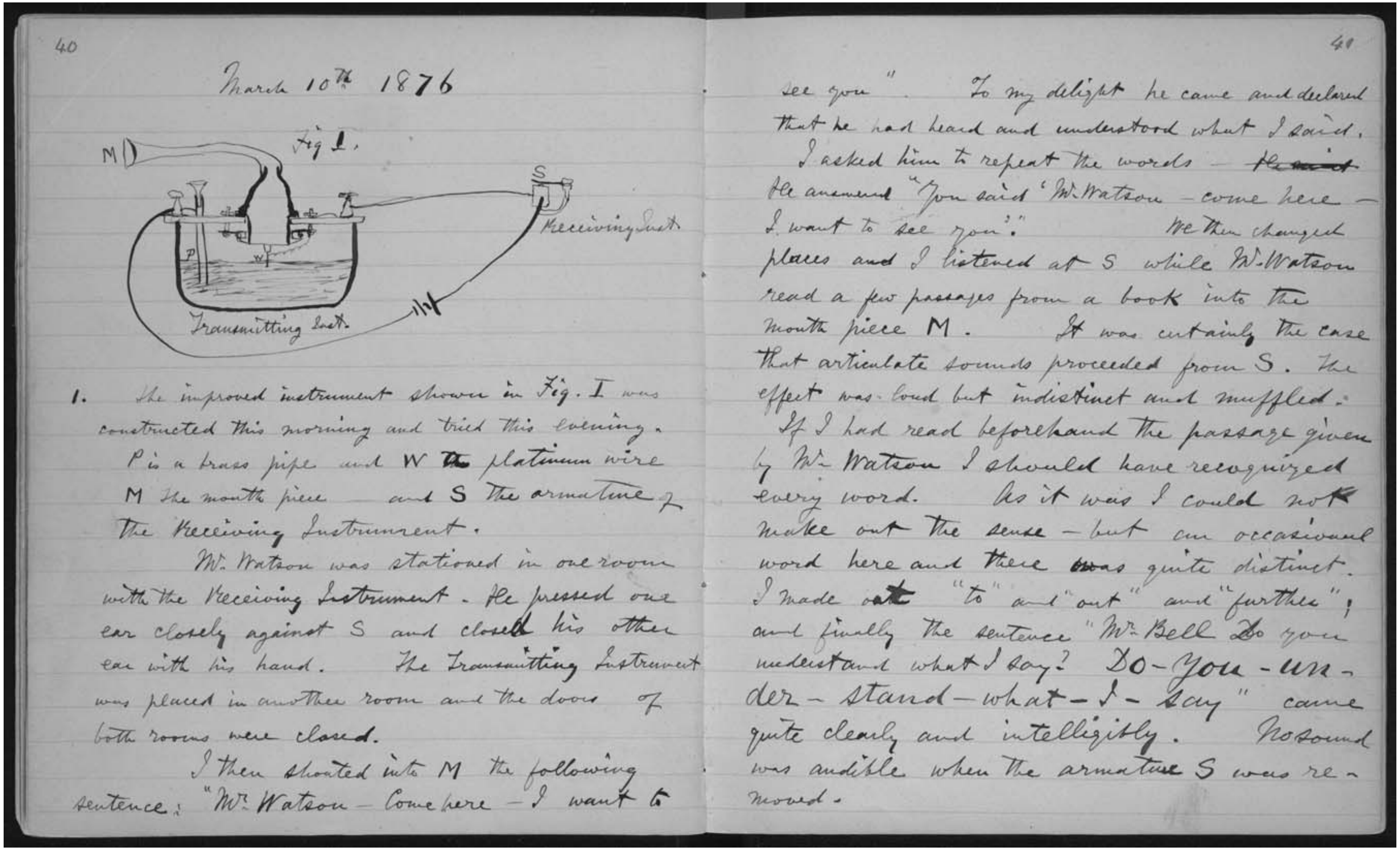
FIZ101E – Lecture 8
Rotation of rigid bodies

Alexandr Jonas

Department of Physics Engineering

Istanbul Technical University

What did we cover last week?



40

March 10th 1876

Fig. 1.

1. The improved instrument shown in Fig. I was constructed this morning and tried this evening.

P is a brass pipe and W the platinum wire M the mouth piece and S the armature of the Receiving Instrument.

Mr. Watson was stationed in one room with the Receiving Instrument. He pressed one ear closely against S and closed his other ear with his hand. The Transmitting Instrument was placed in another room and the doors of both rooms were closed.

I then shouted into M the following sentence: "Mr. Watson — Come here — I want to

41

see you". To my delight he came and declared that he had heard and understood what I said.

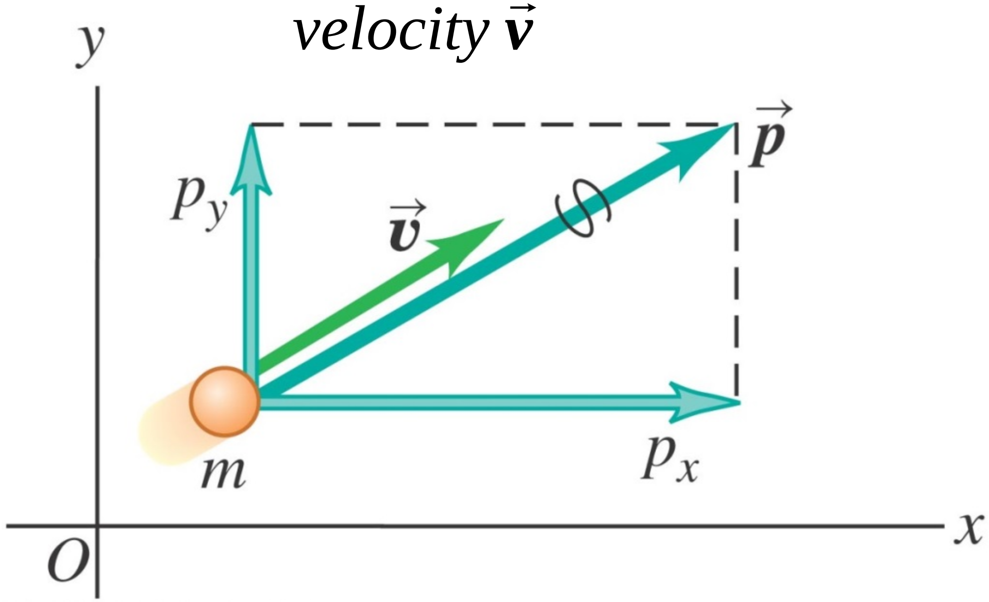
I asked him to repeat the words — ~~He said~~ He answered "You said 'Mr. Watson — come here — I want to see you'." We then changed places and I listened at S while Mr. Watson read a few passages from a book into the mouth piece M. It was certainly the case that articulate sounds proceeded from S. The effect was loud but indistinct and muffled.

If I had read beforehand the passage given by Mr. Watson I should have recognized every word. As it was I could not make out the sense — but an occasional word here and there was quite distinct. I made out "to" and "out" and "further"; and finally the sentence "Mr. Bell Do you understand what I say? Do-you-understand-what-I-say" came quite clearly and intelligibly. No sound was audible when the armature S was removed.

Momentum of a particle

mass m

velocity $\vec{\mathbf{v}}$



Particle momentum $\vec{p}$:

$$\boxed{\vec{\mathbf{p}} = m\vec{\mathbf{v}}}$$

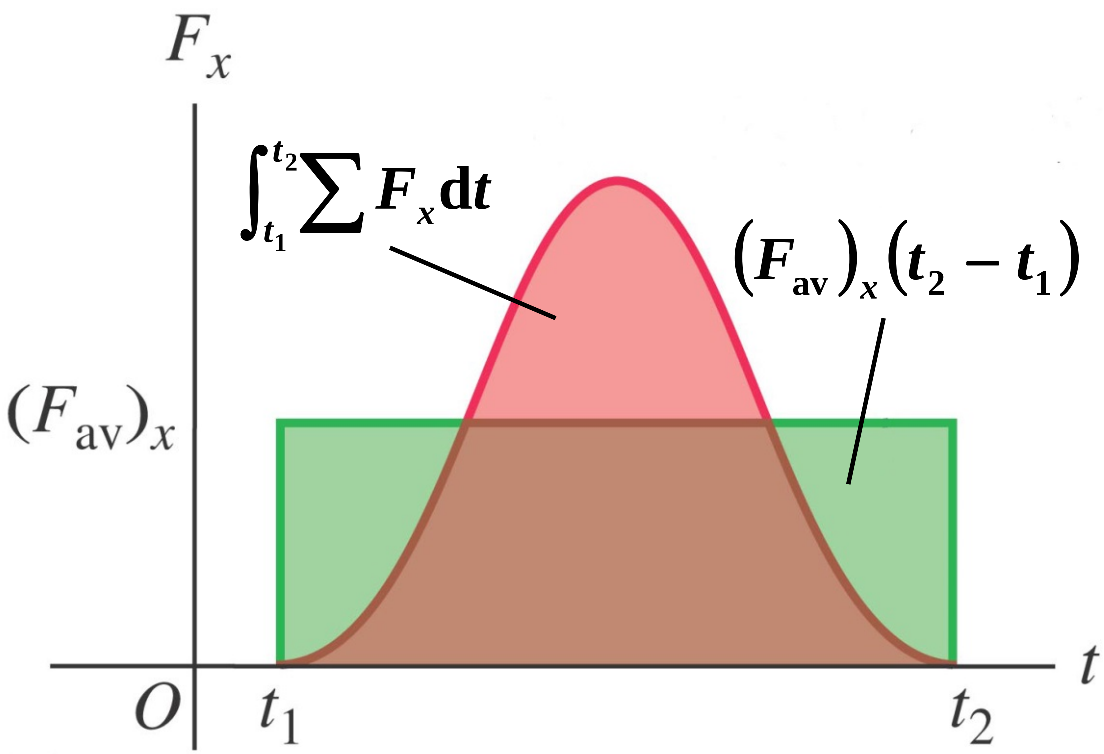
→ vector quantity with the same direction as the particle velocity

The SI unit: kg . m/s

Second Newton’s law formulated in terms of momentum: $\boxed{\sum \vec{F} = \frac{d\vec{\mathbf{p}}}{dt}}$

“The net force on a particle is equal to the rate of change of the particle's momentum”

Impulse and momentum



Impulse of a net force $\Sigma\vec{F}$ acting over a time interval $\Delta t = t_2 - t_1$

Constant net force

$$\boxed{\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F}\Delta t}$$

Variable net force

$$\boxed{\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} \mathrm{d}t = \vec{F}_{av}(t_2 - t_1)}$$

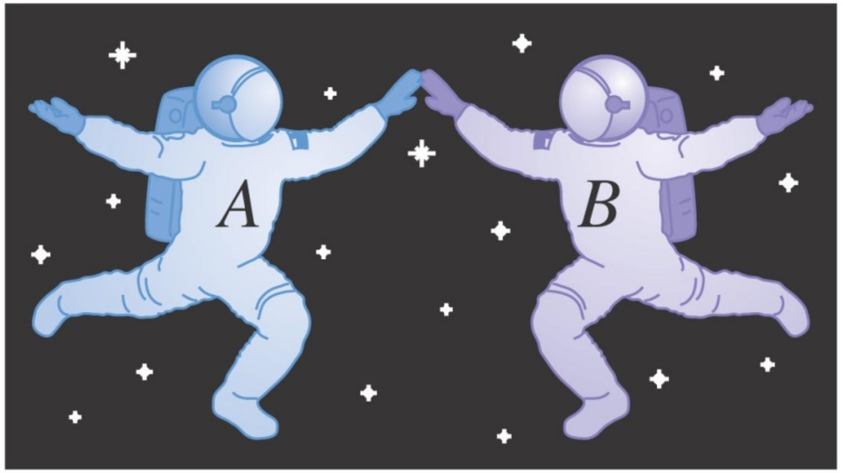
Average net force in time interval (t_1,t_2)

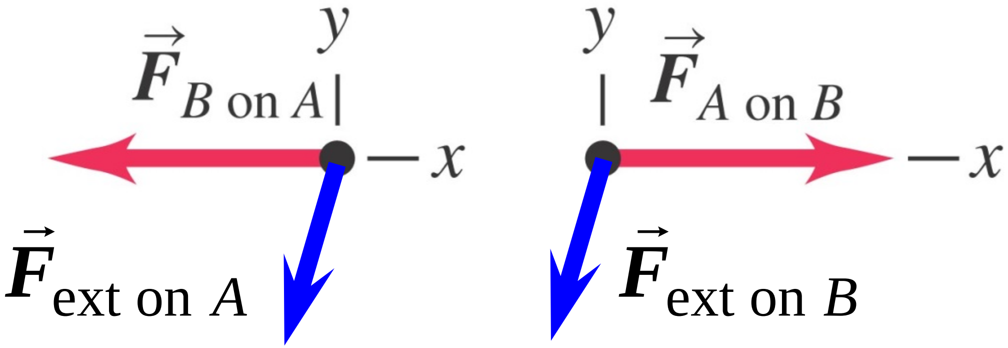
Impulse – momentum theorem

$$\boxed{\vec{J} = \vec{p}_2 - \vec{p}_1}$$

"The change in a particle's momentum during a time interval equals the impulse of the net force that acted on the particle during that time interval"

Conservation of momentum





Newton’s third law for <u>internal</u> forces acting within the system:

$$\boxed{\vec{F}_{B\text{ on }A} = -\vec{F}_{A\text{ on }B} \quad \Rightarrow \quad \sum \vec{F}_{\mathbf{int}} = \mathbf{0}}$$

Net <u>external</u> force acting on the system:

$$\boxed{\sum \vec{F}_{\text{ext}}}$$

<u>Total</u> momentum of the system:

$$\boxed{\vec{P} = \vec{p}_A + \vec{p}_B + \cdots}$$

Conservation of momentum

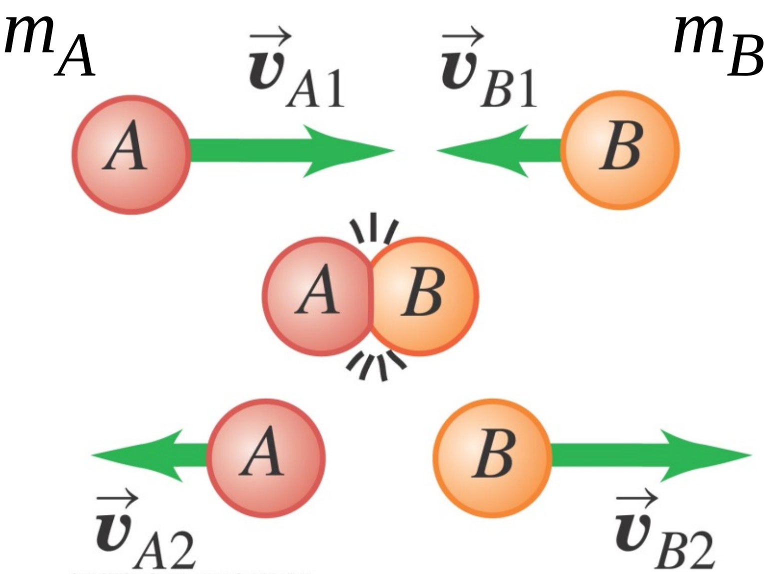
If only internal forces act on a system of particles:

$$\boxed{\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = \mathbf{0} \quad \Rightarrow \quad \vec{P} = \text{const}}$$

If net external force is non-zero:

$$\boxed{\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = \sum \vec{F}_{\text{ext}}}$$

Collisions and conservation of momentum



In collisions of all kinds, the initial and final total momenta are equal:

$$\boxed{m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = m_A\vec{v}_{A2} + m_B\vec{v}_{B2}}$$

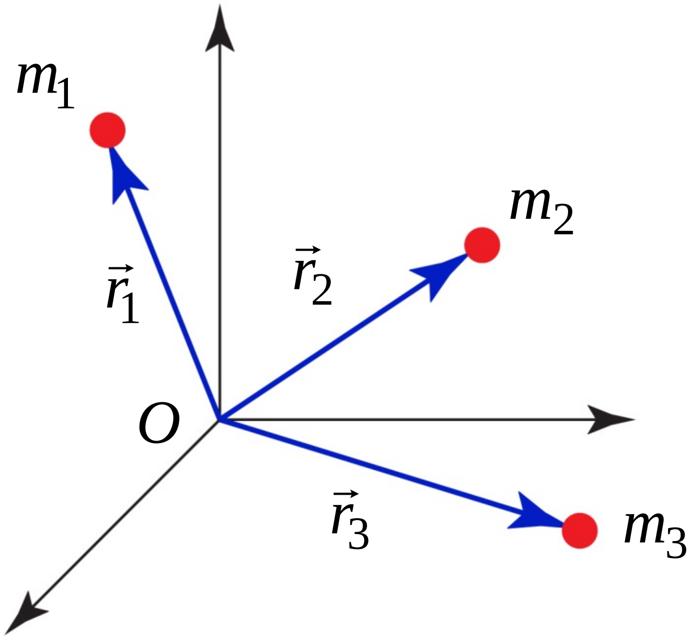
<u>Elastic collisions:</u> the initial and final kinetic energies are equal

$$\boxed{\frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2}$$

<u>Inelastic collisions:</u> the kinetic energy after the collision is smaller than before

<u>Completely inelastic collisions:</u> after the collision, the colliding bodies have the same velocity

Center of mass



The position of the center of mass (CM)

$$\boxed{\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}}$$

total mass *M* of the system

Total momentum of the system of particles 1, 2, 3, …

$$\boxed{\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = M\vec{v}_{cm}}$$

CM velocity

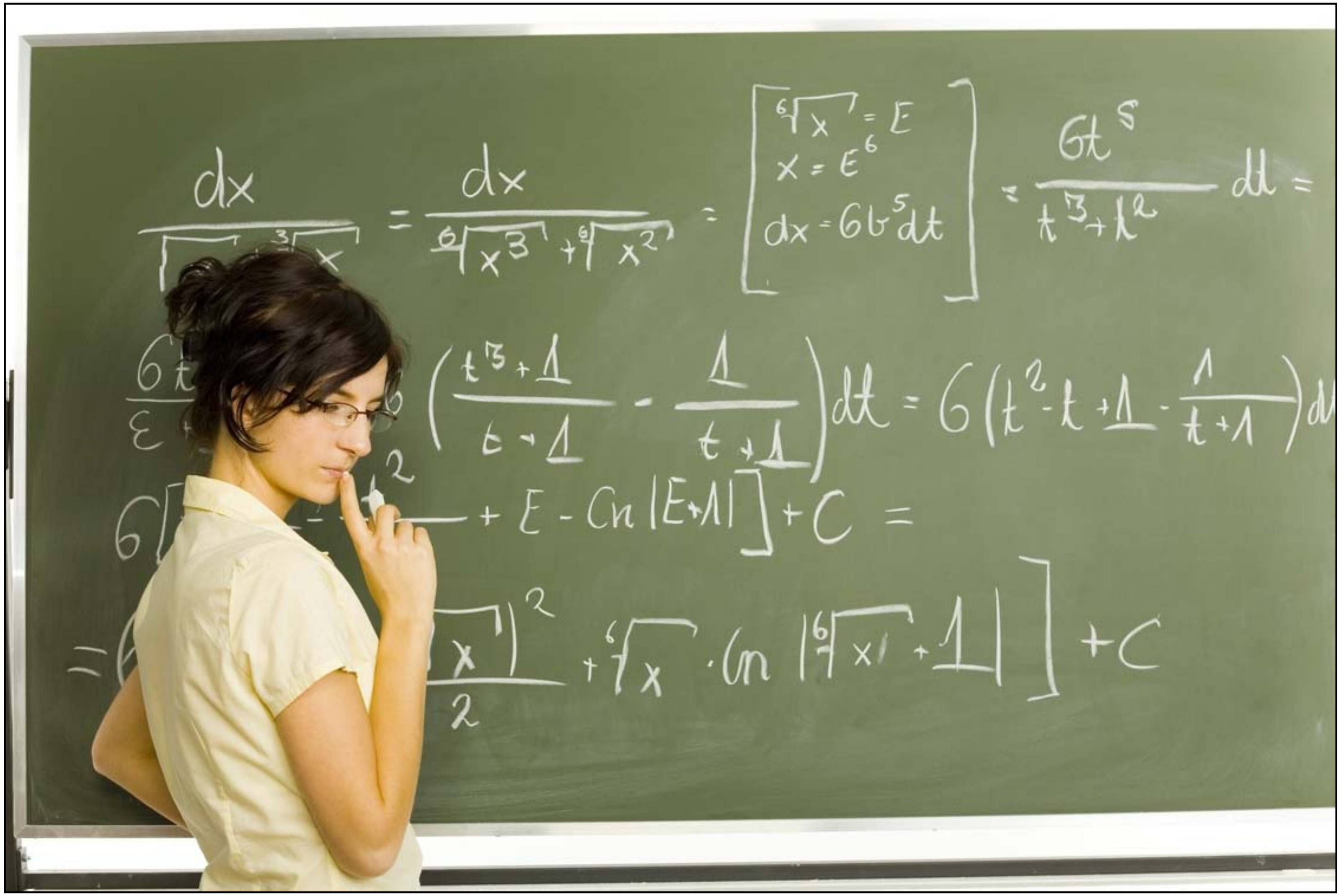
Newton’s second law for the system of particles 1, 2, 3, …

$$\boxed{\sum \vec{F}_{ext} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = M\vec{a}_{cm}}$$

CM acceleration

“Motion of a system of particles or an extended body with the total mass *M* can be represented by the motion of the center of mass of the system / body”

What will we cover today?



Lesson plan

1. Angular velocity and acceleration
2. Rotation with constant angular acceleration
3. Relating linear and angular kinematics
4. Energy in rotational motion
5. Parallel-axis theorem