FIZ101E – Lecture 9
Dynamics of rotational motion

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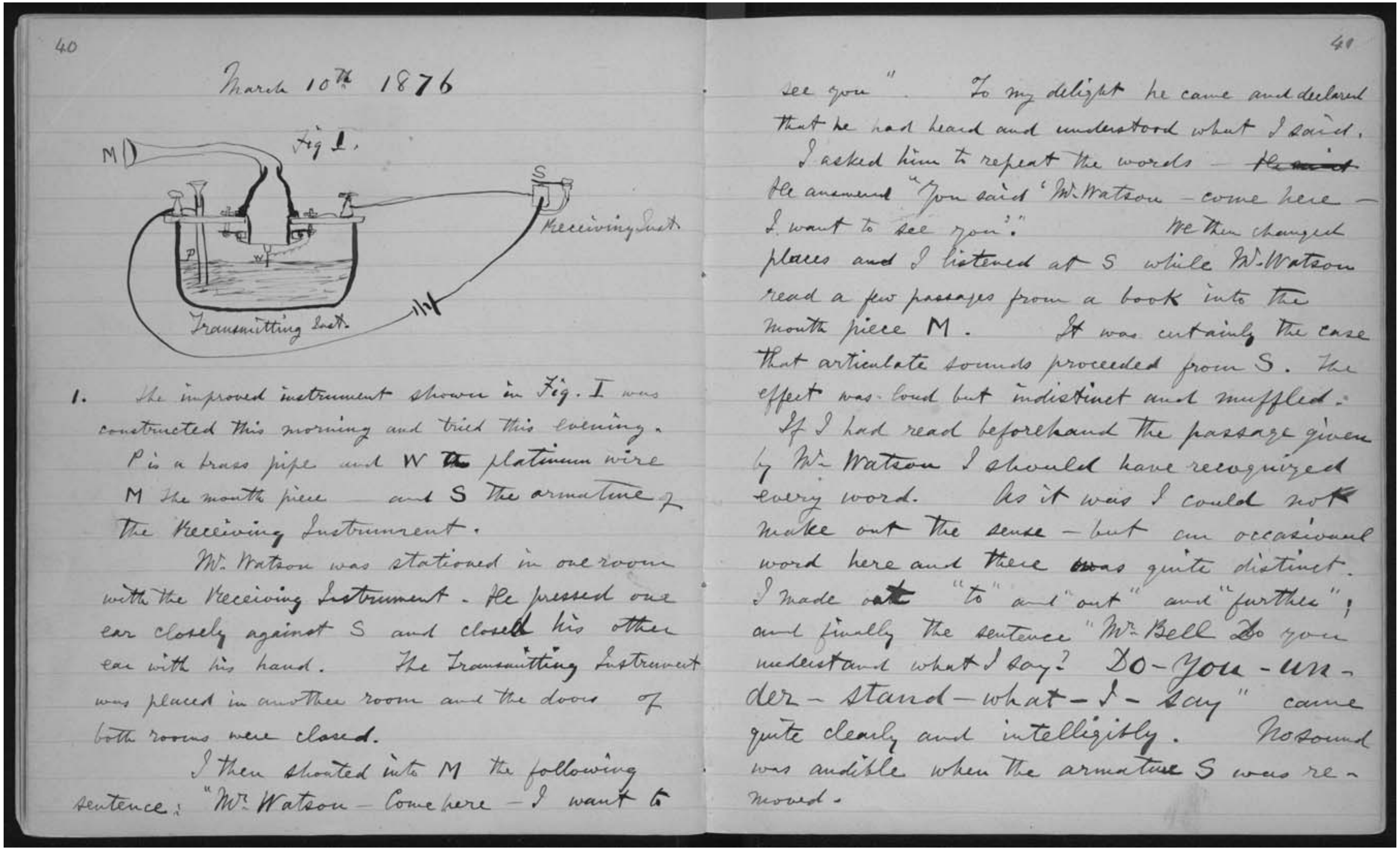
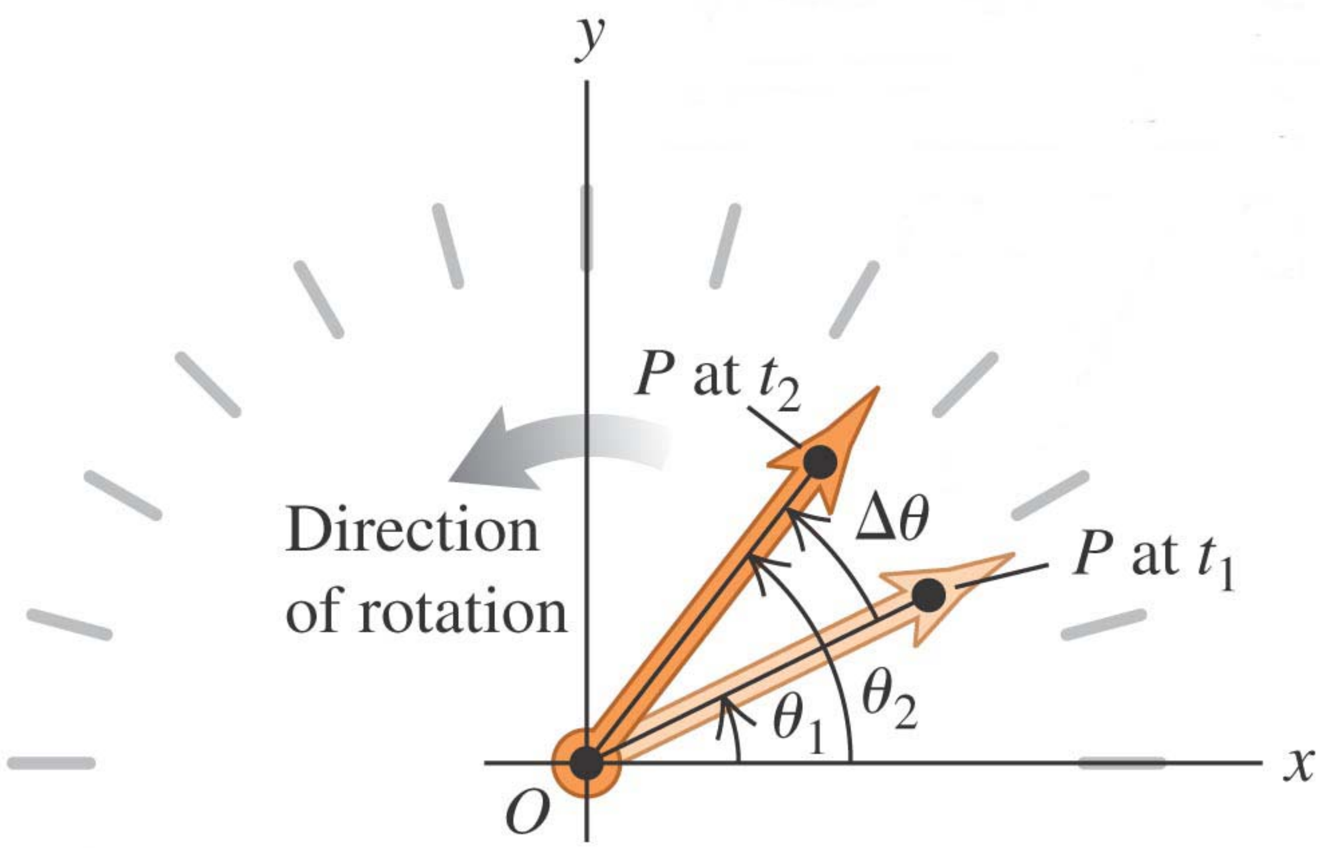
What did we cover last week?



Angular position and displacement

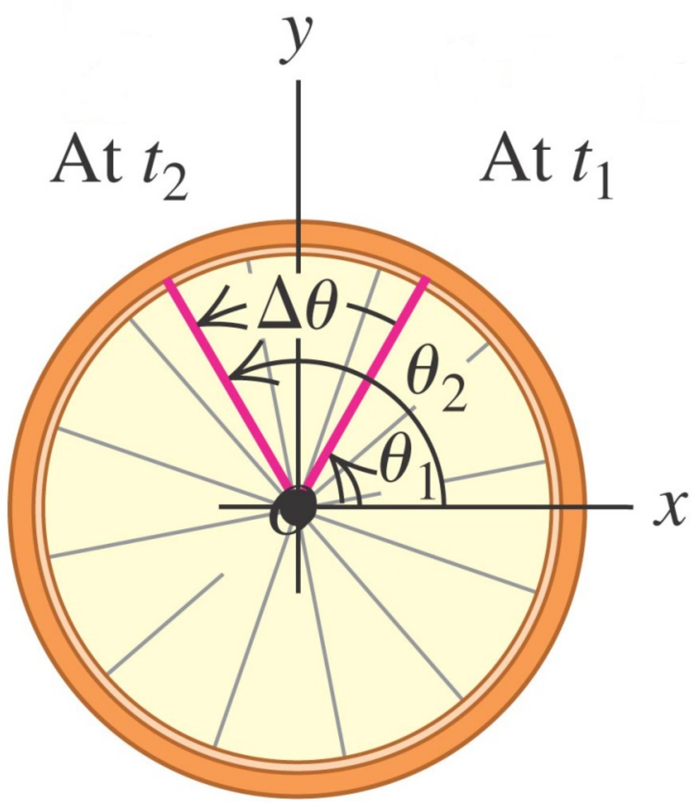


The angle θ from the +x-axis specifies the angular position of a rotating body

Angular position is defined as angle measured in radians

Angular velocity

Angular velocity = rate of change of the angular position



Average angular velocity

$$\omega_{\text{av-z}} = \frac{\theta_2 - \theta_2}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

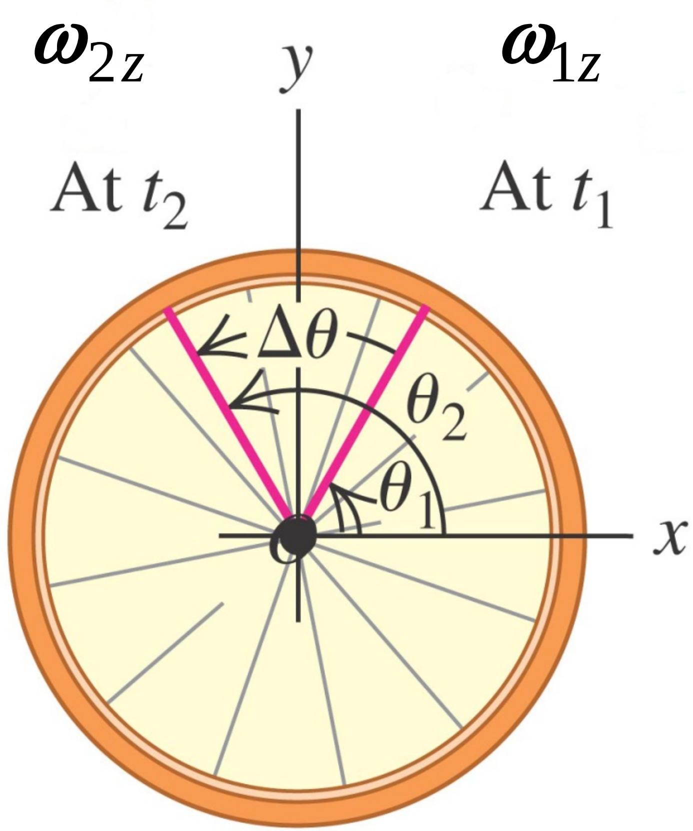
SI units: rad/s

In general, angular velocity $\vec{\omega}$ is a vector directed along the axis of rotation

→ ω_z is the angular velocity component along the z-axis

Angular acceleration

Angular acceleration = rate of change of the angular velocity



Average angular acceleration

$$\alpha_{av\text{-}z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t}$$

Instantaneous angular acceleration

$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2}$$

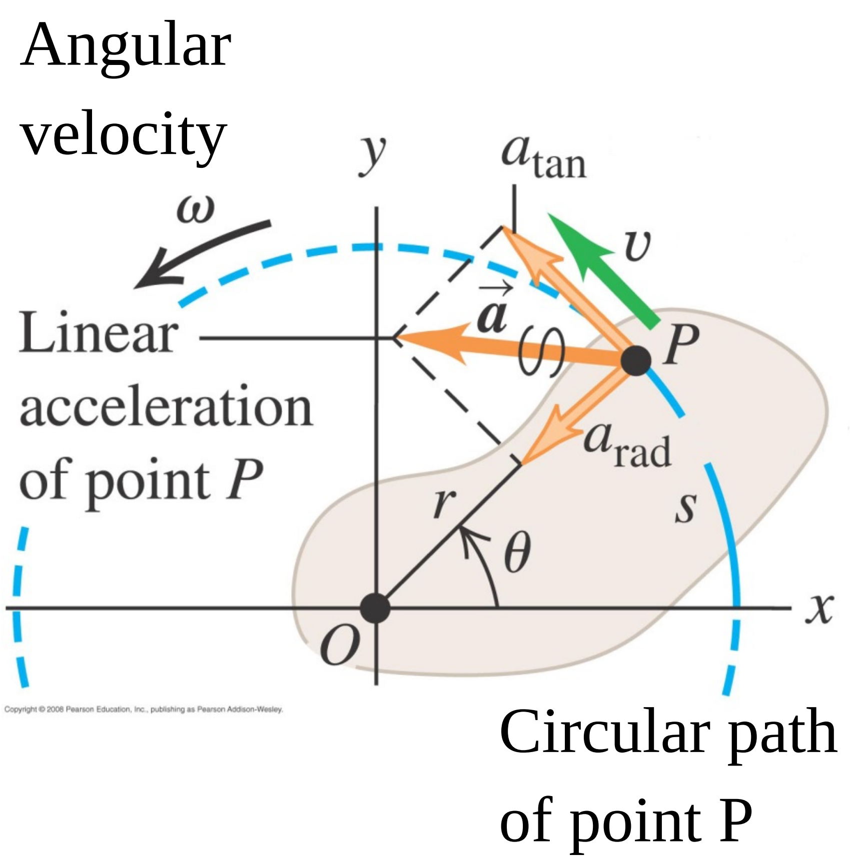
SI units: rad/s^2

In general, angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$ is a vector

→ α_z is the angular acceleration component along the z-axis

Relating angular and linear motion

Consider an arbitrary point *P* in a rotating rigid body located at a distance *r* from the axis of rotation, moving on a circle of radius *r*



Linear velocity of point *P*: $v = r\omega$

→ tangential to the circular path

Linear acceleration of point *P*:

$$\vec{a} = (a_{tan}, a_{rad})$$

with

$$a_{tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

→ tangential component

$$a_{tan} = \frac{v^2}{r} = \omega^2 r$$

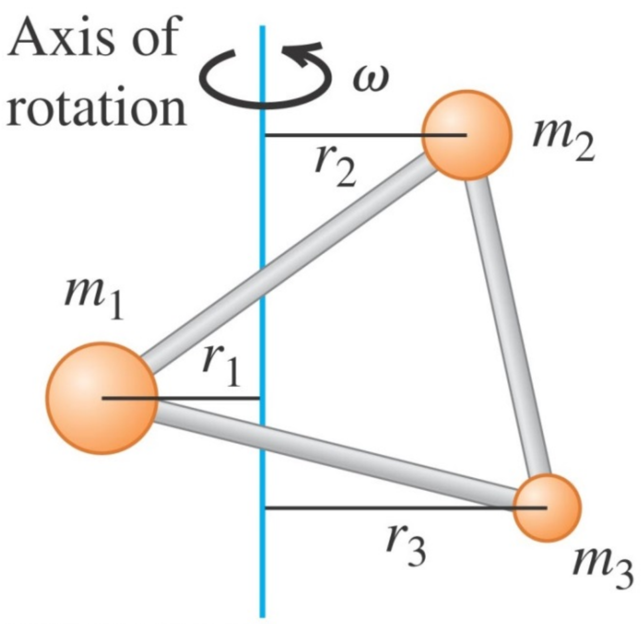
→ radial component

Moment of inertia and rotational kinetic energy

Moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2 = \int_{\text{body}} \mathrm{d}m\, r^2$$

→ describes rotational inertia of a system of particles/continuous body for a specified axis of rotation



Rotational kinetic energy:

$$K = \frac{1}{2} I \omega^2$$

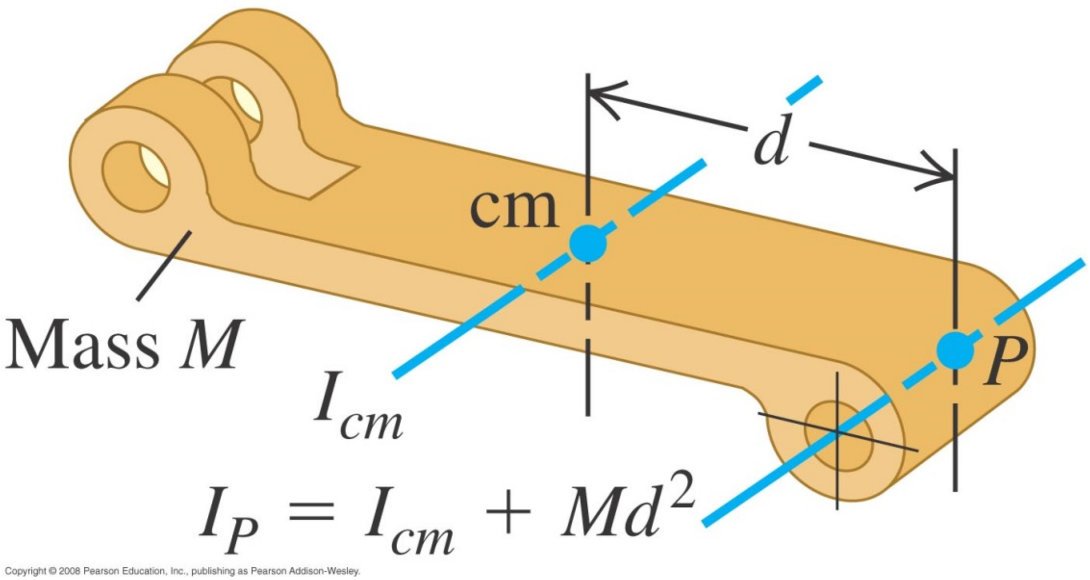
→ kinetic energy of a system of particles / continuous body rotating about a specified axis with angular speed ω

Parallel-axis theorem

Consider rotation of a rigid body about two different parallel axes:

An axis through the center of mass → moment of inertia I_{cm}

An axis through point <u>*P*</u> at a distance <u>*d*</u> from the first axis → moment of inertia I_P



<u>Parallel axis theorem</u>

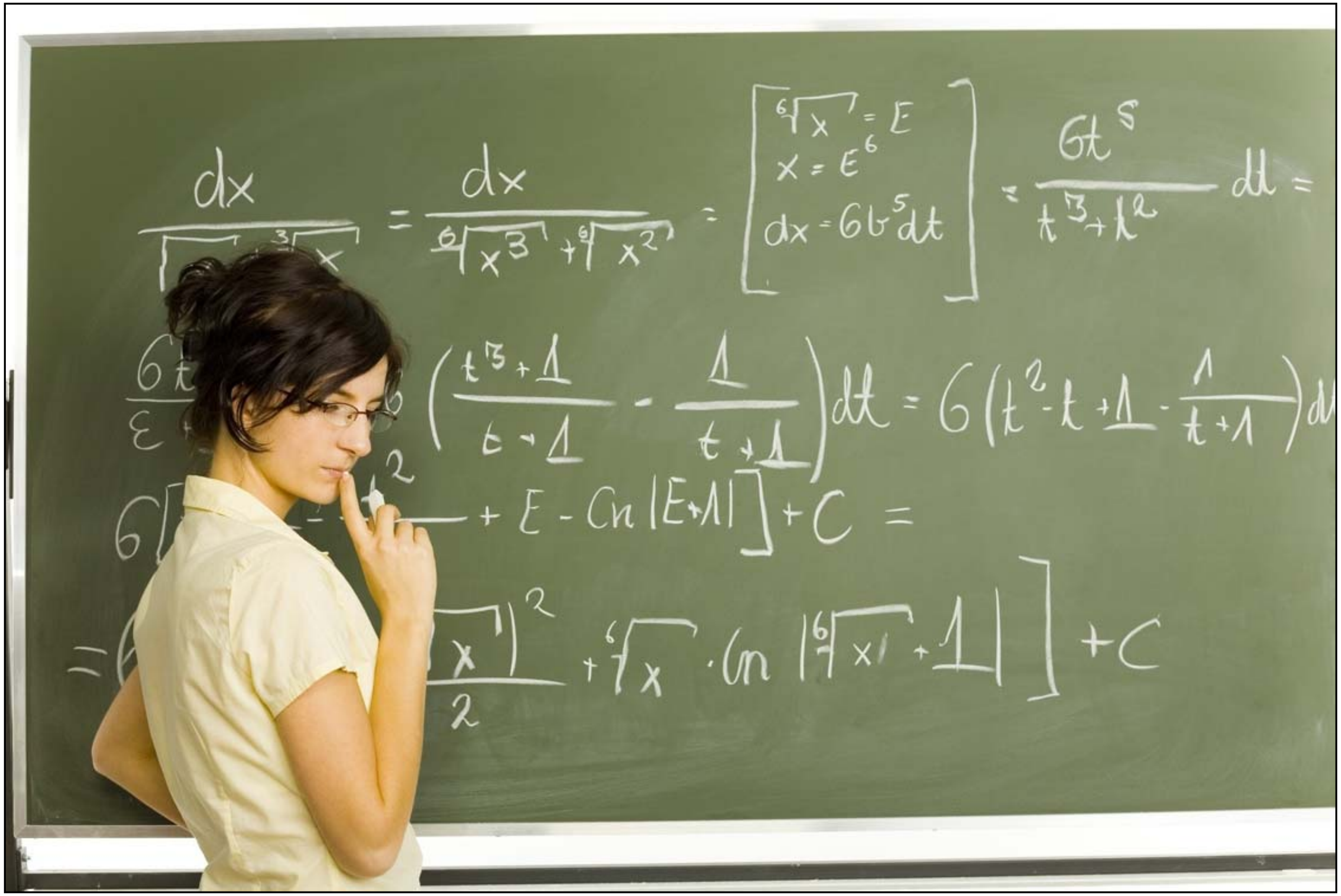
$$I_P = I_{cm} + Md^2$$

M… mass of the body

↓

The smallest possible moment of inertia is about an axis through the center of mass

What will we cover today?



Lesson plan

1. Torque
2. Torque and angular acceleration
3. Rotation about a moving axis
4. Work and power in rotational motion
5. Angular momentum and its conservation