

# **FIZ101E – Lecture 9**

## **Dynamics of rotational motion**



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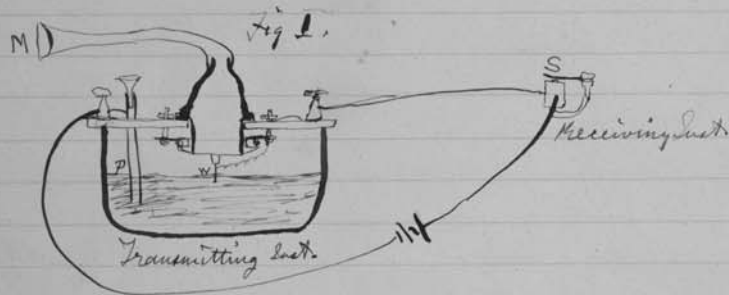
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# What did we cover last week?

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March 10<sup>th</sup> 1876



1. The improved instrument shown in Fig. I was constructed this morning and tried this evening. P is a brass pipe and W the platinum wire M the mouth piece and S the armature of the Receiving Instrument.

Mr. Watson was stationed in one room with the Receiving Instrument. He pressed one ear closely against S and closed his other ear with his hand. The Transmitting Instrument was placed in another room and the doors of both rooms were closed.

I then shouted into M the following sentence: "Mr. Watson - Come here - I want to

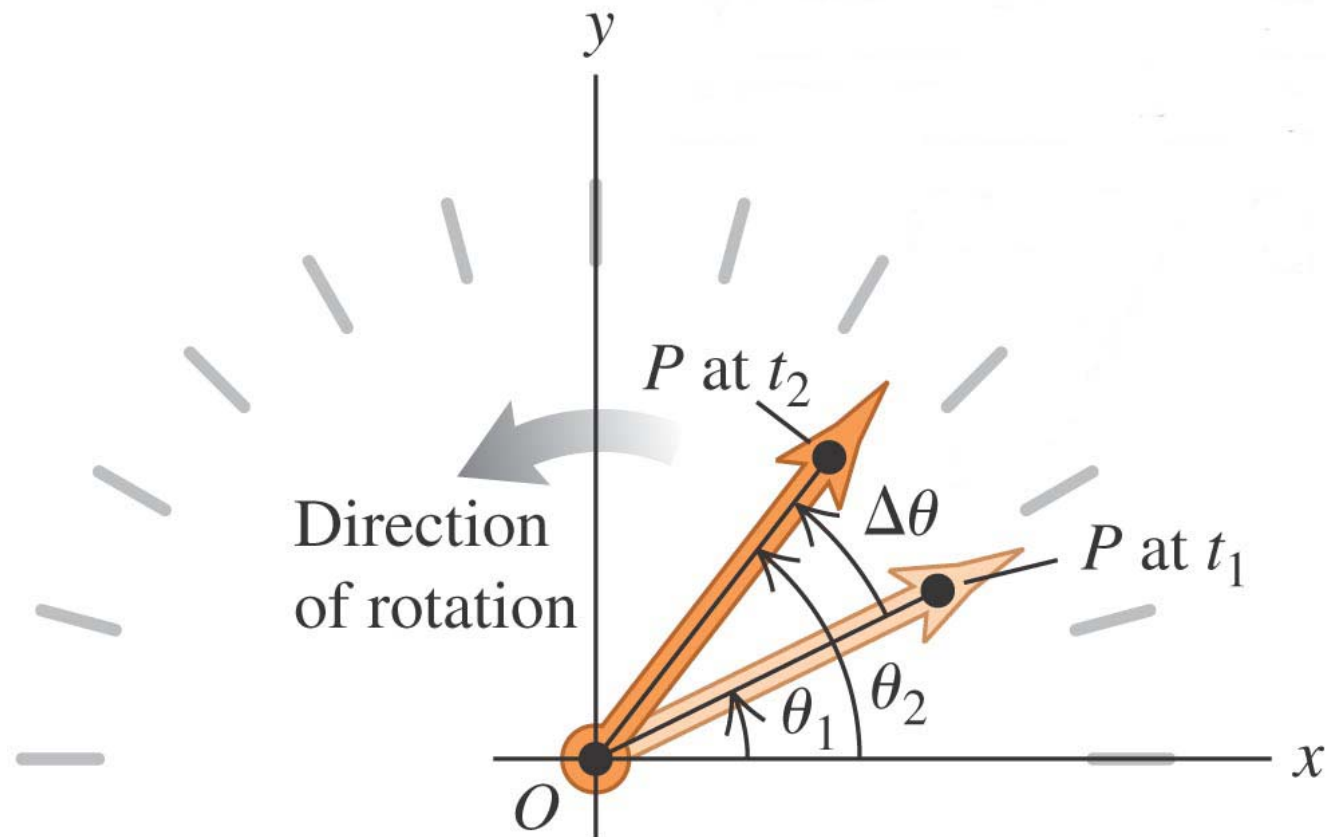
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see you". To my delight he came and declared that he had heard and understood what I said.

I asked him to repeat the words - ~~he said~~ He answered "You said 'Mr. Watson - come here - I want to see you'." We then changed places and I listened at S while Mr. Watson read a few passages from a book into the mouth piece M. It was certainly the case that articulate sounds proceeded from S. The effect was loud but indistinct and muffled.

If I had read beforehand the passage given by Mr. Watson I should have recognized every word. As it was I could not make out the sense - but an occasional word here and there was quite distinct. I made out "to" and "out" and "further"; and finally the sentence "Mr. Bell Do you understand what I say? Do-you-un-der-stand-what-I-say" came quite clearly and intelligibly. No sound was audible when the armature S was removed.

# Angular position and displacement

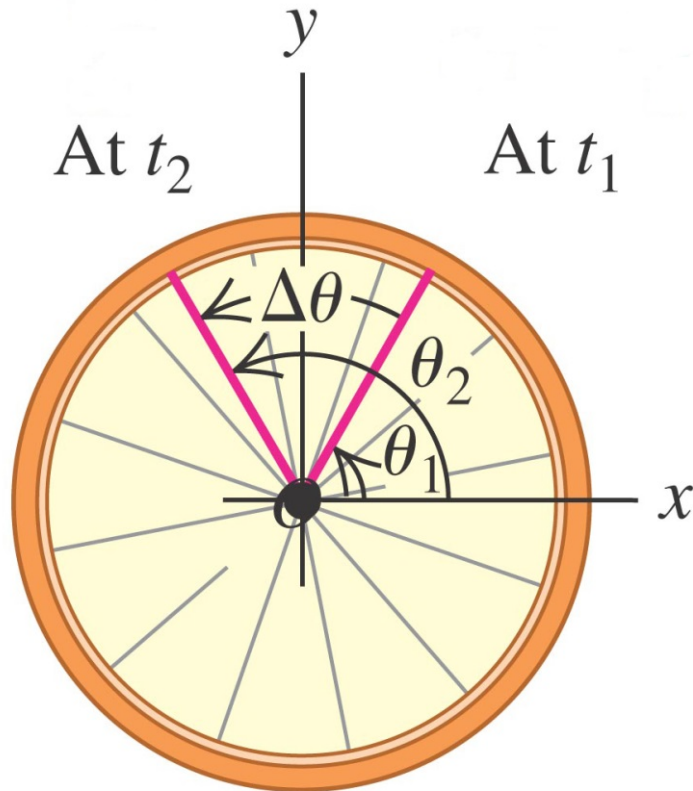


The angle  $\theta$  from the +x-axis specifies the angular position of a rotating body

Angular position is defined as angle measured in radians

# Angular velocity

Angular velocity = rate of change of the angular position



Average angular velocity

$$\omega_{\text{av-z}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity

$$\omega_z = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

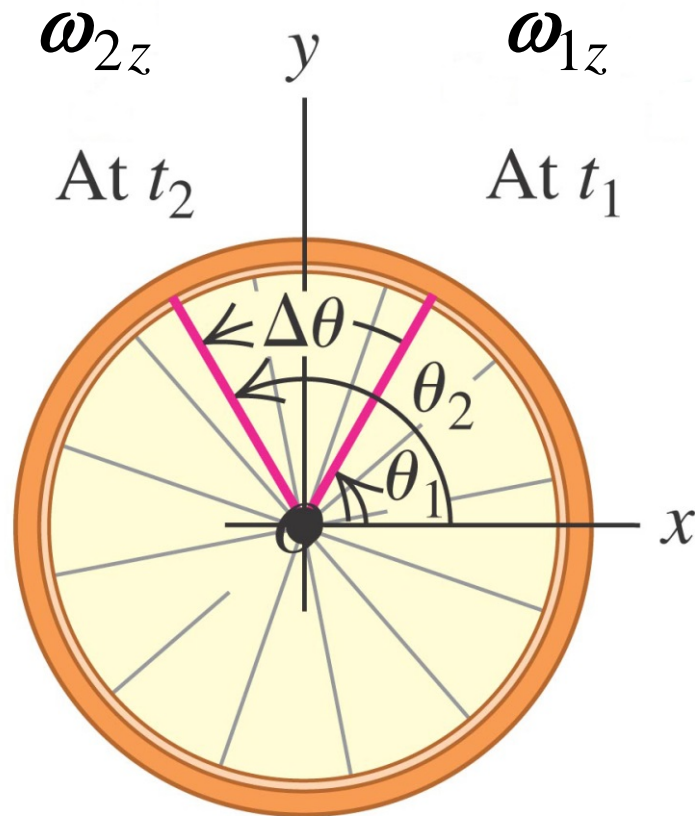
SI units: rad/s

In general, angular velocity  $\vec{\omega}$  is a vector directed along the axis of rotation

→  $\omega_z$  is the angular velocity component along the z-axis

# Angular acceleration

Angular acceleration = rate of change of the angular velocity



Average angular acceleration

$$\alpha_{\text{av-z}} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t}$$

Instantaneous angular acceleration

$$\alpha_z = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2}$$

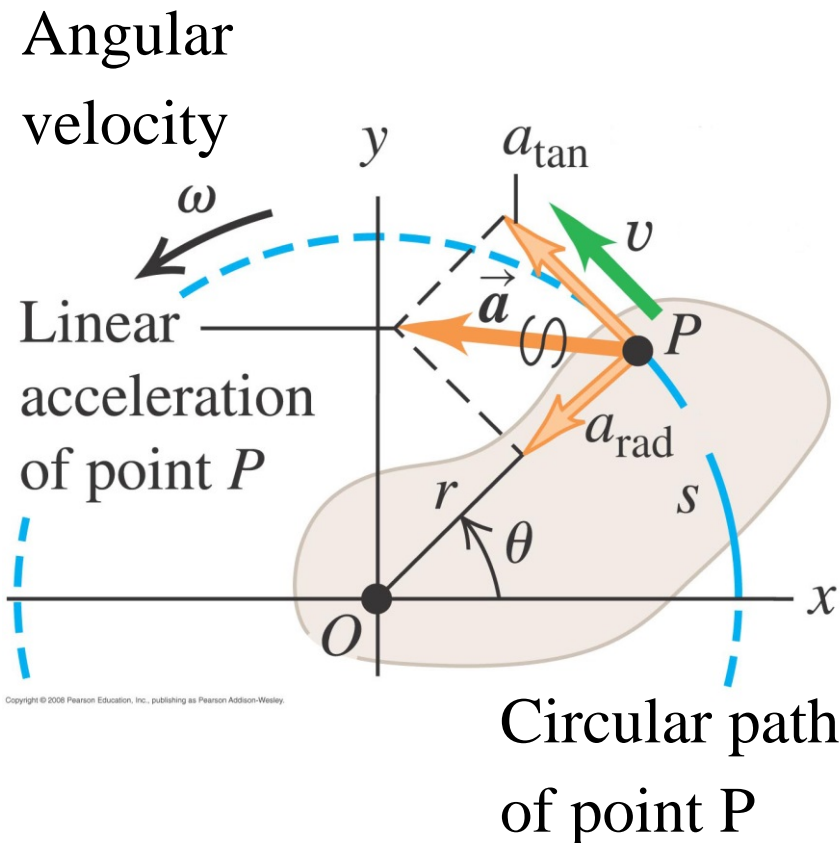
SI units:  $\text{rad/s}^2$

In general, angular acceleration  $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$  is a vector

→  $\alpha_z$  is the angular acceleration component along the z-axis

# Relating angular and linear motion

Consider an arbitrary point  $P$  in a rotating rigid body located at a distance  $r$  from the axis of rotation, moving on a circle of radius  $r$



Linear velocity of point  $P$ :  $v = r\omega$

→ tangential to the circular path

Linear acceleration of point  $P$ :

$$\vec{a} = (a_{\text{tan}}, a_{\text{rad}})$$

with

$$a_{\text{tan}} = \frac{dv}{dt} = r \frac{d\omega}{dt} = r\alpha \quad \rightarrow \text{tangential component}$$

$$a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r \quad \rightarrow \text{radial component}$$

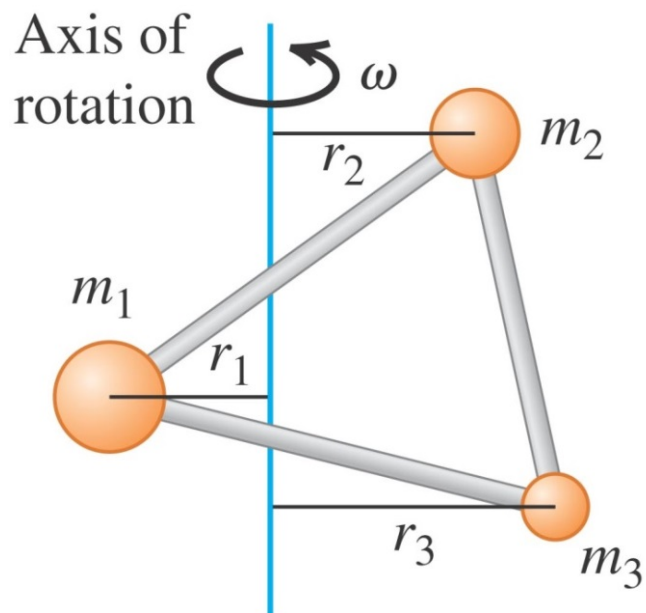


# Moment of inertia and rotational kinetic energy

Moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots = \sum_i m_i r_i^2 = \int_{\text{body}} dm r^2$$

→ describes rotational inertia of a system of particles/continuous body for a specified axis of rotation



Rotational kinetic energy:

$$K = \frac{1}{2} I \omega^2$$

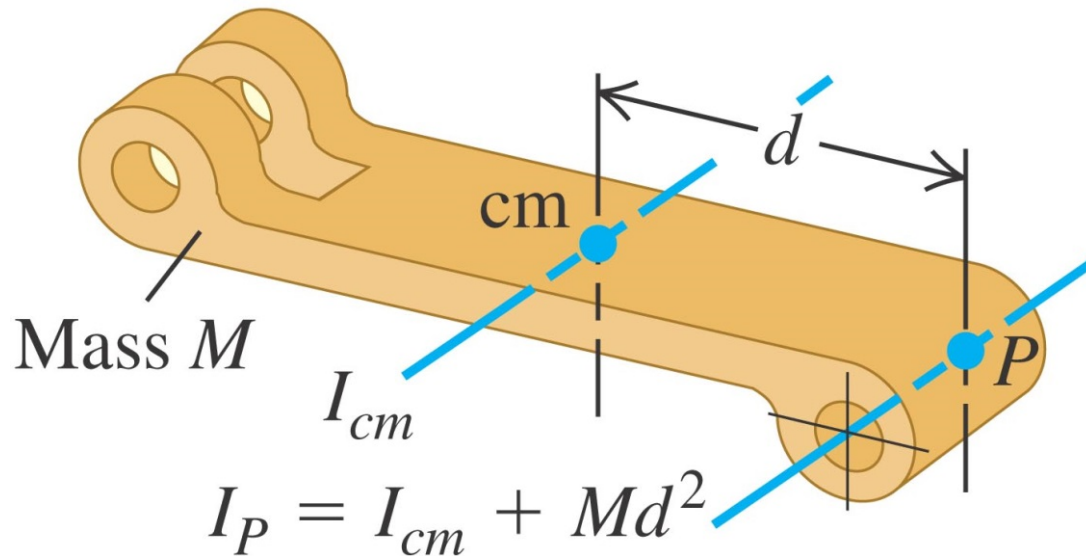
→ kinetic energy of a system of particles / continuous body rotating about a specified axis with angular speed  $\omega$

# Parallel-axis theorem

Consider rotation of a rigid body about two different parallel axes:

An axis through the center of mass  $\rightarrow$  moment of inertia  $I_{cm}$

An axis through point  $P$  at a distance  $d$  from the first axis  $\rightarrow$  moment of inertia  $I_P$



## Parallel axis theorem

$$I_P = I_{cm} + Md^2$$

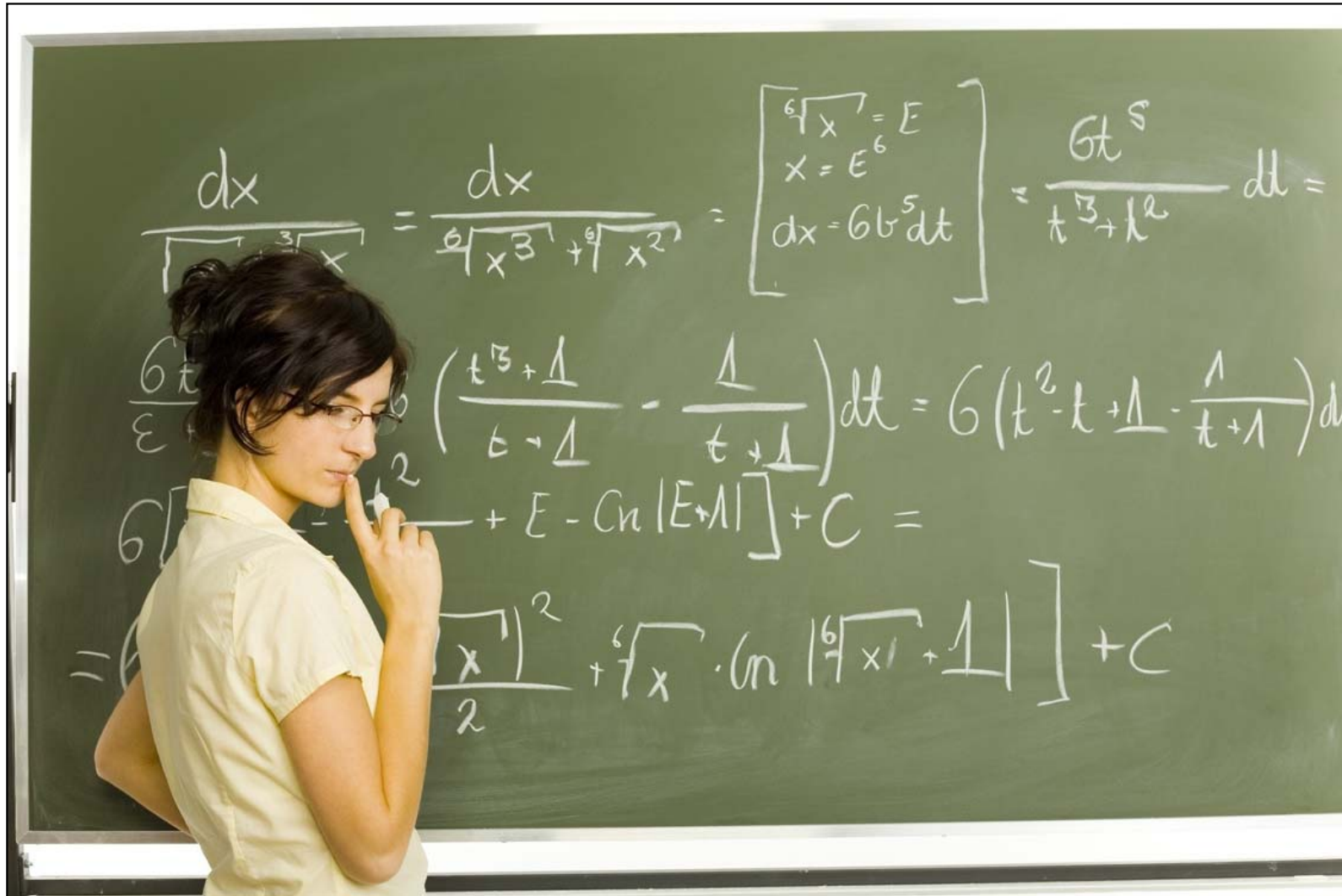
$M$ ... mass of the body



The smallest possible moment of inertia is about an axis through the center of mass



# What will we cover today?



# Lesson plan

- 1. Torque**
- 2. Torque and angular acceleration**
- 3. Rotation about a moving axis**
- 4. Work and power in rotational motion**
- 5. Angular momentum and its conservation**