FIZ101E – Lecture 10
Equilibrium and elasticity

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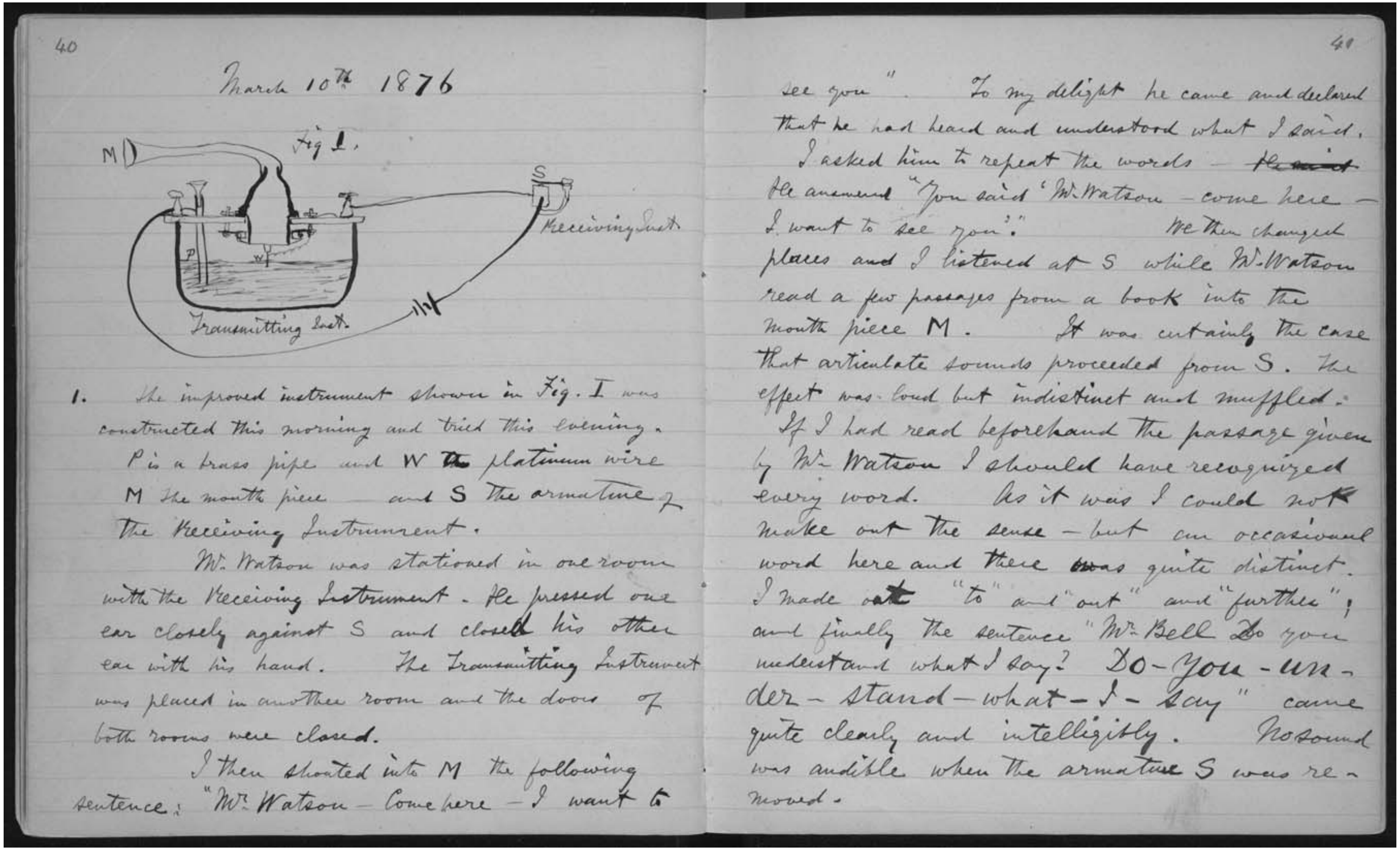
What did we cover last week?



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March 10th 1876

Fig 1.

1. The improved instrument shown in Fig. I was constructed this morning and tried this evening.
P is a brass pipe and W the platinum wire M the mouth piece and S the armature of the Receiving Instrument.

Mr. Watson was stationed in one room with the Receiving Instrument. He pressed one ear closely against S and closed his other ear with his hand. The Transmitting Instrument was placed in another room and the doors of both rooms were closed.

I then shouted into M the following sentence: "Mr. Watson — Come here — I want to

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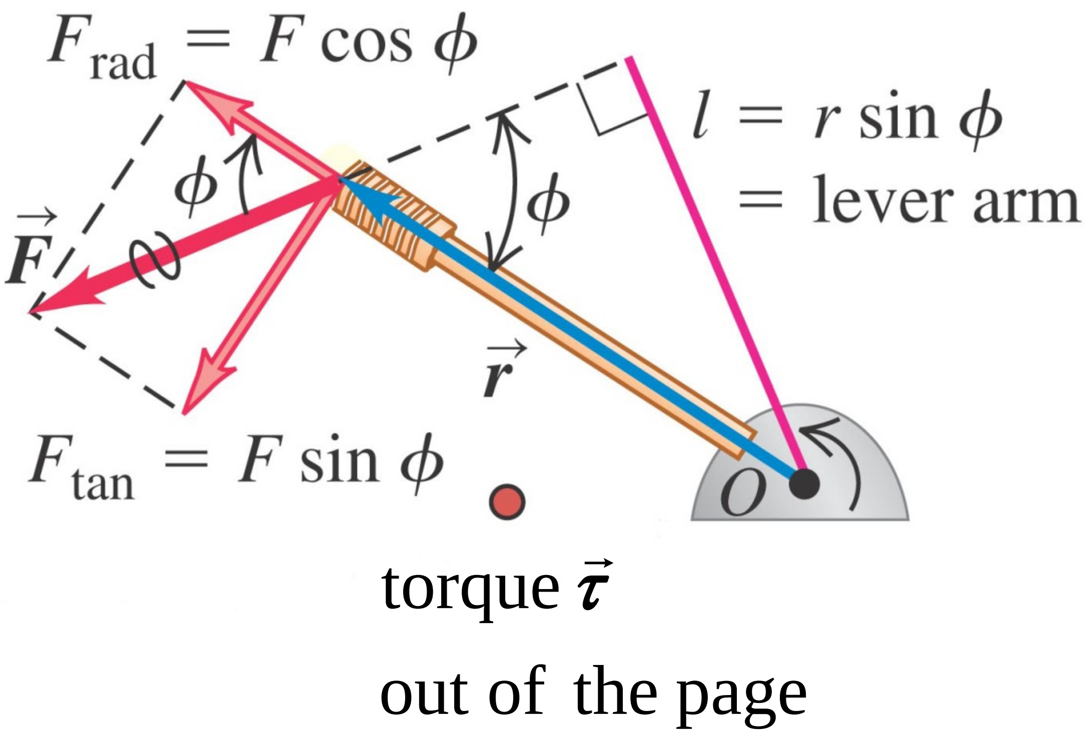
see you". To my delight he came and declared that he had heard and understood what I said.

I asked him to repeat the words — ~~He said~~ He answered "You said 'Mr. Watson — come here — I want to see you'." We then changed places and I listened at S while Mr. Watson read a few passages from a book into the mouth piece M. It was certainly the case that articulate sounds proceeded from S. The effect was loud but indistinct and muffled.

If I had read beforehand the passage given by Mr. Watson I should have recognized every word. As it was I could not make out the sense — but an occasional word here and there was quite distinct. I made out "to" and "out" and "further"; and finally the sentence "Mr. Bell Do you understand what I say? Do — you — un-der — stand — what — I — say" came quite clearly and intelligibly. No sound was audible when the armature S was removed.

Torque of a force

When a force $\vec{F}$ acts on a body, the torque $\vec{\tau}$ of that force with respect to a point *O* is equal to the vector product of the position vector $\vec{r}$ of the point at which the force acts and $\vec{F}$.



Torque

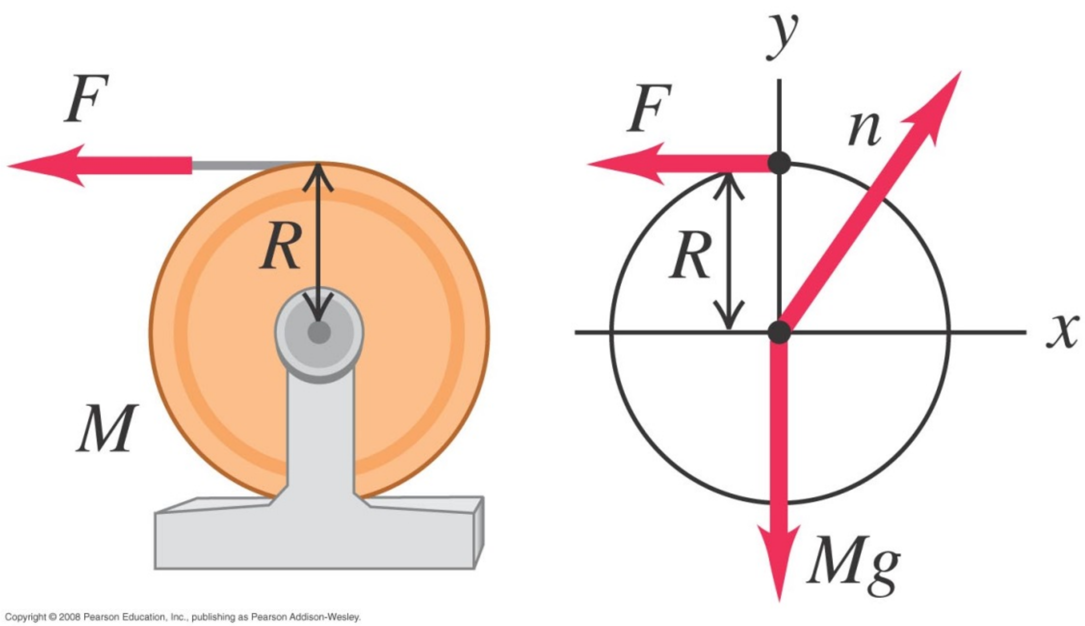
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque magnitude

$$\tau = r\,F \sin\Phi = F\,l$$

Laws of rotational dynamics

The rotational analog of Newton's 2nd law says that the net torque acting on a body equals the product of the body's moment of inertia and its angular acceleration.



$$\sum \tau_z = I\alpha_z$$

$\sum \tau_z$ … net torque

I …… moment of inertia

α_z …… angular acceleration

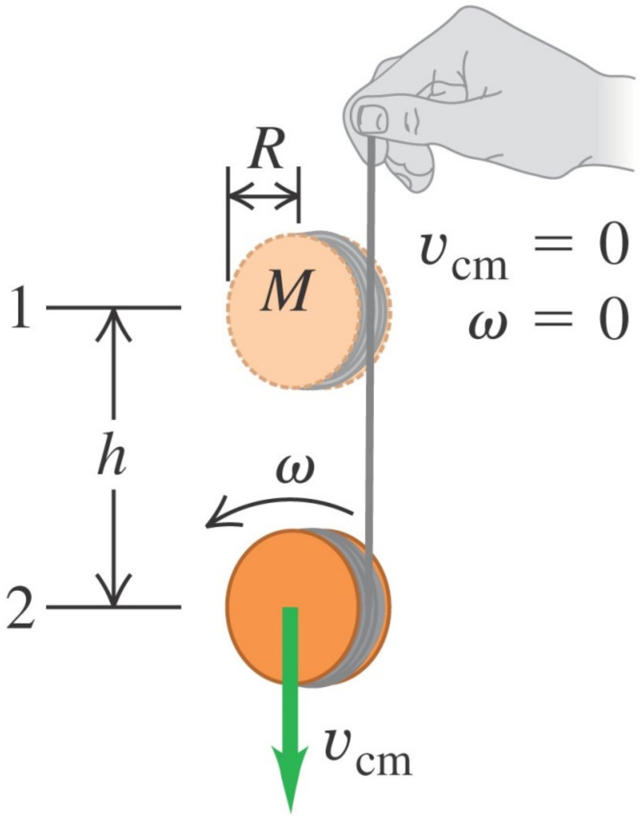
$$\tau_z = R\,F$$

Combined translation and rotation

General motion of a rigid body

Translational motion of the center of mass

+ rotational motion about an axis through the center of mass.



Kinetic energy of combined motion

$$K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

translation — rotation

M … mass of the body

v_{cm} … center-of-mass velocity

I_{cm} … moment of inertia

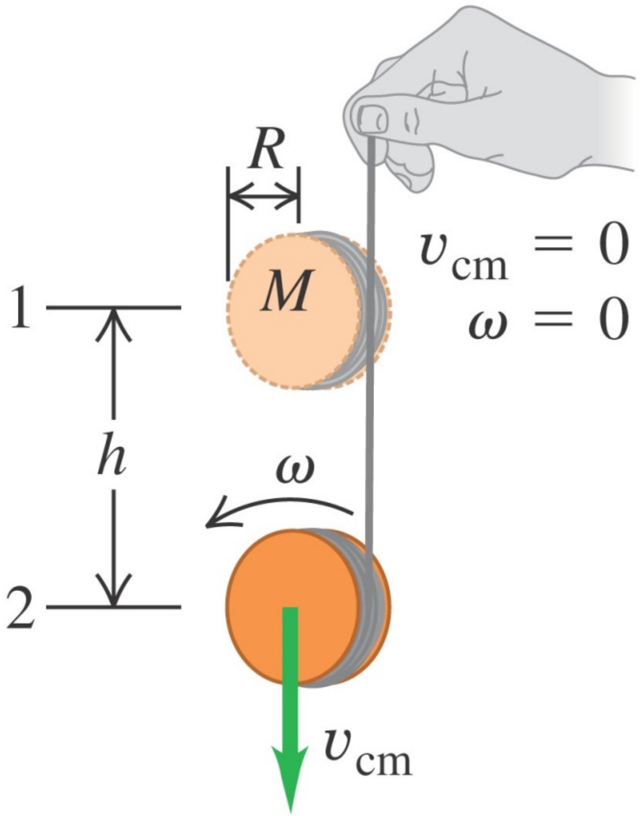
ω … angular velocity

Combined translation and rotation

<u>Dynamics of combined motion</u>

Newton's 2nd law → motion of the center of mass

Rotational equivalent of Newton's 2nd law → rotation about the center of mass.



<u>Translational dynamics</u>

$$\sum \vec{F}_{ext} = M\,\vec{a}_{cm}$$

<u>Rotational dynamics</u>

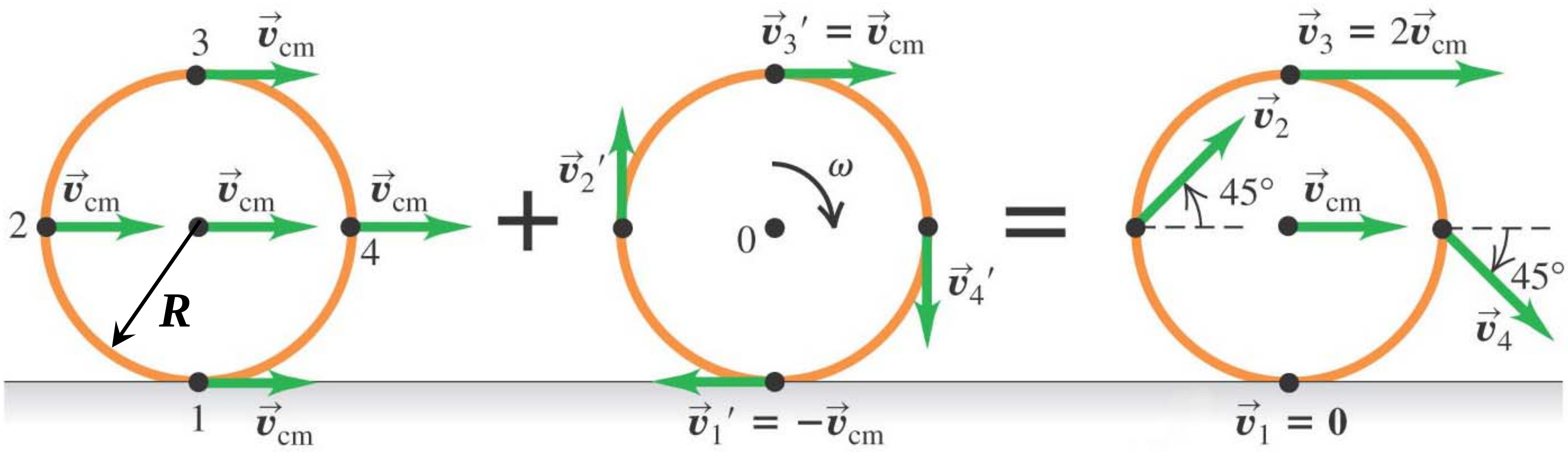
$$\sum \tau_z = I_{cm}\alpha_z$$

→ valid for rotation about an axis of symmetry that does not change direction

Combined translation and rotation

<u>Rolling without slipping</u>

Point 1 on the rolling body contacting the surface on which the body rolls has zero relative velocity with respect to the surface



No-slipping condition: $\boxed{v_{cm} = R\omega}$

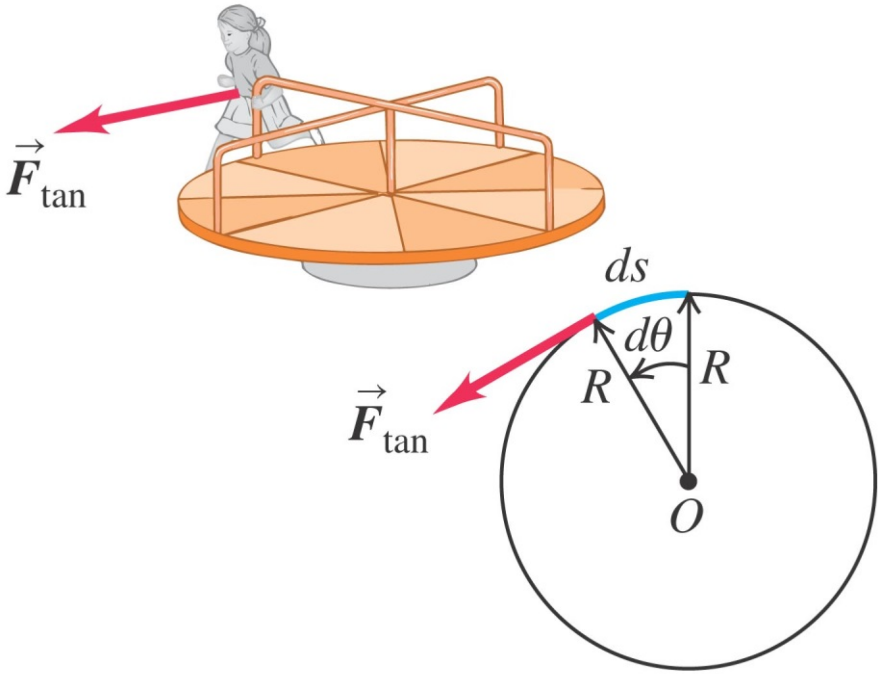
$v_{cm} \ldots$ center-of-mass velocity

$\omega \ldots$ angular velocity

Work done by a torque

Work of a torque on a rotating body
= integral of the torque

$$W = \int_{\theta_1}^{\theta_2} \boldsymbol{\tau}_z \, \mathrm{d}\boldsymbol{\theta} = \int_{\theta_1}^{\theta_2} \boldsymbol{F}_{\mathrm{tan}} \, \boldsymbol{R} \, \mathrm{d}\boldsymbol{\theta}$$



<u>Work - energy theorem:</u>

$$\boldsymbol{W}_{\mathrm{tot}} = \frac{1}{2}\boldsymbol{I}\boldsymbol{\omega}_2^2 - \frac{1}{2}\boldsymbol{I}\boldsymbol{\omega}_1^2$$

→ the total rotational work done
on a rigid body by a torque is equal to
the change in rotational kinetic energy

<u>Power:</u> $\boldsymbol{P} = \boldsymbol{\tau}_z \boldsymbol{\omega}_z$ → product of the torque and the angular velocity

Angular momentum

Angular momentum of a particle
with respect to point *O*:

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

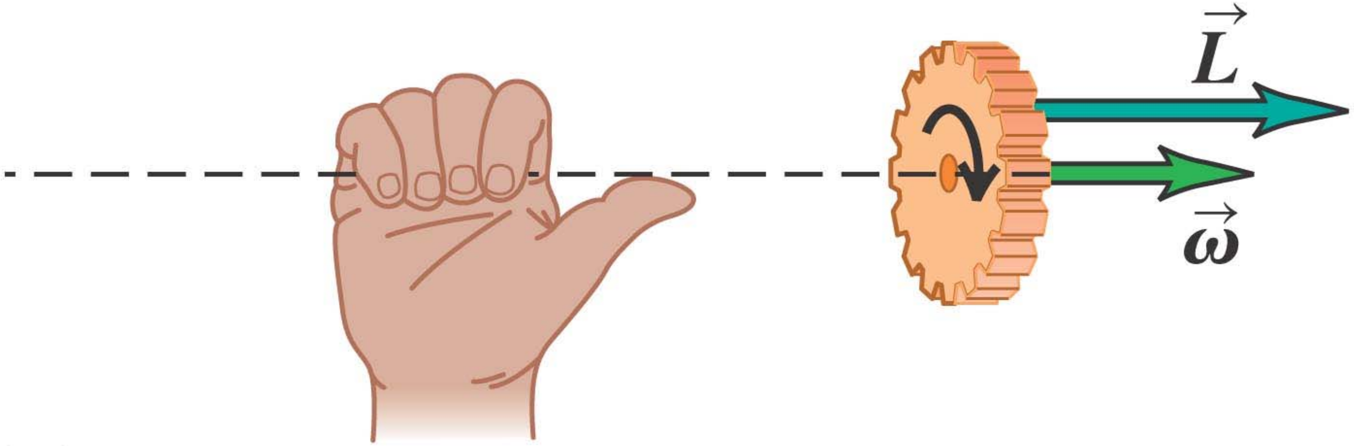
→ vector product of the particle's position vector $\vec{r}$ relative to point *0* and its momentum $\vec{p} = m\vec{v}$

Angular momentum of a rigid body
rotating about a fixed axis of symmetry:

$$\vec{L} = I\,\vec{\omega}$$

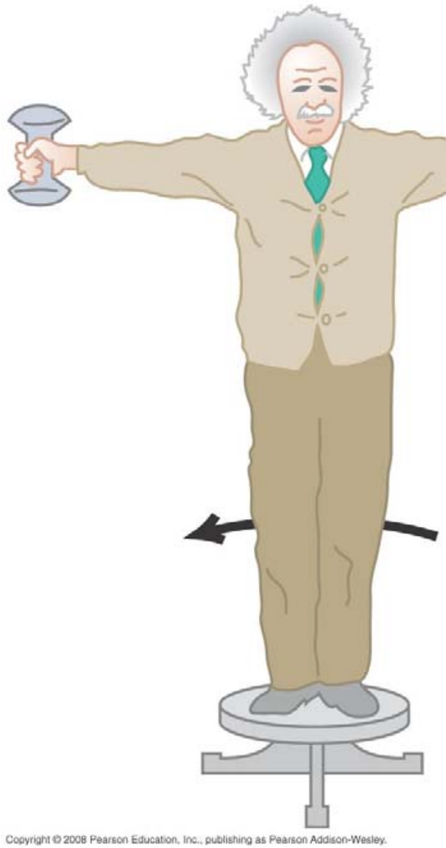
I ... moment of inertia

$\vec{\omega}$ ... angular velocity

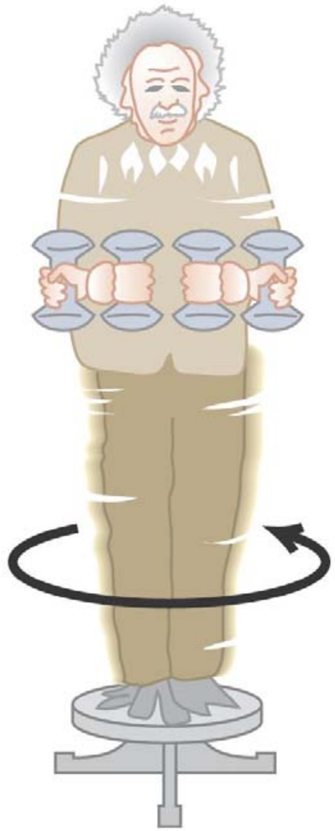


Rotational dynamics and angular momentum

The net external torque on a system is equal to the rate of change of the angular momentum of the system:

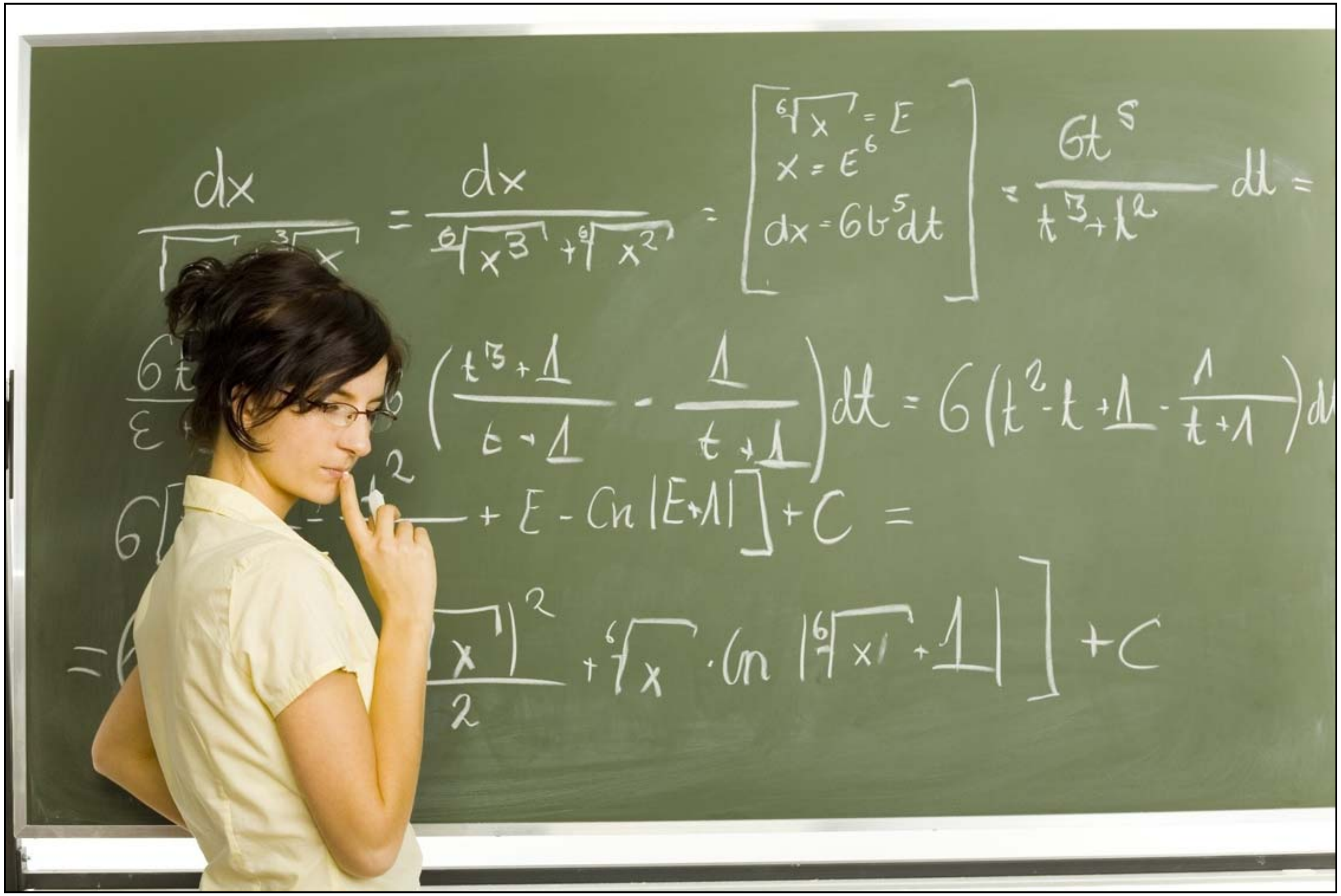


$$\sum \vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}$$



If the net external torque on a system is zero, the total angular momentum of the system is constant (conserved): $\mathrm{d}\vec{L}/\mathrm{d}t = 0$

What will we cover today?



Lesson plan

1. Conditions for equilibrium
2. Center of gravity
3. Rigid-body equilibrium problems
4. Stress, strain, and elastic moduli
5. Elasticity and plasticity