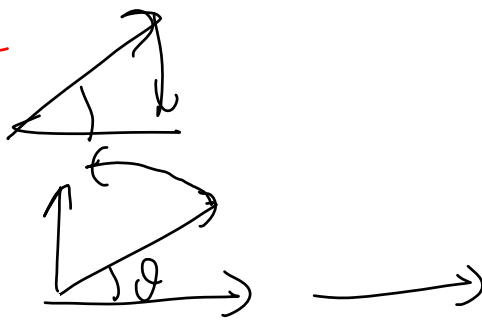


Vector Algebra:

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \times \vec{B} = AB \sin \theta$$



$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Scalar, Vector, Tensor

$$A_{ijk} = i, j, k = 1, \dots, n$$

$$\vec{A} \times \vec{B} = \epsilon_{ijk} A_j B_k \Rightarrow \epsilon_{ijk} = \text{Levi-Civita tensor}$$

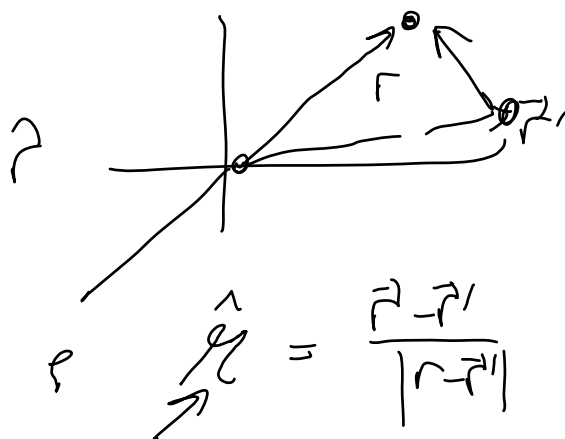
$$\epsilon_{ijk} \epsilon_{ilm} = \delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl}$$

$$\epsilon_{123} = +1$$

$$\epsilon_{213} = -1$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$



$$\vec{A} \rightarrow -\vec{A}$$

$$A_x \rightarrow -A_x$$

$$A_y \rightarrow -A_y$$

$$A_z \rightarrow -A_z$$

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\epsilon_{ijk}$$

pseudo-vector

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = \text{pseudo-scalar}$$

Dif. Calculus

$$f =$$

$$f(x)$$

$$df = \left(\frac{df}{dx} \right) dx$$

∇

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

$$dT \Rightarrow$$

$$T(x, y, z) =$$

$$dT = (\vec{\nabla} T) \cdot d\vec{r}$$

$$\frac{dT}{|d\vec{r}|} = |\vec{\nabla} T|$$

$$|\vec{\nabla} T| = \frac{dT}{|d\vec{r}|} = \frac{dT}{|d\vec{r}| \cos \theta}$$

$$\vec{\nabla} = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$$

$$\vec{\nabla} \times (\vec{A} \times \vec{B})$$

Divergence = $\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl = $\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$

$$\frac{d}{dx}(fg) = f \frac{dg}{dx} + g \frac{df}{dx}$$

$$\frac{d}{dx} \left(\frac{F}{g} \right) = F \frac{d}{dx} \left(\frac{1}{g} \right) + \frac{1}{g} \left(\frac{dF}{dx} \right)$$

$\frac{-dF/dx}{g^2}$
 $\frac{+dF/dx}{g^2}$

$$-F \frac{dF/dx}{g^2} + \frac{1}{g^2} \frac{dF}{dx}$$



216°

360° =



$$-F \frac{dF}{g^2} + \frac{1}{g^2} \frac{dF}{dx} =$$

$$\frac{g \frac{dF}{dx} - F \frac{dF}{dx}}{g^2}$$

↗

$$e \Rightarrow e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\text{Zte!} \quad \left\{ \left(1 + \frac{1}{10}\right) \left(1 + \frac{1}{10}\right) \left(1 + \frac{1}{10}\right) \right\}$$

$$4 \neq 2 \mid \boxed{3} \boxed{5}$$

$$\vec{\nabla} \cdot (\vec{B} \times \vec{C}) = ?$$

$$\epsilon_{ijk} \uparrow \uparrow \uparrow \partial_i \left[B_j \uparrow C_k \uparrow \right]$$

$$A \cdot B = A_i B_i$$

$$A \times B = \epsilon_{ijk} A_j B_k \begin{matrix} A_x B_x + \\ A_y B_y + \\ A_z B_z \end{matrix}$$

$$\epsilon_{ijk} \left[C_k \partial_i B_j + B_j \partial_i C_k \right]$$

$$\epsilon_{ijk} = -\epsilon_{ikj}$$

$$= C_k \underbrace{\epsilon_{ijk} \partial_i B_j}_{\epsilon_{kij} \partial_i B_j} + B_j \underbrace{\epsilon_{ijk} \partial_i C_k}_{\epsilon_{jik} \partial_i C_k}$$

$$= -[-\epsilon_{kij}]$$

$$\epsilon_{kij} \partial_i B_j$$

$$+ \epsilon_{jik} \partial_i C_k$$

$$= \epsilon_{kij}$$

$$C_k (\nabla \times B)_k$$

$$- B_j \epsilon_{jik} \partial_i C_k$$

$$\vec{C} \cdot (\vec{\nabla} \times \vec{B})$$

$$- \vec{B} \cdot (\vec{\nabla} \times \vec{C})$$

$$\vec{\nabla} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{\nabla} \times \vec{B}) - \vec{B} \cdot (\vec{\nabla} \times \vec{C})$$

$$\vec{\nabla} \times (\vec{B} \times \vec{C})$$

$$\int_a^b \left(\frac{df}{dx} \right) dx = f(b) - f(a)$$

$$\int_a^b f(x) dx = f(x) \Big|_a^b = f(b) - f(a)$$

$$\int_a^b (\vec{\nabla} f) \cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$$

Divergence Thm:

$$\int \left(\frac{df}{dx} \right) dx = f(b) - f(a)$$

(Note: The original image contains several scribbles and crossed-out terms like $\int \nabla \cdot \vec{u} d\vec{a}$ and $\int \vec{u} \cdot d\vec{a}$ in this section.)

$$\int (\vec{\nabla} \cdot \vec{u}) d\tau = \oint \vec{u} \cdot d\vec{a}$$

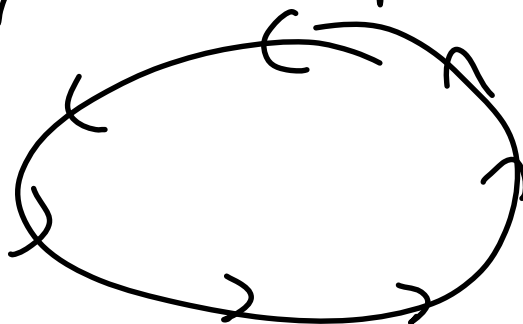
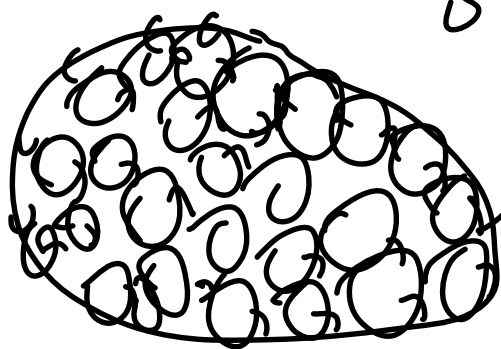
(Note: The original image includes a diagram of a volume element with dimensions dx, dy, dz and a surface element $d\vec{a}$ in this section.)

Curl:

Stokes

Stokes

$$\int_S (\vec{\nabla} \times \vec{u}) \cdot d\vec{a} = \oint_P \vec{u} \cdot d\vec{l}$$



John Poynting

Killing vector

Pointing

$$\int_a^b \frac{d}{dx} (fg) = \int_a^b f \frac{dg}{dx} + \int_a^b g \frac{df}{dx}$$

$$(fg) \Big|_a^b = \int_a^b f \frac{dg}{dx} dx + \int_a^b g \frac{df}{dx} dx$$

$$\int_a^b f \frac{dg}{dx} dx = \underbrace{(fg) \Big|_a^b}_{\text{circled}} - \int_a^b g \frac{df}{dx} dx$$

$$\int_0^b x e^{-x} dx$$

-e^{-x}

$$g = -e^{-x} \quad f = x$$

$$-x e^{-x} - \int e^{-x} dx$$

$$\int x^n e^{-x} dx = -x^n e^{-x} - \int x^{n-1} e^{-x} dx$$

$$\begin{aligned}
 & \int_V F(\underbrace{\vec{\nabla}_i \cdot \vec{A}_i}_{dF}) d\tau \\
 & \underbrace{\int_V \vec{A} \cdot (\vec{\nabla} F) d\tau}_{\text{}} - \oint_S F \vec{A} \cdot d\vec{a} \Big|_a^b \\
 & \underbrace{\int_V F \left(\frac{dg}{dx} \right) dx}_{\text{}} = \int g \left(\frac{dF}{dx} \right) dx
 \end{aligned}$$