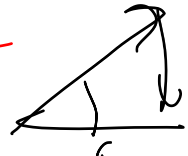
Vector Algebra:

$\vec{A} \cdot \vec{B} = AB\cos\theta$



$\vec{A} \times \vec{B} = AB\sin\theta$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Scalar, Vector, Tensor $\quad A_{ijk}$

$A_{ij} \quad i,j = 1, \dots, n$

$\underline{\vec{A} \times \vec{B} = \epsilon_{ijk} A_j B_k} \Rightarrow \quad \epsilon_{ijk} =$ Levi-Civita tensor

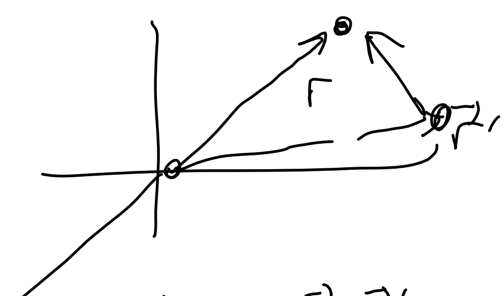
$\epsilon_{123} = +1$

$\epsilon_{213} = -1$

$\epsilon_{ijk}\,\epsilon_{ilm} = \delta_{jl}\,\delta_{km} - \delta_{jm}\,\delta_{kl}$

$\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$

$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$



?

$\hat{\mathscr{r}} = \dfrac{\vec{r} - \vec{r}'}{|r - \vec{r}'|}$

$\vec{A} \to -\vec{A}$

$A_x \to -A_x$

$A_y \to -A_y$

$\vec{A} \times \vec{B} = \vec{C}$

$A_z \to -A_y$

ϵ_{ijk} pseudo-vector

$(\vec{A} \times \vec{B}) \cdot \vec{C} =$ pseudo-scalar

Dif. Calculus

$f \Rightarrow$

$df = \left(\frac{df}{dx}\right)(dx)$

$f(x)$

$\nabla \Delta$

$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

dT

$dT \Rightarrow T(x, y, z) \Rightarrow$

$dT = (\vec{\nabla} T) \cdot d\vec{l}$

$\frac{dT}{|dl|} = |\vec{\nabla} T|$

$|\vec{\nabla} T| \, |dl| \cos\theta$

1

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\vec{\nabla} \times (\vec{A} \times \vec{B})$$

Divergence = $\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

$\partial_i v_i$

Curl = $\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

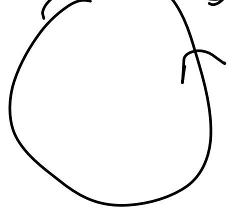
$$\frac{d}{dx}\left(\frac{f}{g}\right) = \underbrace{f\frac{d}{dx}\left(\frac{1}{g}\right)}_{-\frac{dg/dx}{g^2}} + \underbrace{\frac{1}{g}\left(\frac{df}{dx}\right)}_{+\frac{df/dx}{g}}$$

$$-f\frac{dg/dx}{g^2} + \frac{df}{g\,dx}$$

$$-\frac{f}{g^2}\frac{dg}{dx} + \frac{g}{g^2}\frac{df}{dx} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$$

216°

360° =



$e \Rightarrow \quad e^x = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$

$2.7e^1 \quad \{5 \quad \left(1 + \frac{1}{10}\right)\left(1 + \frac{1}{10}\right)\left(1 + \frac{1}{10}\right)$

4 7 2 1 3 6

$\vec{\nabla} \cdot (\vec{B} \times \vec{C}) = ?$ $\quad A \cdot B = A_i B_i$

$A_x B_x + A_y B_y + A_z B_z$

$\epsilon_{ijk}\, \partial_i \left[B_j C_k \right]$ $\quad A \times B = \epsilon_{ijk} A_j B_k$

$\epsilon_{ijk} \left[C_k\, \partial_i B_j + B_j\, \partial_i C_k \right]$ $\quad \epsilon_{ijk} = -\epsilon_{ikj}$

$= C_k \underbrace{\epsilon_{ijk}\, \partial_i B_j} + B_j \underbrace{\epsilon_{ijk}\, \partial_i} C_k$ $\quad = -[-\epsilon_{kij}]$

$\underbrace{\epsilon_{kij}\, \partial_i B_j} + \epsilon_{jik}$ $\quad = \epsilon_{kij}$

$C_k (\nabla \times B)_k \quad - B_j\, \epsilon_{jik}\, \partial_i C_k$

$\vec{C} \cdot (\vec{\nabla} \times \vec{B}) \quad - \vec{B} \cdot (\vec{\nabla} \times \vec{C})$

$\vec{\nabla} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{\nabla} \times \vec{B}) - \vec{B} (\vec{\nabla} \times \vec{C})$

$\underline{\vec{\nabla}} \times (\underline{\vec{B} \times \vec{C}})$

$$\int_a^b \left(\frac{df}{dx}\right) dx = f(b) - f(a)$$

$$\int_a^b 6\, dx = f(x)\Big|_a^b = f(b) - f(a)$$

$$\int_a^b (\vec{\nabla} f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Thm:

$$\int (\vec{\nabla} \cdot \vec{v})\, d\tau = \oint_S \vec{v} \cdot d\vec{a}$$

$d\tau = dx\, dy\, dz$

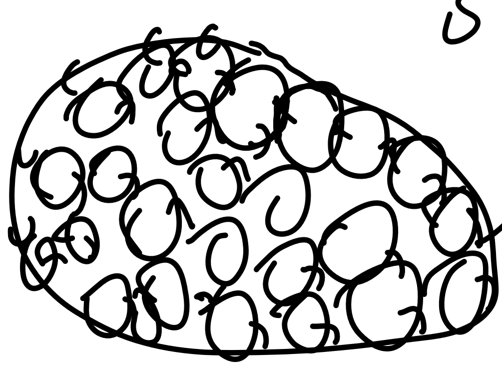
$$\int \left(\frac{df}{dx}\right) dx = f(b) - f(c)$$

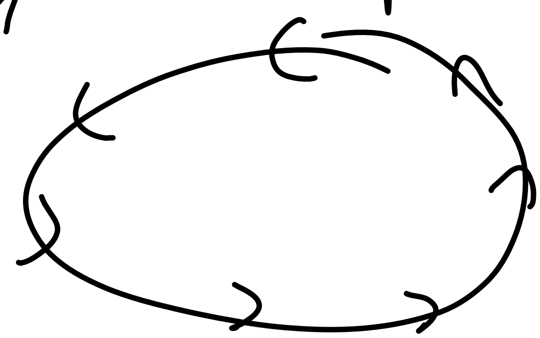
$$\iiint (\vec{\nabla} \cdot \vec{v})\, d\vec{a}\, dx$$

<u>Curl:</u>

$$\text{Stokes'} \int_S (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_P \vec{v} \cdot d\vec{l}$$

Stokes'





John Poyntng Killing vector

Pointing

$$\int_a^b \frac{d}{dx}(fg) = \int_a^b f\frac{dg}{dx} + \int g\frac{df}{dx}$$

$$(fg)\Big|_a^b = \int_a^b f\frac{dg}{dx}dx \int_a^b g\frac{df}{dx}dx$$

$$\int_a^b f\frac{dg}{dx}dx = fg\Big|_a^b - \int_a^b g\frac{df}{dx}dx$$

$$\int_0^b x e^{-x} dx \qquad g = -e^{-x} \qquad f = x$$

$$-e^{-x}$$

$$-xe^{-x} - \int e^{-x} dx$$

$$\int x^n e^{-x} dx = -x^n e^{-x} \int x^{n-1} e^{-x} dx$$

$$\int_V f\,(\vec{\nabla}_i \cdot \vec{A}_i)\,\underset{\sim}{d\tau} \qquad df$$

$$\int_V \vec{A}\cdot(\vec{\nabla} f)\,d\tau \;-\; \oint_S f\,\vec{A}\cdot d\vec{a} \quad -(fg)\Big|_a^b$$

$$\int f\left(\frac{dg}{dx}\right)dx = \int g\left(\frac{df}{dx}\right)dx$$