ITU PHYSICS ENGINEERING. STUDENT, **now21, 16**
IDENTITY NUMBER :
NAME LASTNAME :
FIZ 411E QUIZ – 5 POINT :

[**Question.1**] (*70/100 Pnts*) A (perfect) dipole $\mathbf{p}$ is situated a distance z above an infinite grounded conducting plane (in the following Figure). The dipole makes an angle θ with the perpendicular to the plane.
Find the torque on $\mathbf{p}$.

Hint-1: Use the image method, sketch the figure.
Hint-2: Remember! the perfect dipole important property to express its electric field.
Hint-3: Express the perfect dipole in the polar coordinate.
Hint-4: Take the direction out of page for torque with $-\hat{\boldsymbol{\phi}}$ (by taking $\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}} \ = \ \hat{\boldsymbol{\phi}}$).

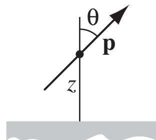

FIG. 1: perfect dipole configuration in the system

[**Answer.1**] With the help of Hint-1,2,3,4:

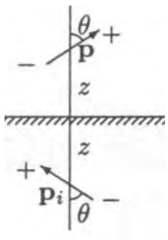

FIG. 2: perfect dipole image configuration in the system

$$\mathbf{N} \ = \ \mathbf{p} \times \mathbf{E}_{\text{image}} \ , \tag{1}$$

here, the electric field expression for perfect dipole image and the perfect dipol expression in the polar coordinate are given, respectively:

$$\mathbf{E}_{\text{image}} \ = \ k_e \frac{p}{(2z)^3} \left(\hat{\mathbf{r}} \ 2cos\theta + \hat{\boldsymbol{\theta}} \ sin\theta\right) \ ,$$
$$\mathbf{p} \ = \ p \left(\hat{\mathbf{r}} \ cos\theta + \hat{\boldsymbol{\theta}} \ sin\theta\right) \ .$$

By substituting these into eq.(1),

$$\mathbf{N} \ = \ \frac{k_e \ p^2}{8z^3} \left((\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}}) \ cos\theta \ sin\theta \ + \ (\hat{\boldsymbol{\theta}} \times \hat{\mathbf{r}}) \ 2 \ sin\theta \ cos\theta)\right) \ ,$$
$$\therefore \mathbf{N} \ = \ - \ \hat{\boldsymbol{\phi}} \frac{k_e \ p^2}{16z^3} \ sin \ 2\theta \ . \tag{2}$$

here we used the trigonometrical identity as $2sin\theta \ cos\theta \ = \ sin2\theta$ and $\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}} \ = \ \hat{\boldsymbol{\phi}}$.

The direction of torque is out of the page.

[**Question.2**] (*30/100 Pnts*) A sphere of radius R carries a polarization $\mathbf{P}(\mathbf{r}) = \alpha\mathbf{r}$, where α is a constant and $\mathbf{r}$ is the vector from the center. Calculate the expressions σ_b and ρ_b for the bound charges.

Hint-5: Remember! Divergence operator for spherical coordinates.

[**Answer.2**] We know that

$$\sigma_b \;=\; \mathbf{P}\cdot\hat{\mathbf{n}} \text{ and } \rho_{\mathbf{b}} \;=\; -\boldsymbol{\nabla}\cdot\mathbf{P}\,. \tag{3}$$

Then for this question, by applying Hint-5 we obtain their values as

$$\begin{aligned} \sigma_b \;&=\; \alpha\, R\,(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}) \quad \text{and} \quad \rho_b \;=\; -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\;\alpha\; r)\;(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}), \\ \therefore \sigma_b \;&=\; \alpha\, R \quad \text{and} \quad \rho_b \;=\; -3\,\alpha\,. \end{aligned} \tag{4}$$