5.2 Statistical Methods and Using Diagrams in the Prediction of Ship’s Resistance

Both the diagrams published and the statistical methods resulted in regression formulas are based on systematical experimental measurements and previous design data obtained from existing ships.

a) Using Diagrams in Power Prediction

Some of the methods using diagrams are given in the following. Only Guldhammer & Harvald’s Diagrams are given in full detail.

i) Taylor’s and Gertler’s Diagrams

First, Taylor (1933) established his diagrams depending on systematic hull series and related experimental data. Then Gertler (1954) reanalyzed and improved Taylor’s test data.

ii) Lap’s Diagrams

Depending on a large number of tests performed between 1935-1955 in NSMB (Netherland’s Ship Model Basin), Lap made an attempt to establish a calculation method by means of diagrams. In Lap’s method the resistance of the ship (without any roughness effect) is given by

\[ R = \left( C_{Fh} \zeta R \frac{A}{S} \right) \left( \frac{P}{2} SV^2 \right) \]

where \( \zeta R \) is obtained from the diagrams given as function of B/T, prismatic coefficient \( \phi \) and of a special speed-length ratio \( (V/\sqrt{\phi L}) \). Lap introduces roughness correction and service condition increase as well. (See: Lap, A. J. W., (1956, 1957) “Resistance (Fundamentals of Ship and Propulsion)”, International Shipbuilding Progress, Vol. 3, No. 24, 25, 28, Vol. 4, No. 29)

iii) Guldhammer’s and Harvald’s Diagrams

In 1965, Guldhammer and Harvald by assembling and coordinating the test data in DTU (Technical University of Denmark), organised diagrams in \( L/\sqrt{h} \) to give residual resistance as function of speed and prismatic coefficient. A revision was made in 1974 by Guldhammer and Harvald. Residual resistance coefficient is determined from the diagrams. The dashed lines in the diagrams indicate that they are based on very few test data or obtained by extrapolation and therefore uncertainty is relatively higher in those regions. Since the diagrams are given for the standard ship in terms of B/T, LCB, character of the sectional curves and the bow geometry, a series of corrections are required regarding with these characteristics. Appendage resistance and other conditional increments should also be taken into account. Here is the summary of the application of the method:
1st; make ready the following characteristics of the ship, 
$L_{pp}$ and $L_{WL}$, $(Fr = V / \sqrt{gL})$, $B$, $T$, $\Delta$, $\delta(C_B)$, $\phi(C_p)$, $B(C_M)$, 
$L/\sqrt{V^{1/3}}$, $LCB$ and $\Delta LCB = (LCB_{Actual} - LCB_{Standard})$, shape of sections and bow. 

Then read the $C_R$ value from the diagrams. (Interpolations may be necessary.)

2nd; $B/T$ correction: since the diagrams are prepared for standard $B/T=2.5$, a correction $C_R$ is required for the actual $B/T$ as:

$$10^3 C_R = 10^3 C_{R(Standard)} + 0.16(B/T - 2.5)$$

3rd; $LCB$ correction: The standard $LCB$ position can be read from the following figure (Harvald (1991)):

Indeed, there is no need to make $LCB$ correction for Froude numbers less than 0.15. The deviation in $LCB$ is then determined by $\Delta LCB = LCB_{Actual} - LCB_{Standard}$ (used as $LCB$ in % of $L$).

Then, the corrected residual resistance:

$$10^3 C_R = 10^3 C_{R(Standard)} + \frac{\partial 10^3 C_R}{\partial LCB} |\Delta LCB|$$

where the second term in the r.h.s. of the expression can be obtained from the following figure (Harvald (1991)):
There is no correction for LCB when the position of LCB remains aft of the standard value.

4th; Hull form corrections: The forms used in towing tank in this method are neither distinctly U-shaped nor V-shaped. If the sections of the actual ship are distinctly U or V shaped, the following corrections to $10^3 C_R$ should be made:

<table>
<thead>
<tr>
<th></th>
<th>Extreme U</th>
<th>Extreme V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fore Body</td>
<td>-0.1</td>
<td>+0.1</td>
</tr>
<tr>
<td>Aft Body</td>
<td>+0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

These corrections are recommended for the speed range of $0.2 \leq Fr \leq 0.25$. With regard to bulb application, standard form does not have a bulbous bow, so that a correction is required if the actual ship has a bulb. For a vessel with bulbous bow having $A_{BT}/A_x \geq 0.1$ (where $A_{BT}$ is the sectional area of the bulbous bow at the fore perpendicular and $A_x$ is the area of the midship section) the following corrections to $10^3 C_R$ are recommended:

<table>
<thead>
<tr>
<th>$C_p(\varphi)$</th>
<th>Fr=0.15</th>
<th>0.18</th>
<th>0.21</th>
<th>0.24</th>
<th>0.27</th>
<th>0.30</th>
<th>0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>+0.2</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>+0.2</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>+0.20</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>+0.1</td>
<td>0.0</td>
<td>-0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For $0 < A_{BT}/A_x < 0.1$, the corrections may be assumed to be proportional with size of the bulb. Note that these corrections are valid for loaded conditions only.

5th; Appendage correction on $C_R$.

a) There is no need to make corrections to $C_R$ due to rudders and bilge keels
b) For full ships add 3-5 % to $C_R$ due to bossing.
c) For fine ships add 5-8 % to $C_R$ due to shaft brackets and shafts.
6th; Roughness correction: For model-ship correlation due to roughness, following is suggested:

For vessels with
\[ L \leq 100 \text{m} \quad 10^3 C_A = 0.4 \]
\[ L \leq 150 \text{m} \quad 10^3 C_A = 0.2 \]
\[ L \leq 200 \text{m} \quad 10^3 C_A = 0 \]
\[ L \leq 250 \text{m} \quad 10^3 C_A = -0.2 \]
\[ L \leq 300 \text{m} \quad 10^3 C_A = -0.3 \]

An alternative way of calculating \( C_A \) is also given by:

<table>
<thead>
<tr>
<th>Displacement</th>
<th>( C_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 ton</td>
<td>( 0.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>10000 ton</td>
<td>( 0.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>100000 ton</td>
<td>0.0</td>
</tr>
<tr>
<td>1000000 ton</td>
<td>( -0.6 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

7th; Frictional resistance coefficient can be calculated from
\[ C_F = \frac{0.075}{(\log \text{Re} - 2)^2} \]

Meantime correction to \( C_F \) due to appendages is made simply by increasing \( C_F \) according to
\[ C_F' = C_F \frac{S_f}{S} \]
where \( S \) is the bare wetted surface of the hull and \( S_f \) is the wetted surface of the hull and appendages.

8th; Increase due to air resistance and steering resistance. As the magnitude of the air resistance is of minor importance on the one hand and it is not clear how to determine the wind direction and speed on the other; naval architects usually employ a general formula of \( 10^3 C_{AA} = 0.07 \), if required. The correction for steering resistance may be taken as;
\[ 10^3 C_{AS} = 0.04 \]

For ships having satisfactory directional stability (course keeping capability), \( C_{AS} \) may be omitted.
Thus, $C_T$ is the sum of the corrected $C_R$, $C_F$, $C_A$ and other components if any. The following diagrams of Guldhammer & Harvald are for $C_R$ values for the standard ship.
\[
\frac{L}{V^{1/3}} = 5.5
\]
\( \frac{L}{V^{1/2}} = 7.5 \)
\[
\frac{L}{\sqrt[3]{f}} = 8.0
\]
Example:

Main characteristics of a ship with design speed of $V=12.15$ kn are given in the following:

$L_{WL}=55.50$ m, $B=9.00$ m, $T=3.10$ m, $\Delta=1122.7$ ton, $S=711.6$ m$^2$, $S_l/S=1.01$, $C_B(\delta)=0.708$, $C_P(\varphi)=0.742$, $C_M(B)=0.954$ and there is shaft bossing, LCB=0.326 m (0.59 %) backward, $\rho=1025$ kg/m$^3$, $\nu=1.19 \times 10^{-6}$ m$^2$/s.

Let’s proceed, first, by calculating required parameters:

$V=12.15$ kn=6.25 m/s; $(V/\sqrt{gL}) = 0.268$; $\Delta=1122.7$ ton, $V=\Delta/\rho=1095.34$ m$^3$.

$\therefore \quad L/V^{1/3}=5.0$ and $B/T=2.9$.

Using diagrams – Fig. 5.5.7 and 5.5.8 of Harvald (1991):

$L/V^{1/3} = 5.0; \quad 10^3 C_R = 3.75$ (at $Fr=0.268$ and $\varphi=0.74$)
\[ L / \sqrt[3]{V} = 5.5 ; \quad 10^3 C_R = 3.10 \text{ (at Fr=0.268 and } \phi=0.74) \]

Linear interpolation for \( L / \sqrt[3]{V} = 5.38 \) gives \( 10^3 C_R = 3.256 \).

**Corrections:**

- **B/T correction:** \( 0.16(B/T-2.5) = 0.16(2.9-2.5) = 0.064 \) should be added to \( 10^3 C_R \).
- **LCB correction:** First, determine \( \text{LCB}_{\text{Standard}} \) from the figure that \( \text{LCB}_{\text{Standard}} = 2.3 \% \text{ aft}(-) \)
  \[
  \Delta \text{LCB} = \text{LCB} - \text{LCB}_{\text{Standard}} = (-0.59 - (-2.3)) = 1.71 \%
  \]
  From the figure which gives \( \frac{\partial 10^3 C_R}{\partial \text{LCB}} = 0.42 \) (at \( \phi=0.74 \))
  \[
  \therefore \text{Addition to } 10^3 C_R : \quad \frac{\partial 10^3 C_R}{\partial \text{LCB}} |_{\Delta \text{LCB}} = 0.42 \cdot 1.71 = 0.72
  \]
- **We may skip hull form corrections in this sample problem.**
- **Due to bossing increase the \( C_R \) by 4 \%: \( 0.04 \times 3.256 = 0.13 \)**
  \[
  \therefore \text{The resultant } C_R : \quad 10^3 C_R = 3.256 + 0.064 + 0.72 + 0.13 = 4.17
  \]
- **Frictional resistance** \( C_f = \frac{0.075}{(\log \text{Re} - 2)^2} = 1.795 \times 10^{-3} \)
- **Roughness correction:** \( \text{since } L \leq 100 \text{m}, \text{then } 10^3 C_A = 0.4 \)
- **Appendage effect** \( 10^3 C_{F'} = C_F \frac{S_l}{S} = 1.795(1.01) = 1.813 \)
  \[
  \therefore \text{Total resistance coefficient} : \quad 10^3 C_T = 1.813 + 4.17 + 0.4 = 6.383
  \]
  \[
  \therefore \text{Total resistance} : \quad R_T = C_T \left( \frac{\rho}{2} SV^2 \right) = 0.006383(14245.90) = 90.93 \text{ kN}
  \]
  **Effective power** (at \( V=12.15\text{kn} \)) : \( P_E = R_T V = 90.93(6.25) = 568.3 \text{ kW} \)

**b) Statistical Methods by Regression Formulae**

Statistical methods use regression equations which can be obtained mathematically by least squares methods, based on a series of result from towing tests. The regression equation, therefore, links the form data of ships to the resistance data (results) by minimizing the error between the result of the proposed equation and the experimental resistance data. Doust (1962), in Trondheim, pioneered to develop a regression formula that expresses ship resistance for certain basic form parameters of a particular ship type.

In the following, we first mention well-known statistical methods and then focus on Holtrop & Mennen’s (1982) method which may be regarded as most popular and reliable for conventional ship forms.
For small displacement hulls such as tugs and trawlers with $0.52 \leq C_P \leq 0.70$, $3.4 \leq L/B \leq 6.2$, $1.9 \leq B/T \leq 3.4$, $0.73 \leq C_M \leq 0.98$. Speed range: $0.05 \leq Fr \leq 0.5$. It is reported that the error range is generally less than 12%.

For BSRA Series ship hulls valid within BSRA series’ constraints.

For displacement ships with $0.525 \leq C_B \leq 0.725$, $+1(\%L) \leq LCB \leq -4(\%L)$. Speed range : $0.18 \leq Fr \leq 0.30$. Standard error estimated is around 2%.

For general cargo ships with $20 \leq L \leq 450m$, $0.50 \leq C_B \leq 0.875$, $3.5 \leq L/B \leq 9.0$, $2.0 \leq B/T \leq 5.0$. Speed range is: $0.1 \leq Fr \leq 0.34$.

For displacement hulls with $0.55 \leq C_P \leq 0.85$, $3.9 \leq L/B \leq 14.9$, $2.1 \leq B/T \leq 4.0$. Speed range is: $0.05 \leq Fr \leq 1.0$. Regarded as a complete and reliable methods especially for cruiser stern ships. The method may result in an under-prediction in transom stern ships.

For semi displacement hulls with transom stern, round bilge/hard chine craft. Good correlation within the limits of original data, it can also be used for high-speed displacement craft.


For planning craft with constant deadrise less than 30°. Generally the method over-predicts with L/B > 5.0 and under-predicts with low deadrise.


For single and twin-screw displacement ships with 0.601 ≤ C_B ≤ 0.830 (for single-screw), 0.512 ≤ C_B ≤ 0.775 (for twin-screw), 4.71 ≤ L/B ≤ 7.106 (for single-screw), 3.96 ≤ L/B ≤ 7.13 (for twin-screw), 1.989 ≤ B/T ≤ 4.002 (for single-screw), 2.308 ≤ B/T ≤ 6.110 (for twin-screw).

The speed range is defined as a function of C_B for single-screw and twin-screw cases.

It is reported that for single-screw ships, the method shows similar errors with those of the other well-known methods, but performs better in case of twin-screw ships.


As a latest method; suitable especially for tankers and transom stern ships.

Now let’s review the outlines of the Holtrop & Mennen (1982) method: In this method, total resistance of a ship treated as:

\[ R_p = R_f (1+k_1) + R_{App} + R_w + R_g + R_{TR} + R_A \]

\( R_f \): frictional resistance (ITTC-1957 formula)

\( (1+k_1) \): form factor of the hull form

\[ =c13 \left\{ 0.93 + c_{12} \left( B/L_p \right)^{0.92497} (0.95 - C_p)^{-0.521448} (1 - C_p + 0.0225L_{CB})^{0.6906} \right\} \]

where LCB is forward of 0.5L as percentage of L,

\[ L_p/L = 1 - C_p + 0.06C_p(LCB)/(4C_p - 1) \]

\( C_{12} = T/L^{0.2228446} \) when T/L>0.05

\( C_{12} = 48.20(T/L-0.02)^{2.078} + 0.479948 \) when 0.02<T/L<0.05

\( C_{12} = 0.479948 \) when T/L<0.02

\( C_{13} = 1+0.003C_{stem} \) when \( C_{stem} = \begin{cases} -10 & \text{for V shaped sec.} \\ 0 & \text{for normal sec.} \\ +10 & \text{for U shaped sec.} \end{cases} \)
The ship’s resistance is given by

\[ R_{\text{App}} = \frac{1}{2} \rho V^2 S_{\text{App}} (1 + k_2) c_p S \]  

(C_F ship’s friction resistance) and \((1+k_2)\) are for streamlined appendages:

<table>
<thead>
<tr>
<th>Appendage Configuration</th>
<th>(1+k_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rudder behind skeg</td>
<td>1.5-2.0</td>
</tr>
<tr>
<td>rudder behind stern</td>
<td>1.3-1.5</td>
</tr>
<tr>
<td>twin screw balanced rudders</td>
<td>2.8</td>
</tr>
<tr>
<td>shaft brackets</td>
<td>3.0</td>
</tr>
<tr>
<td>skeg</td>
<td>1.5-2.0</td>
</tr>
<tr>
<td>strut bossings</td>
<td>3.0</td>
</tr>
<tr>
<td>hull bossings</td>
<td>2.0</td>
</tr>
<tr>
<td>shafts</td>
<td>2.0-4.0</td>
</tr>
<tr>
<td>stabilizers fins</td>
<td>2.8</td>
</tr>
<tr>
<td>dome</td>
<td>2.7</td>
</tr>
<tr>
<td>bilge keels</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Combination of appendages gives:

\[(1+k_2)_{eq} = \frac{\sum (1+k_2)S_{\text{App}}}{\sum S_{\text{App}}}\]

If there is a bow thruster opening: \(\rho V^2 \pi d^2 C_{BTc}\) should be added to appendage resistance where \(C_{BTc} \approx 0.003-0.012\).

\[ R_w = c_1 c_2 c_3 \sqrt{\rho g} \exp( \frac{m_1 F r^{0.9} + m_2 \cos(\lambda Fr^{-2})}{m_1} ) \]

where

\[ c_1 = 2223105c_7^{3.70613} (T / B)^{1.07961} (90 - i_E)_{-1.37565} \]

\[ c_7 = 0.229577 (B / L)^{0.33333} \text{ when } B / L < 0.11 \]

\[ c_7 = B / L \text{ when } 0.11 \leq B / L < 0.25 \]

\[ c_7 = 0.5 - 0.0625 (L / B) \text{ when } B / L > 0.25 \]

\[ c_2 = \exp[-1.89 \sqrt{c_3}] \]

\[ c_3 = 0.56A_{BT}^{1.5} [(BT) (0.31 \sqrt{A_{BT}} + T_F - h_b)] \]

where \(A_{BT}\) is the transversal area of the bulb section at F.P. and \(h_b\) is the centroid of the area \(A_{BT}\) from the keel line. \(T_F\): draft at F.P.

\[ c_4 = 1 - 0.8A_r / (BTC_m) \]: \(A_T\) denotes the immersed part of the transom area.

\[ \lambda = 1.446C_p - 0.03L / B \quad \text{when } L / B < 12 \]

\[ \lambda = 1.446C_p - 0.36 \quad \text{when } L / B > 12 \]

\[ m_1 = 0.01440407L / T - 1.75254 \sqrt[1/3]{L} - 4.79323B / L - c_{16} \]
\[ c_{16} = 8.07981C_p - 13.8673C_p^2 + 6.984388C_p^3 \] when \( C_p < 0.80 \)

\[ c_{16} = 1.73014 - 0.7067C_p \] when \( C_p > 0.80 \)

\[ m_2 = c_{15}C_p^2 \exp[-0.1F \gamma^{-2}] \]

\[ C_{15} = -1.69385 \text{ for } \frac{L}{\gamma} < 512 \]

\[ C_{15} = 0.0 \text{ for } \frac{L}{\gamma} > 1727 \]

\[ C_{15} = -1.69385 + \frac{(L/\gamma^{1/3} - 8.0)/2.36}{512 < \frac{L}{\gamma} < 1727} \]

\( i_E \): half angle of entrance in degrees (also given by an empirical formula in Holtrop & Mennen’s paper).

Existence of a bulbous bow near the free surface requires the following additional resistance:

\[ R_B = 0.11 \exp[-3P_B^{2}]F_r^3A_{ht}\rho g/(1 + F_r^2) \]

where \( P_B = 0.56 \sqrt{A_{ht}}/(T_f - 1.5h_B) \)

\[ F_r = V/\sqrt{g/T_f - h_B - 0.25 \sqrt{A_{ht}} + 0.15V^2} \]

Similarly, additional resistance due to the immersed transom:

\[ R_{TR} = 0.5\rho V^2A_f c_6 \]

\[ c_6 = 0.2(1 - 0.2F_{rT}) \] when \( F_{rT} < 5 \)

\[ c_6 = 0.0 \] when when \( F_{rT} \geq 5 \)

\( F_{rT} \) is defined as: \( F_{rT} = V/\sqrt{2gA_f/(B + BC_{WP})} \) \( C_{WP} \) waterplane area coefficient

The model –ship correlation is given by:

\[ R_A = 0.5\rho SV^2C_A \]

\[ C_A = 0.06(L + 100)^{0.16} - 0.00205 + 0.003\sqrt{L/7.5C_g^4c_2}(0.04 - c_4) \]

\[ c_4 = T_f/L \] when \( T_f/L \leq 0.04 \)

\[ c_4 = 0.04 \] when \( T_f/L > 0.04 \)

(At this point ITTC-1978 \( C_A \) formula can alternatively be used).

By the use of the above formulae given, one may obtain the total resistance and subsequently the effective power. Holtrop & Mennen’s paper also gives the propulsion formulae from which the break or shaft power of the main engine can be calculated.