Istanbul Technical University Faculty of Mechanical Engineering

MAK422E Engineering Design and CAD Midterm Exam, Nov.14, 2012

Instructor: Hikmet Kocabas Time: 90 minutes

1. (10) Write down the input devices in CAD.

Keyboard (bluetooth projection keyboard),
mouse,
joystick (pin stick),
lightpen,
microphone,
scanner,
camera,
glove (hand),
touchpad,
touchscreen

2. (10) How to calculate the transformation matrix of a mirror about an arbitrary axis that is not through the origin?

$$Mirx := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Rz(\theta) := \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\theta = \operatorname{atan}\left(\frac{P2_y - P1_y}{P2_x - P1_x}\right) \qquad T = \begin{pmatrix} 1 & 0 & 0 & P1_x \\ 0 & 1 & 0 & P1_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad T1 = \begin{pmatrix} 1 & 0 & 0 & -P1_x \\ 0 & 1 & 0 & -P1_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P0' = T \cdot Rz(\theta) \cdot Mirx \cdot Rz(-\theta) \cdot T1 \cdot P0$$

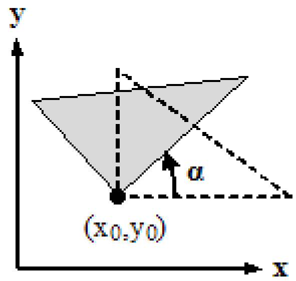
3. (10) What are the differences between **Hermite**, **Bezier** and **Spline** curves.

Hermite curve is defined two end points (P_1 , P_2) and two tangents at the ends (P_1' , P_2'). Curve degree is limited to 3.

Bezier curve uses control points for approximate form, or points on curve for interpolate form. Curve degree is (n-1), where n is the number of points.

Spline curve is the same of Bezier, except curve degree is defined by user.

4. **(20)** Consider a 2D problem in which it is desired to create a rotation transformation that will cause an object to be rotated about a point defined by (x0,y0). Determine the necessary 2D transformation matrix, T, that will accomplish this.

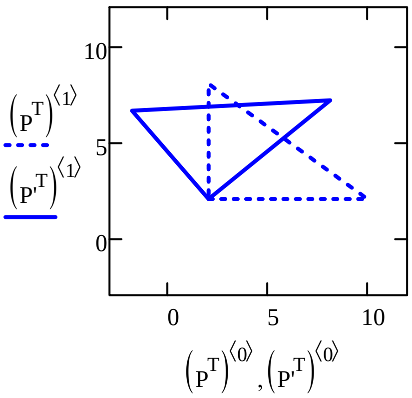


Sample: Rotate **α=40º** the 3 line segments formed by points $\mathbf{P_0=(2,2)}$, $\mathbf{P_1=(10,2)}$ and $\mathbf{P_2=(2,8)}$ around the $\mathbf{P_0=(2,2)}$.

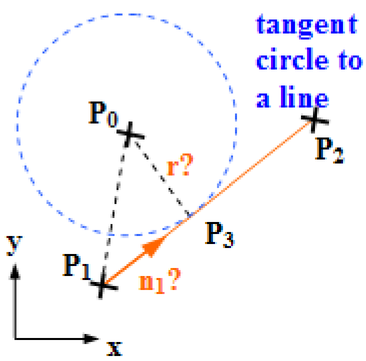
$\alpha := 40\text{deg}$ $\qquad Rz(\alpha) := \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\qquad x_0 := 2 \qquad y_0 := 2$

$T := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad T1 := \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \qquad P := \begin{pmatrix} 2 & 10 & 2 & 2 \\ 2 & 2 & 8 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$P' := T \cdot Rz(\alpha) \cdot T1 \cdot P \qquad P' = \begin{pmatrix} 2 & 8.128 & -1.857 & 2 \\ 2 & 7.142 & 6.596 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix}$



5. **(25)** Find the tangent point $\mathbf{P_3}$ and radius $\mathbf{r}$ of circle defined by center $\mathbf{P_0}$**=(2,6)** to line between $\mathbf{P_1}$**=(1,2)**, $\mathbf{P_2}$**=(7,6)** ?



$$P_0 := \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix} \qquad P_1 := \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \qquad P_2 := \begin{pmatrix} 7 \\ 6 \\ 0 \end{pmatrix}$$

$$n_1 := \frac{(P_2 - P_1)}{|P_2 - P_1|} \qquad n_1 = \begin{pmatrix} 0.832 \\ 0.555 \\ 0 \end{pmatrix} \qquad |n_1| = 1$$

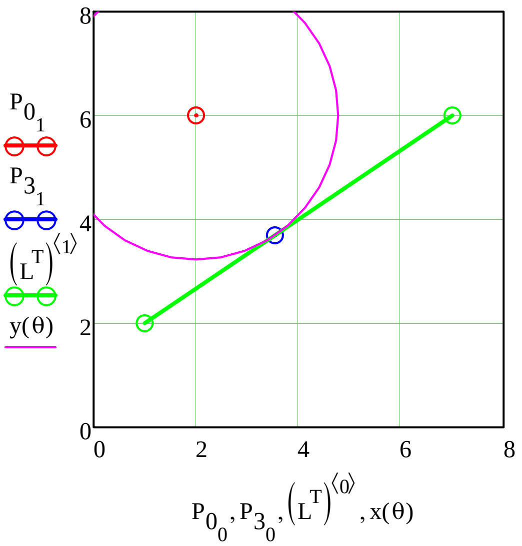
$$r := |n_1 \times (P_0 - P_1)| \qquad r = 2.774$$

$$P_3 := P_1 + n_1 \cdot [n_1 \cdot (P_0 - P_1)] \qquad P_3 = \begin{pmatrix} 3.538 \\ 3.692 \\ 0 \end{pmatrix}$$

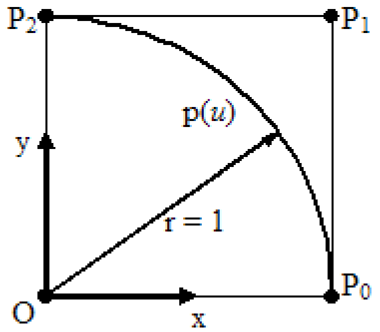
$$L := \text{augment}(P_1, P_2)$$

$$\theta := 0\text{deg}, 10 \cdot \text{deg} .. 360 \cdot \text{deg}$$

$$x(\theta) := P_{0_0} + r \cdot \cos(\theta) \qquad y(\theta) := P_{0_1} + r \cdot \sin(\theta)$$



6. (25) Consider the following cubic Bezier curve composed of three control points as following in the xy plane: P0=(1,0), P1=(1,1), P2=(0,1). Compute the point p(1/2). Check the radius of the arc curve at this point.



$n := 2$

$C(n,i) := \dfrac{n!}{i!\cdot(n-i)!}$

$B(i,n,u) := C(n,i)\cdot u^{i}\cdot(1-u)^{n-i}$

$C(n,i) \to \dfrac{2}{(2-i)!\cdot i!}$

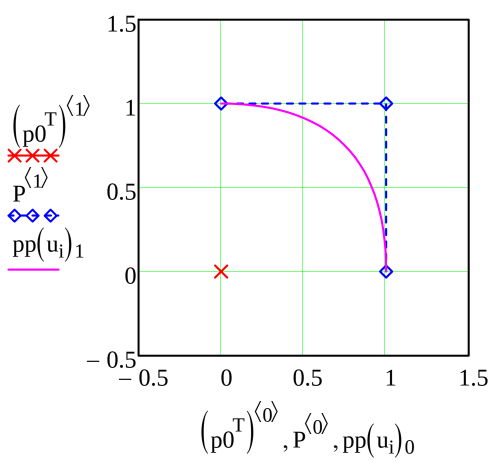
$$\sum_{i=0}^{n}\left(B(i,n,u)\cdot p_i\right) \to (u-1)^2\cdot p_0 + u^2\cdot p_2 - 2\cdot u\cdot(u-1)\cdot p_1$$

$u := \dfrac{1}{2}$ $\qquad \sum_{i=0}^{n}\left(B(i,n,u)\cdot p_i\right) \to \dfrac{p_0}{4} + \dfrac{p_1}{2} + \dfrac{p_2}{4}$ $\qquad p0 := \begin{pmatrix}0\\0\end{pmatrix}$

$p_0 := \begin{pmatrix}1\\0\end{pmatrix}$ $\quad p_1 := \begin{pmatrix}1\\1\end{pmatrix}$ $\quad p_2 := \begin{pmatrix}0\\1\end{pmatrix}$ $\quad P := \mathrm{stack}\left(p_0^{T}, p_1^{T}, p_2^{T}\right)$ $\quad P = \begin{pmatrix}1 & 0\\1 & 1\\0 & 1\end{pmatrix}$

$pp(u) := \sum_{i=0}^{n}\left(B(i,n,u)\cdot p_i\right)$ $\qquad pp\left(\dfrac{1}{2}\right) \to \begin{pmatrix}\dfrac{3}{4}\\ \dfrac{3}{4}\end{pmatrix}$ $\qquad pp(u) = \begin{pmatrix}0.75\\0.75\end{pmatrix}$

$r12 := \left|pp\left(\dfrac{1}{2}\right) - p0\right|$ $\qquad r12 = 1.061$ $\qquad i := 0..30$ $\qquad u_i := \dfrac{i}{30}$



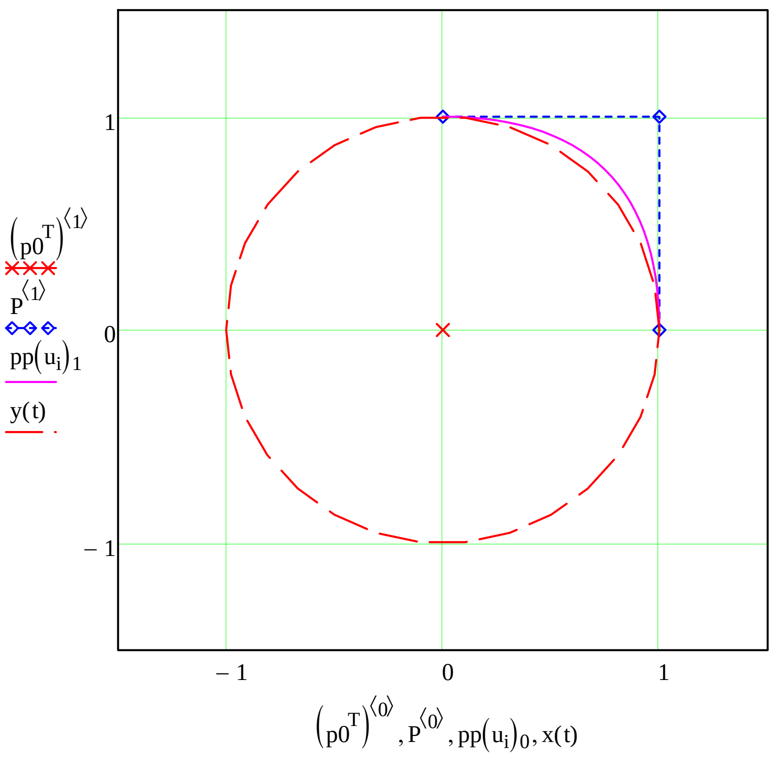
$pp(u_i)_0 =$	$pp(u_i)_1 =$
1	0
0.999	0.066
0.996	0.129
0.99	0.19
0.982	0.249
0.972	0.306
0.96	0.36
0.946	0.412
0.929	0.462
0.91	0.51
...	...

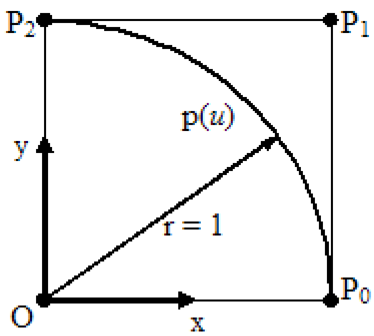
Plotting a Circle

$x_c := 0$ $\quad y_c := 0$ $\quad r := 1$

$n := 30 \qquad j := 0..n \qquad i := 0..n \qquad t_j := \dfrac{j \cdot 2 \cdot \pi}{n}$

$x(t) := r \cdot \cos(t) + x_c \qquad y(t) := r \cdot \sin(t) + y_c$





$n := 8$

$C(n,i) := \dfrac{n!}{i! \cdot (n-i)!}$

$B(i,n,u) := C(n,i) \cdot u^i \cdot (1-u)^{n-i}$

$P_0 := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad P_1 := \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad P_2 := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad P_3 := \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad P_4 := \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad P_5 := \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$P_6 := \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad P_7 := \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad P_8 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$P := \mathrm{stack}\left(p_0^{T}, p_1^{T}, p_2^{T}, p_3^{T}, p_4^{T}, p_5^{T}, p_6^{T}, p_7^{T}, p_8^{T}\right)$$

$$pp(u) := \sum_{i=0}^{n} \left(B(i,n,u)\cdot p_i\right) \qquad pp\left(\frac{1}{2}\right) \rightarrow \begin{pmatrix} -\frac{41}{64} \\ 0 \end{pmatrix} \qquad pp(u) = \begin{pmatrix} 4.333 \\ 3.109 \times 10^{-15} \end{pmatrix}$$

$$i := 0..30 \qquad u_i := \frac{i}{30}$$

$$P = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ -1 & 1 \\ -1 & 0 \\ -1 & -1 \\ 0 & -1 \\ 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$pp(u_i)_0 =$	$pp(u_i)_1 =$
1	0
0.971	0.237
0.892	0.423
0.775	0.564
0.631	0.666
0.47	0.732
0.302	0.766
0.133	0.768
-0.03	0.742
-0.18	0.69
-0.314	0.615
-0.429	0.518
...	...