
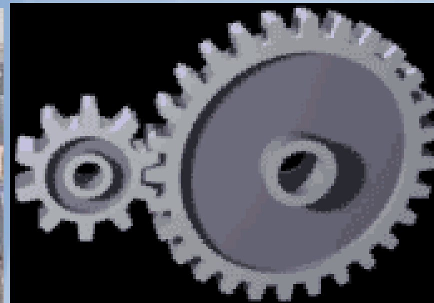

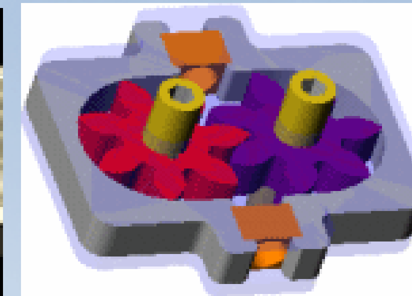
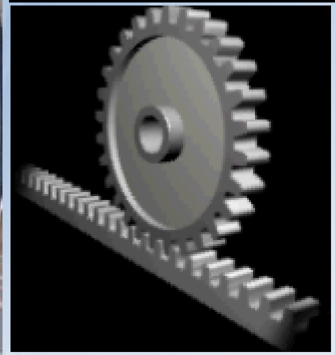

Machine Desing II
Transmission Mechanisms
Makina Elemanları II
Hareket İletim Mekanizmaları

Prof. Dr Hikmet Kocabaş
İ.T.Ü. Makina Fakültesi
ITU Faculty of Mechanical Engineering

Referanslar

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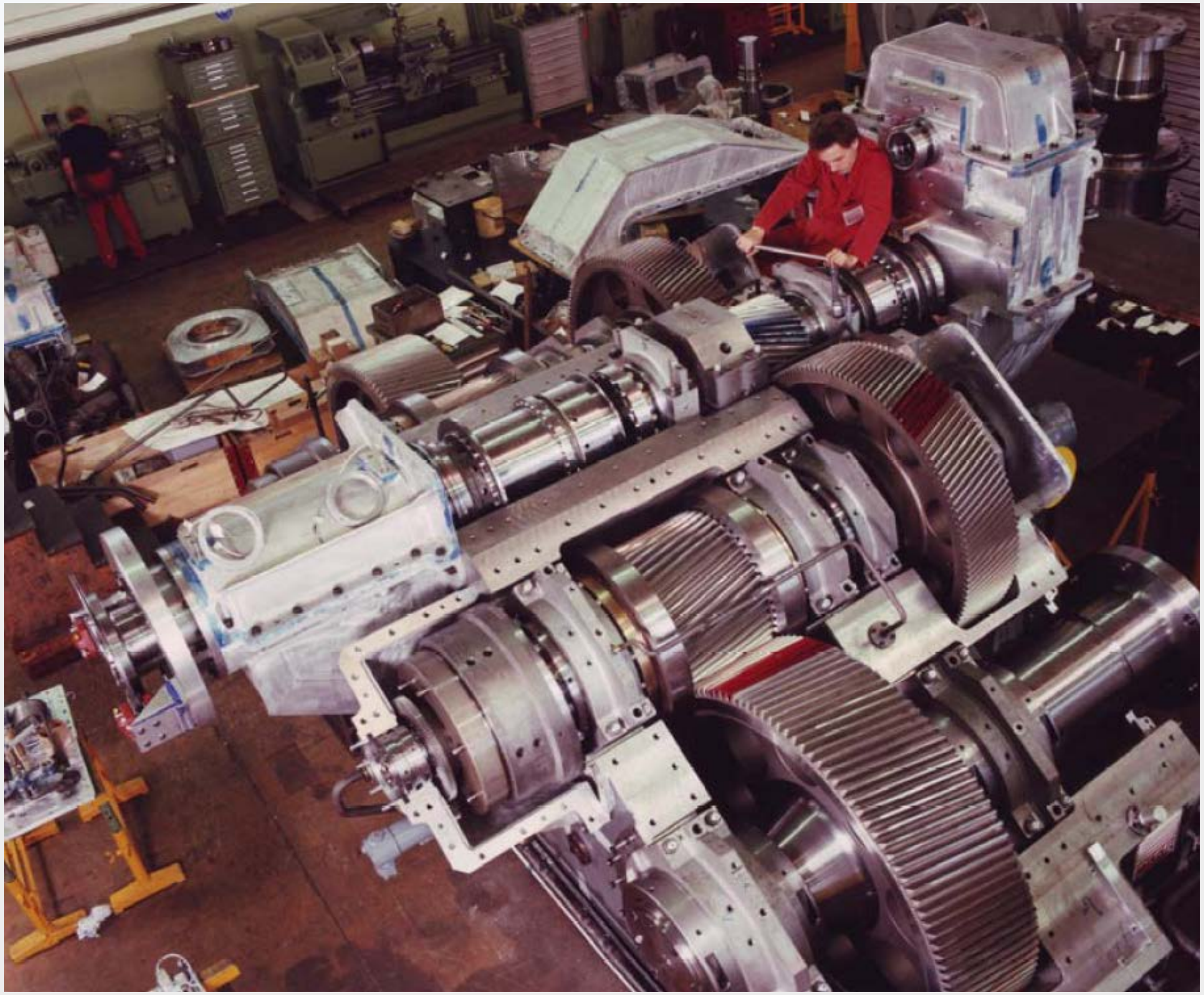
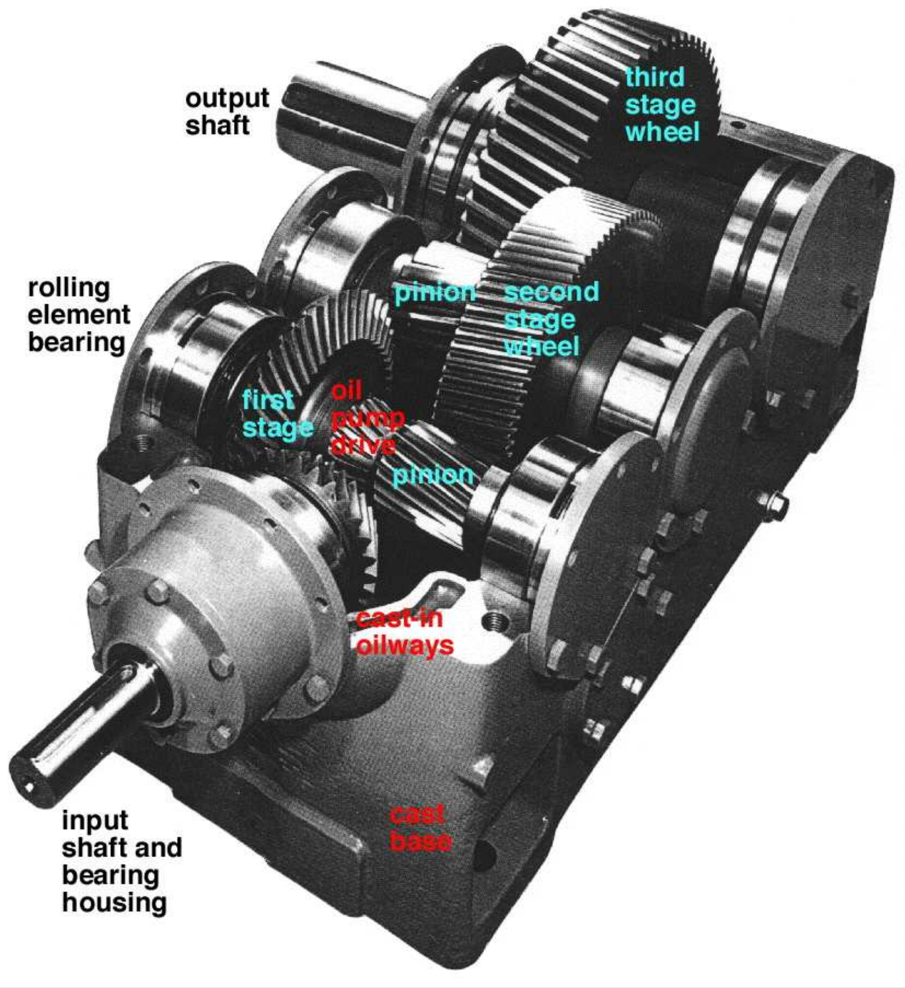
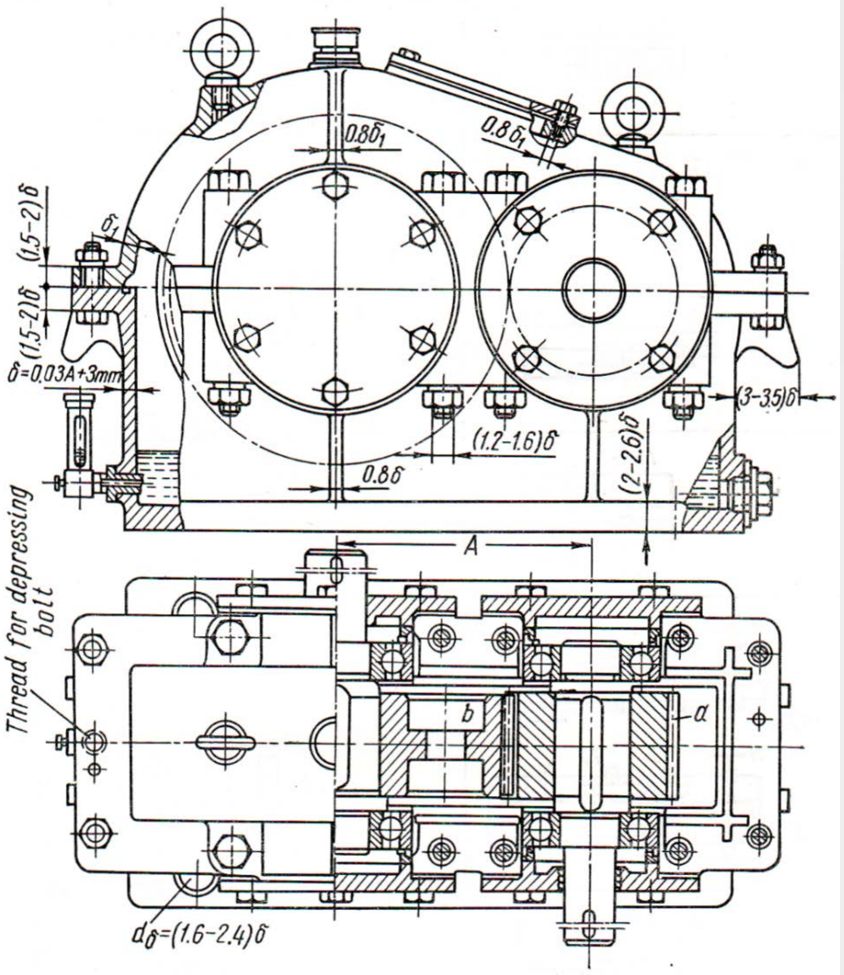
Gemi redüktörü



Dişli kutusu (redüktör) tasarım

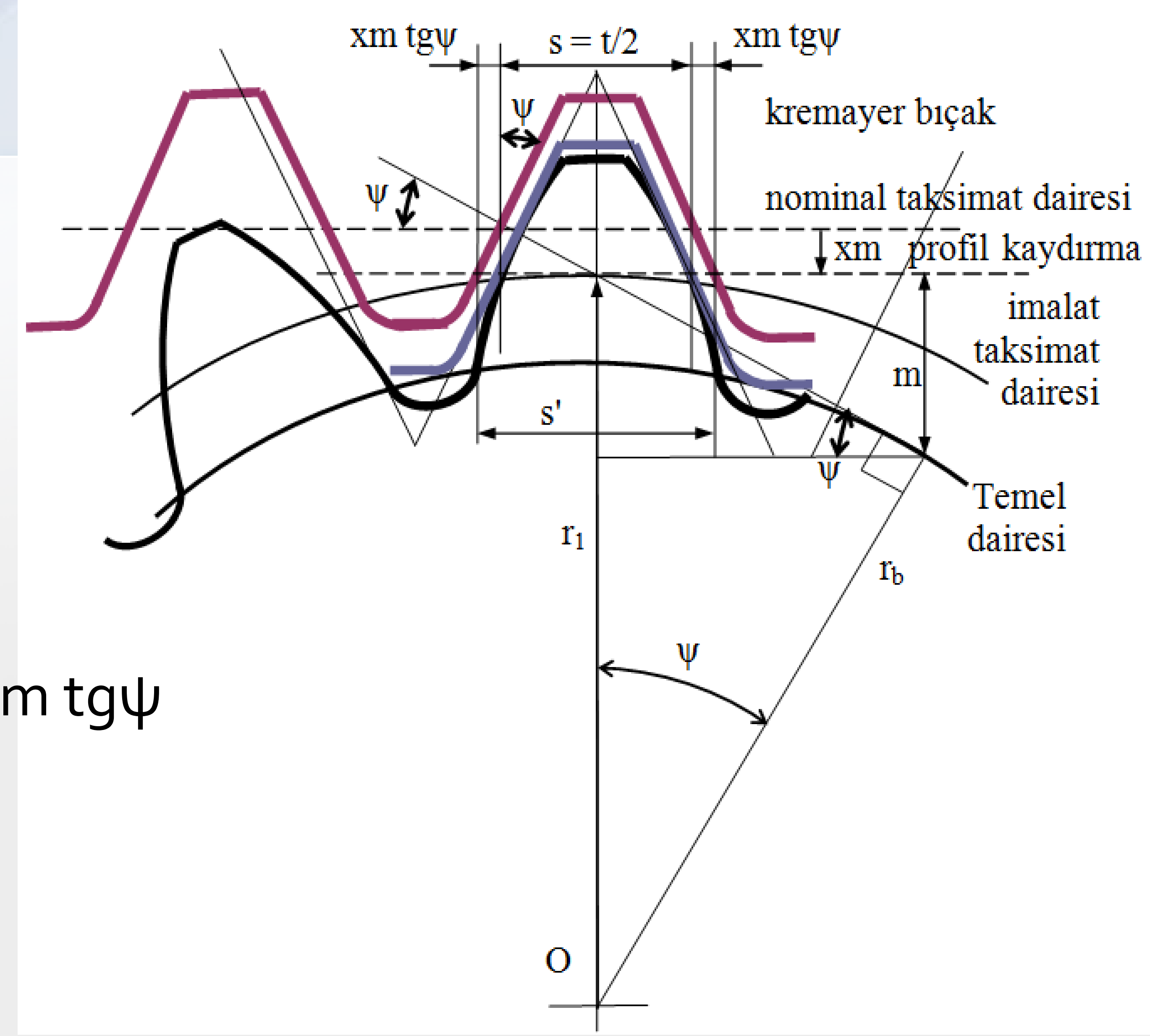




Profil Kaydırma

imalat yuvarlanma (taksimat) dairesi üzerindeki kalınlık

$s' = t / 2 + 2\,xm\,tg\psi$



Profil Kaydırma

İmalat yuvarlanma (taksimat) dairesi üzerindeki kalınlık

$s' = t / 2 + 2\, xm\, tg\psi$

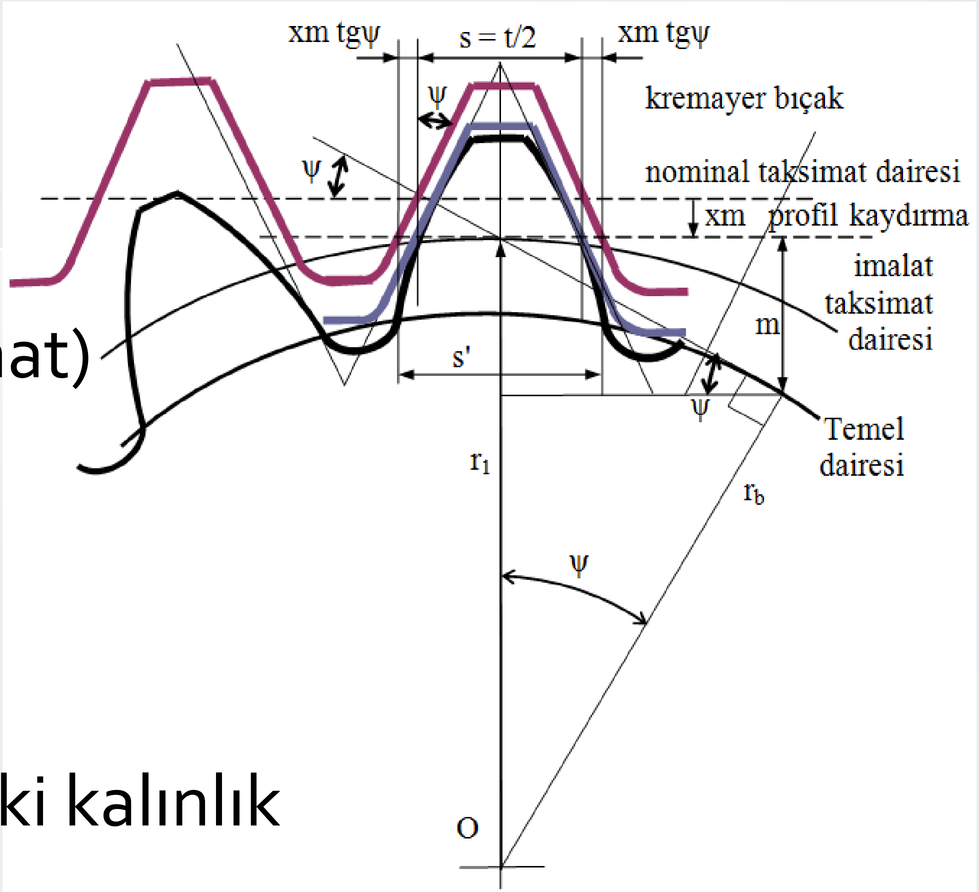
$s_2 = 2\, r_2\, [\, s_1/2r_1 - (\varphi_2 - \varphi_1)]$

herhangi bir r_y yarıçapındaki kalınlık

$s_y = 2\, r_y\, [\, s/2r - (\varphi_y - \varphi)]$

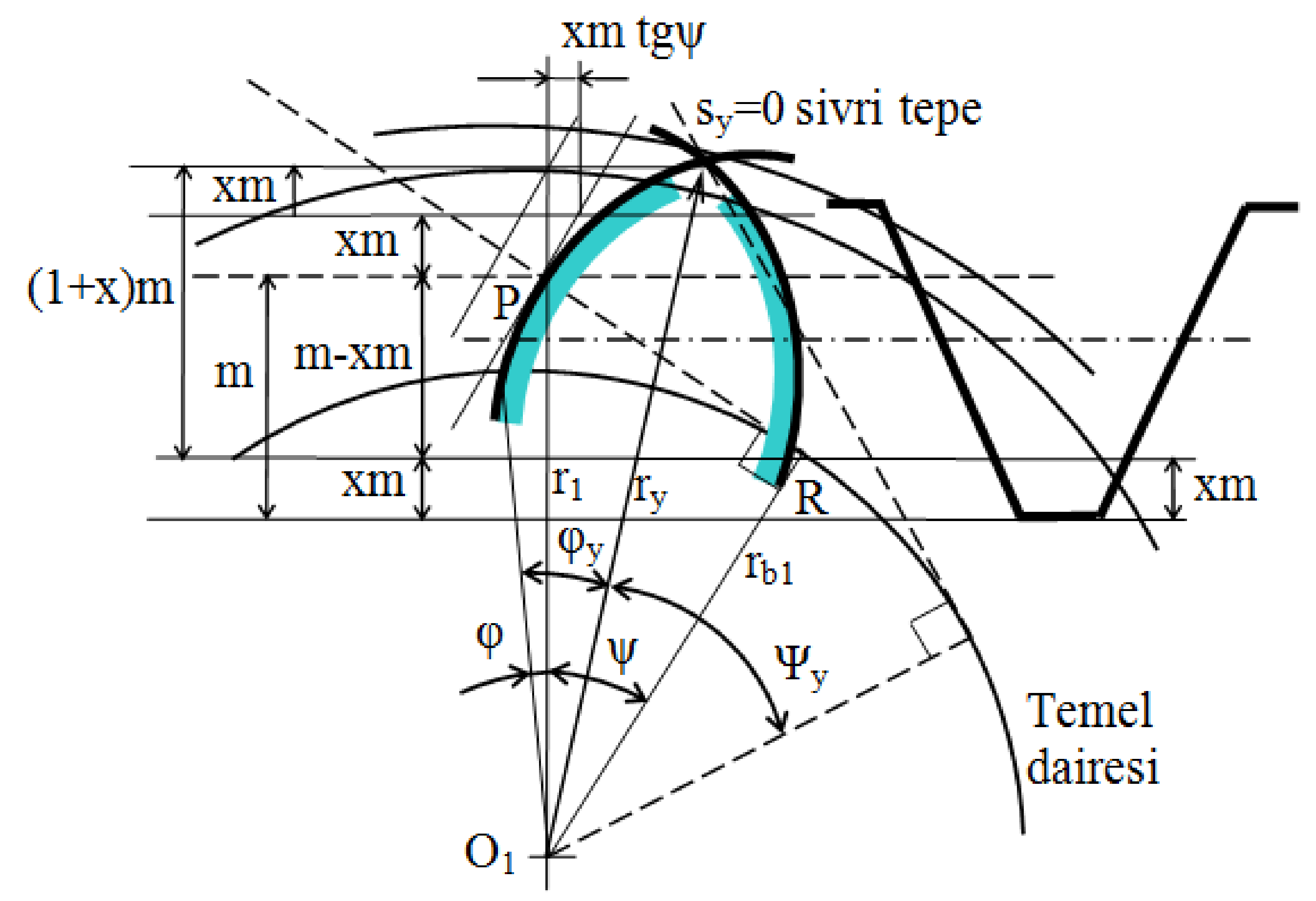
$s_y = 2\, r_y\, [\, s'/mz - (\varphi_y - \varphi)]$

$s_y = 2\, r_y\, [\, 1/z\, (\pi / 2 + 2\, x\, tg\psi) - (\varphi_y - \varphi)]$



Profil kaydırma alt sınırı

$PR = r_1 \sin\psi = (m\, z_1 / 2) \sin\psi = (m - xm) / \sin\psi$



Profil kaydırma alt sınırı

$PR = r_1 \sin\psi \; = (m\, z_1 / 2) \sin\psi \; = (m - xm) / \sin\psi$

$m - xm = (m\, z_1 / 2) \sin^2\psi$

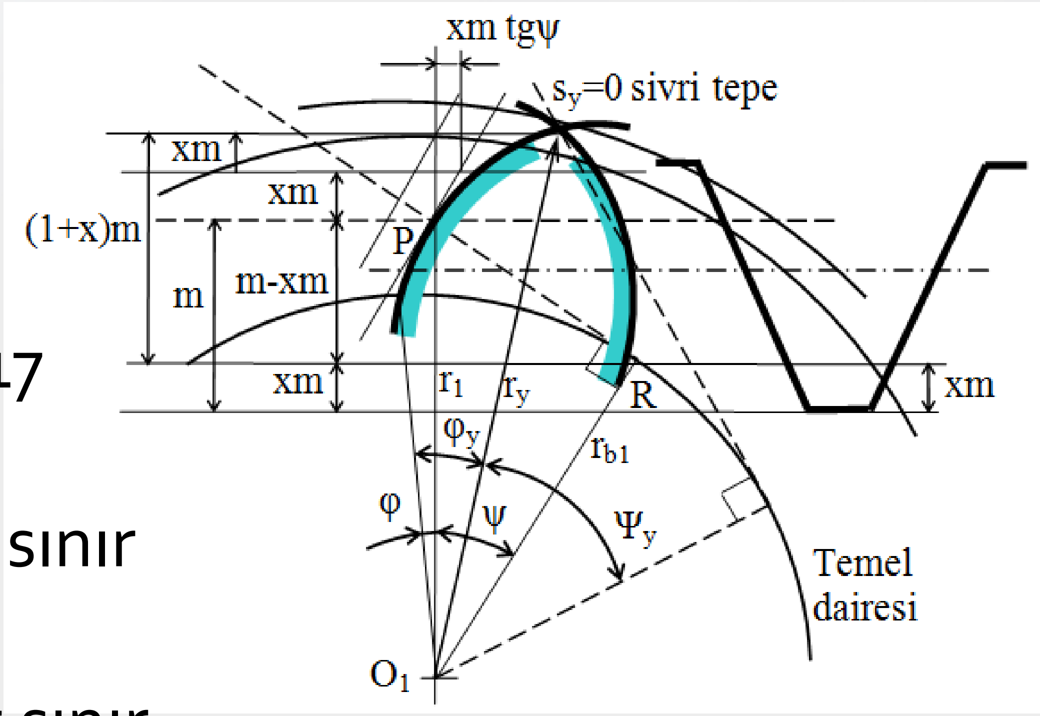
$xm = m - (m\, z_1 / 2) \sin^2\psi$

$x \; = 1 - z_1 / (2/\sin^2\psi)$

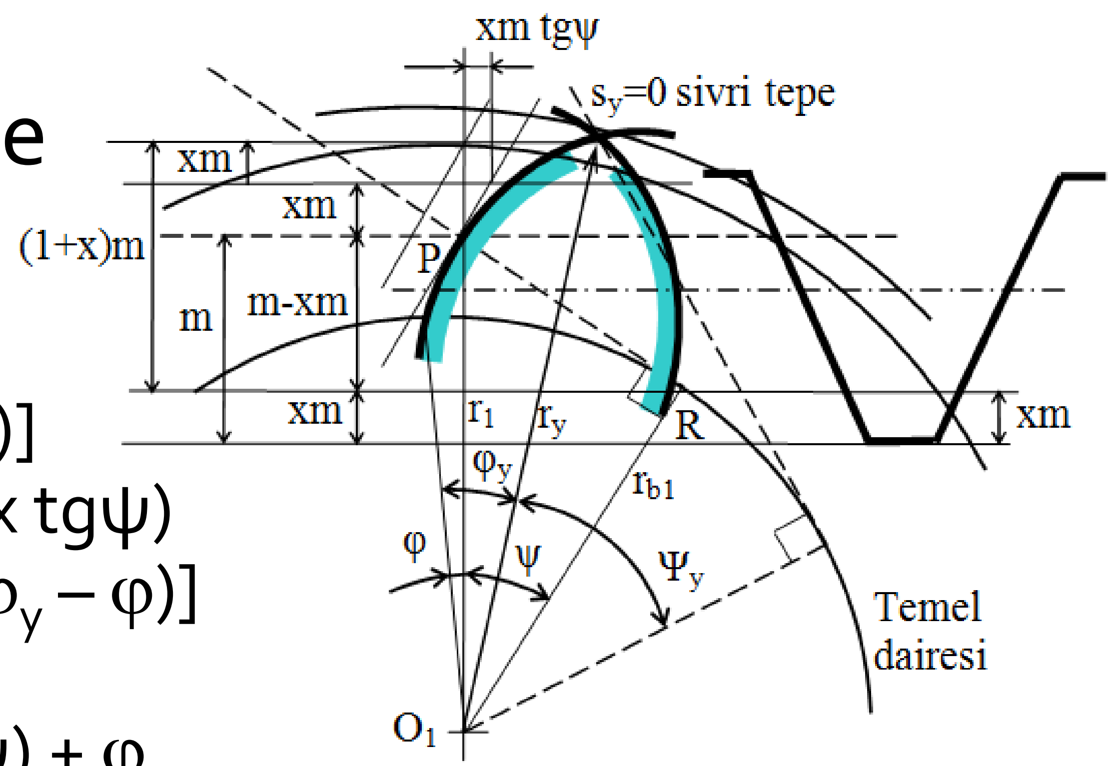
$\psi = 20°$ için $\rightarrow$ $2/\sin^2\psi = 17$

$x = (17 - z_1) / 17$ teorik alt sınır

$x = (14 - z_1) / 14$ pratik alt sınır



Profil kaydırma üst sınırı, sivri tepe



$$s_y = 2\, r_y\, [\, s/2r - (\varphi_y - \varphi)]$$

$$s_y = 2\, r_y\, [\, 1/z\, (\pi / 2 + 2\, x\, tg\psi) - (\varphi_y - \varphi)]$$

$$s_y = 0$$

$$\varphi_y = 1/z\, (\pi / 2 + 2\, x\, tg\psi) + \varphi$$

$$r_y = r \cos\psi / \cos\psi_y = r_t' = (m\, z / 2) \cos\psi / \cos\psi_y$$

$$r_y = r + (1 + x)\, m$$

$$r_y = m\, (z / 2 + (1 + x)) = m\, (z/2) \cos\psi / \cos\psi_y$$

$$(z / 2 + (1 + x)) = (z / 2) \cos\psi / \cos\psi_y$$

Profil kaydırma (Addendum Modification)

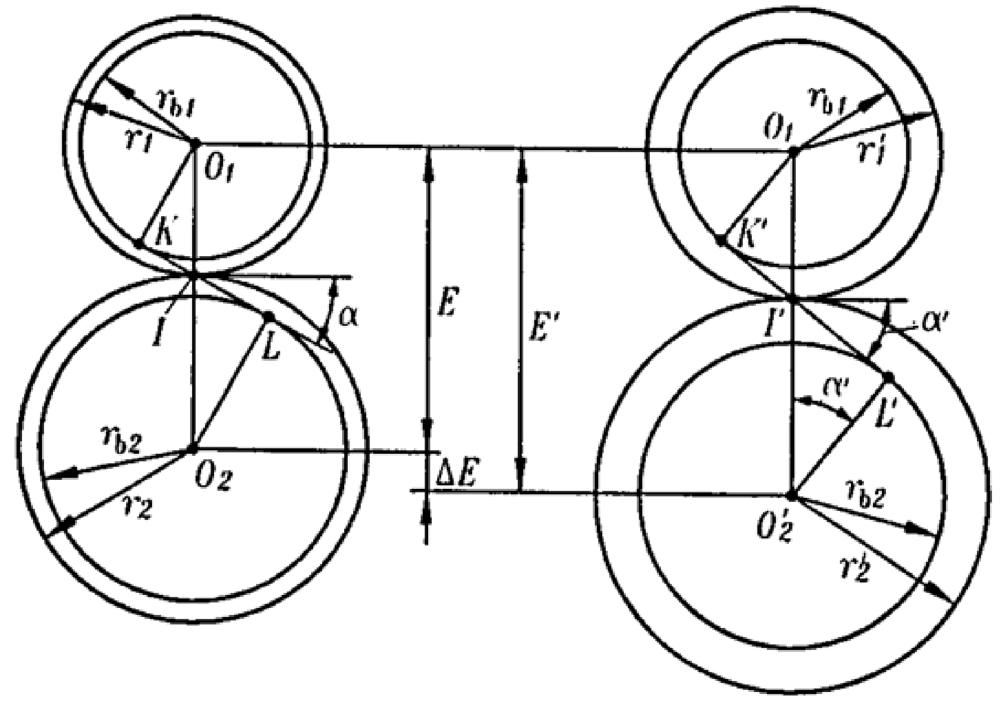
Profil kaydırma faktörlerinin seçimine bağlı olarak,

sıfır $x_1=0$, $x_2=0$, $a_v=a$,

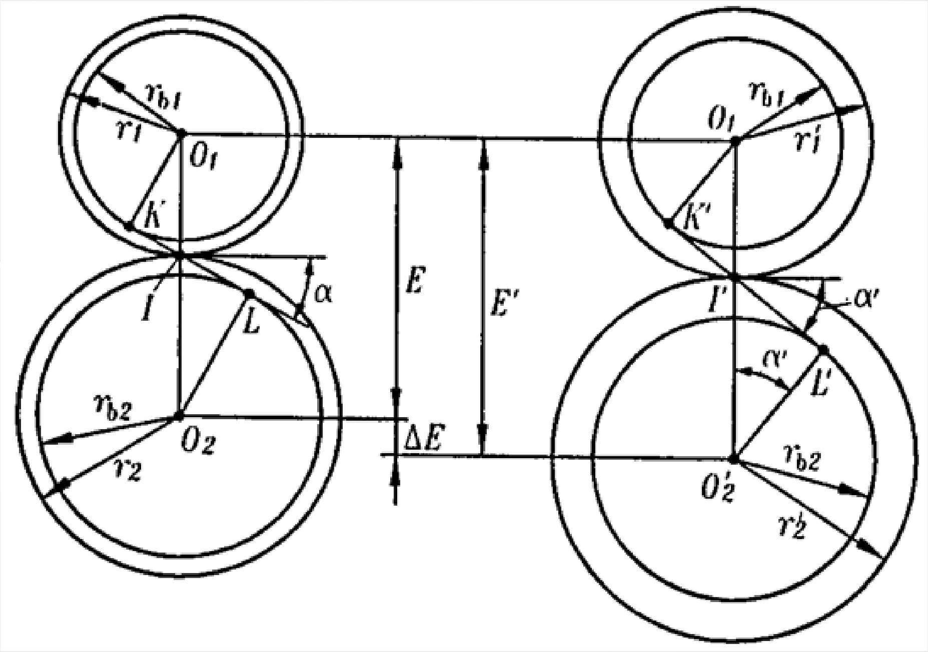
V-Sıfır $x_2+x_1=0$, $x_1= -x_2$ (sadece biri pozitif, $a_v=a$)

ve genel V-mekanizmaları $x_1 \neq x_2$ ($x_2-x_1>0$),

sıfır dişliler, eş çalışma kavrama açısı, imalat kavrama açısından farklıdır.



V mekanizması



Sıfır dişlisi

$x_1 = 0$ $x_2 = 0$ $a_v = a$

$r_{eş1} = r_1$ $r_{eş2} = r_2$ $\psi_v = \psi$

V-Sıfır mekanizması

$x_2 = - x_1$ $a_v = a$

$r_{eş1} = r_1$ $r_{eş2} = r_2$ $\psi_v = \psi$

V mekanizması

Eş çalışma kavrama açısı, imalat kavrama açısından farklıdır. Merkez mesafeleri de farklıdır. Eş çalışmayı aşağıdaki denklemler tayin eder.

$$s_{eş1} = 2\, r_{eş1} \left[1/z_1 \left(\pi / 2 + 2\, x_1 \, tg\psi\right) - \left(\varphi_v - \varphi\right)\right]$$

$$s_{eş2} = 2\, r_{eş2} \left[1/z_2 \left(\pi / 2 + 2\, x_2 \, tg\psi\right) - \left(\varphi_v - \varphi\right)\right]$$

V mekanizması

$2\, r_{eş1} = z_1\, t_{eş} / \pi$ ve $2\, r_{eş2} = z_2\, t_{eş} / \pi$

$s_{eş1} + s_{eş2} = t_{eş}$

$$t_{eş} = 2\, r_{eş1} \left[1/z_1 (\pi / 2 + 2\, x_1\, tg\psi) - (\varphi_v - \varphi)\right] + 2\, r_{eş2} \left[1/z_2 (\pi / 2 + 2\, x_2\, tg\psi) - (\varphi_v - \varphi)\right]$$

$$t_{eş} = (z_1\, t_{eş} / \pi) \left[1/z_1 (\pi/2 + 2\, x_1\, tg\psi) - (\varphi_v - \varphi)\right] + (z_2\, t_{eş} / \pi) \left[1/z_2 (\pi/2 + 2\, x_2\, tg\psi) - (\varphi_v - \varphi)\right]$$

V mekanizması

$$t_{eş} = (z_1\, t_{eş} / \pi)\,(1/z_1)\,(\pi / 2)$$
$$+ (z_1\, t_{eş} / \pi)\,(1/z_1)\,(2\, x_1\, tg\psi) – (z_1\, t_{eş} / \pi)\,(\varphi_v – \varphi)$$
$$+ (z_2\, t_{eş} / \pi)\,(1/z_2)\,(\pi / 2)$$
$$+ (z_2\, t_{eş} / \pi)\,(1/z_2)\,(2\, x_2\, tg\psi) – (z_2\, t_{eş} / \pi)\,(\varphi_v – \varphi)$$

$$t_{eş} = (t_{eş} /2) + (t_{eş} /2) + (t_{eş} / \pi)\, 2\,(x_1 + x_2)\, tg\psi$$
$$– (z_1 + z_2)\,(t_{eş} / \pi)\,(\varphi_v – \varphi)$$

V mekanizması

$$(z_1 + z_2)\,(t_{eş} / \pi)\,(\varphi_v - \varphi) = (t_{eş} / \pi)\,2\,(x_1 + x_2)\,tg\psi$$

$$\varphi_v = 2\,[(x_1+x_2)/(z_1+z_2)]\;tg\psi + \varphi$$

$$x_1+x_2 = (z_1+z_2)\,(\varphi_v - \varphi) / (2\,tg\psi)$$

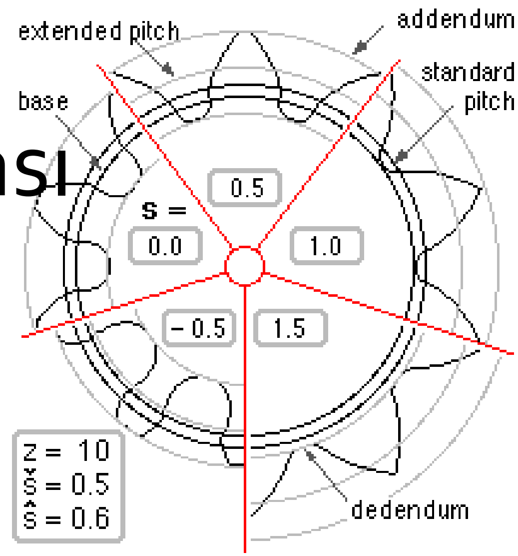
$$r_{eş1} = r_1 \cos\psi / \cos\psi_v$$

$$r_{eş2} = r_2 \cos\psi / \cos\psi_v$$

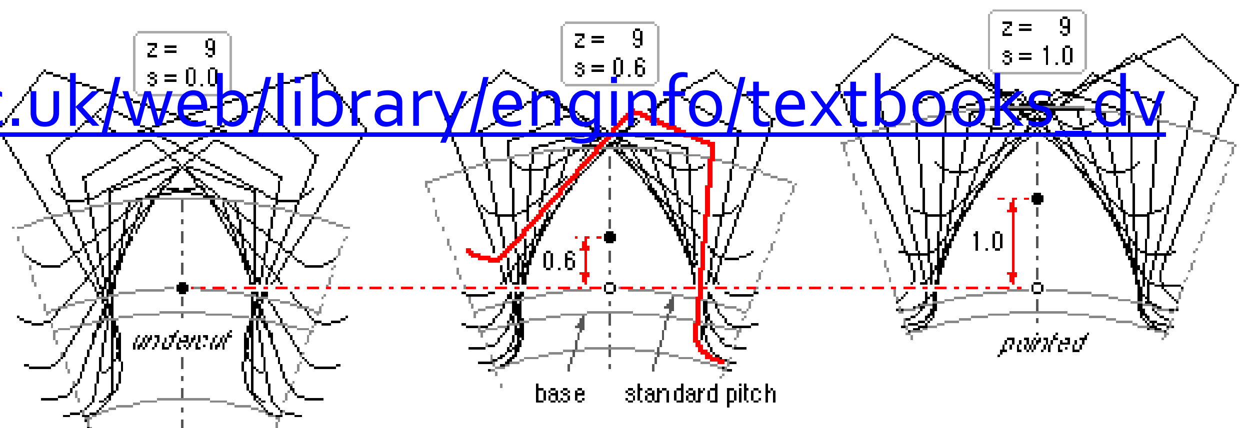
$$a_v = (r_1 + r_2)\;\cos\psi / \cos\psi_v$$

V mekanizması

Gear tooth Generation

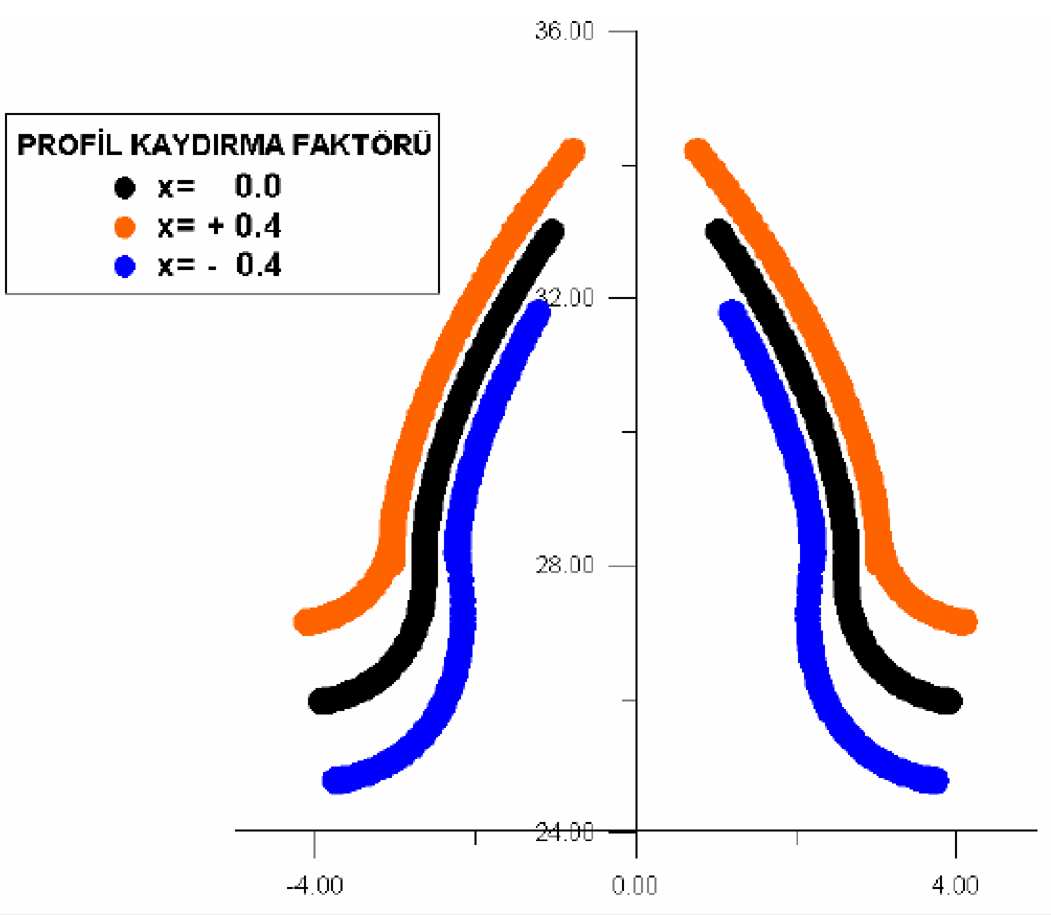


http://www-mdp.eng.cam.ac.uk/web/library/enginfo/textbooks_dvd_only/DAN/

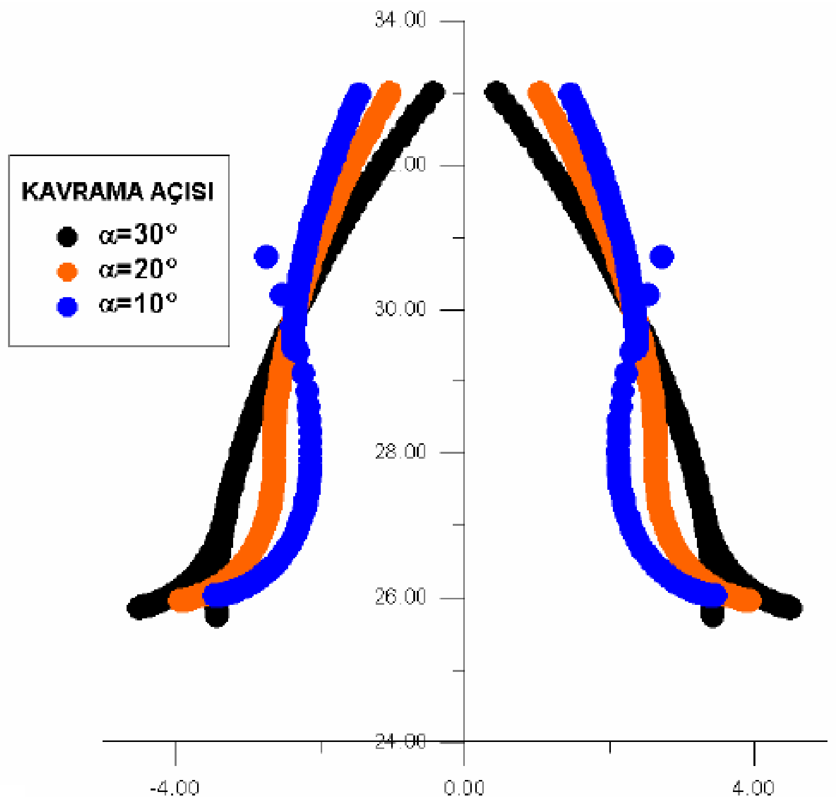


http://www-mdp.eng.cam.ac.uk/web/library/enginfo/textbooks_dvd_only/DAN/gears/generation/generation.html

Profil Kaydırmanın Etkisi



Kavrama Açısının Etkisi



Eş çalışan dişli boyutları

Diş dibi yarıçapları:

$r_{d1} = r_1 – (m + s_k) + m\ x_1$

$r_{d2} = r_2 – (m + s_k) + m\ x_2$

Diş başı yarıçapları:

$r_{t1}' = r_1 + m + m\ x_1$

$r_{t2}' = r_2 + m + m\ x_2$

https://www.sdp-si.com/resources/elements-of-metric-gear-technology/page3.php

s_k diş boşluğunu sağlayan diş başı yarıçapları

$r_{t1} = a_v - (r_{d2} + s_k)$
$r_{t2} = a_v - (r_{d1} + s_k)$
$r_{t1} = a_v - r_2 + m\,(1 - x_2)$
$r_{t2} = a_v - r_1 + m\,(1 - x_1)$

$k\,m = r_{t1}' - r_{t1} = r_{t2}' - r_{t2} = m\,(x_1 + x_2) - (a_v - a)$

dişlilerin uçlarında eş çalışmada boşluk için kısaltma oranı
$k = x_1 + x_2 - (a_v - a) / m$

Profil Kaydırmanın Etkisi

https://www.sdp-si.com/resources/elements-of-metric-gear-technology/page3.php

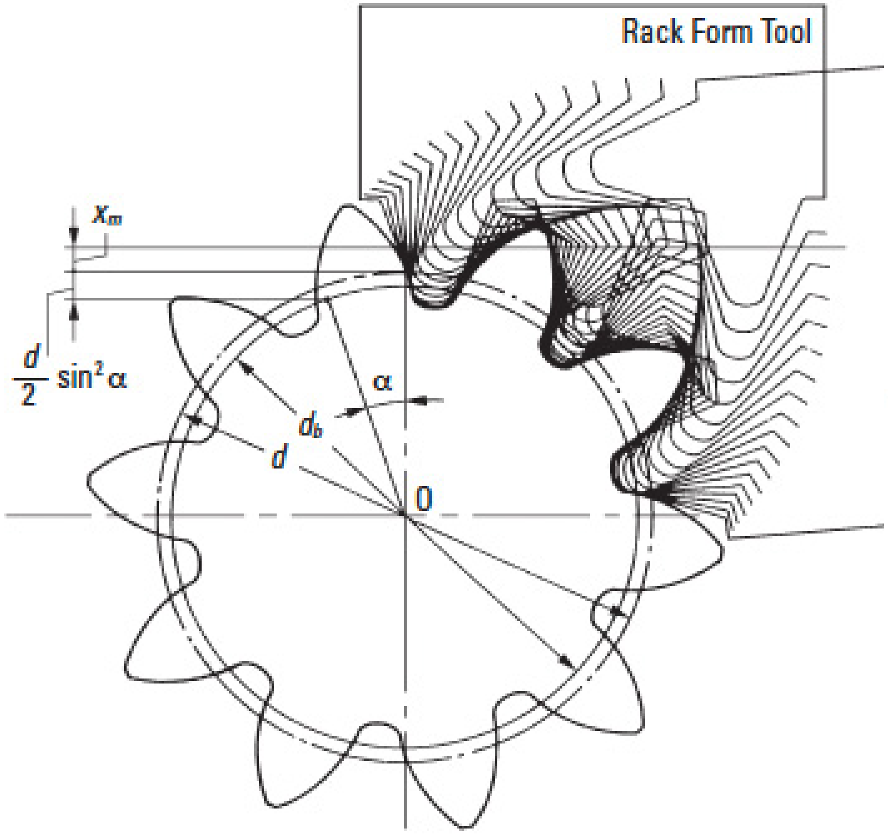


Fig. 4-6 Generating of Positive Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = +0.5$)

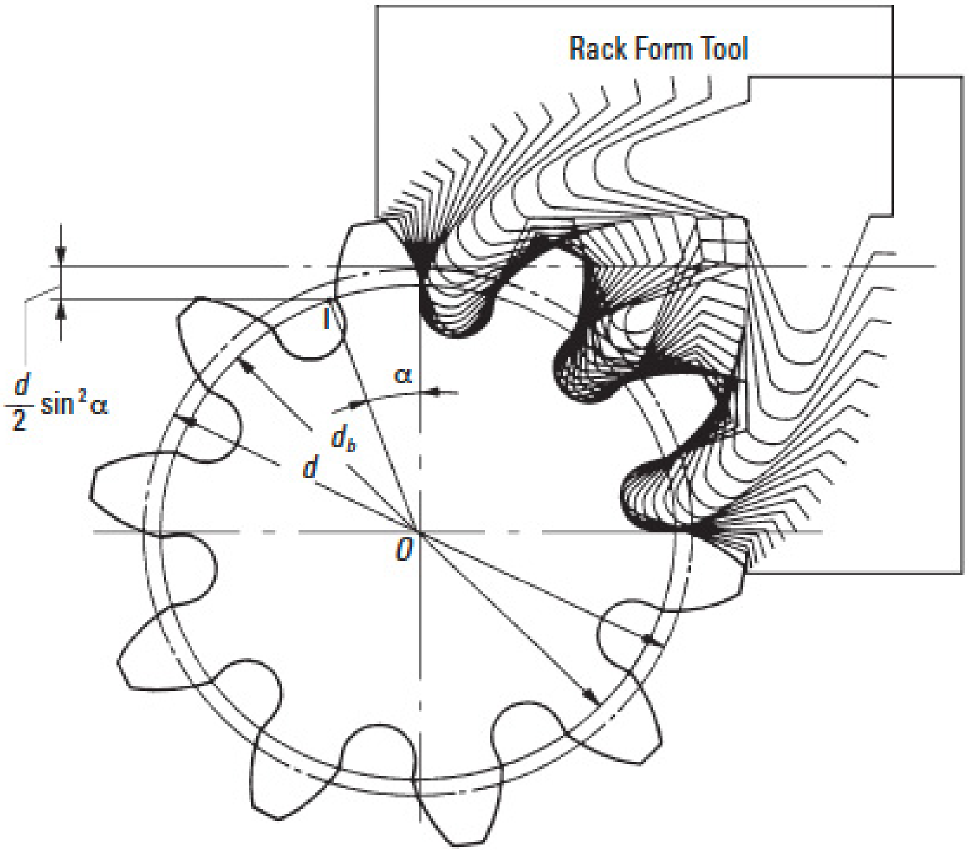


Fig. 4-2 The Generating of a Standard Spur Gear
($\alpha = 20°$, $z = 10$, $x = 0$)

Profil Kaydırmanın Etkisi

https://www.sdp-si.com/resources/elements-of-metric-gear-technology/page3.php

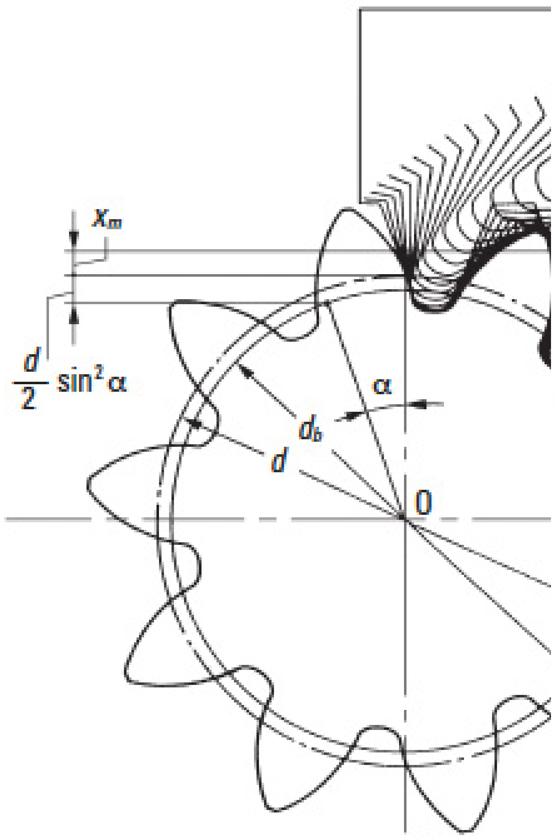


Fig. 4-6 Generating of Positive Shifted
($\alpha = 20°$, $z = 10$, $x = +0.5$)

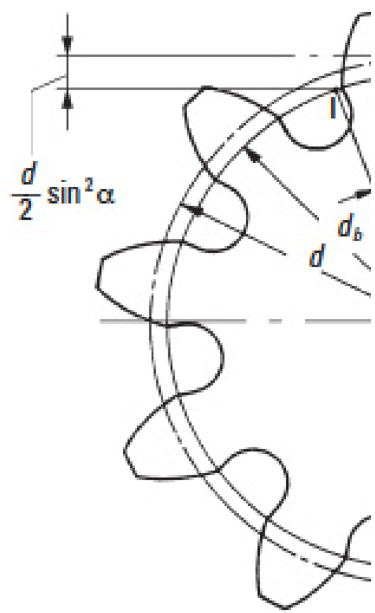


Fig. 4-2 The Gener
(α = :

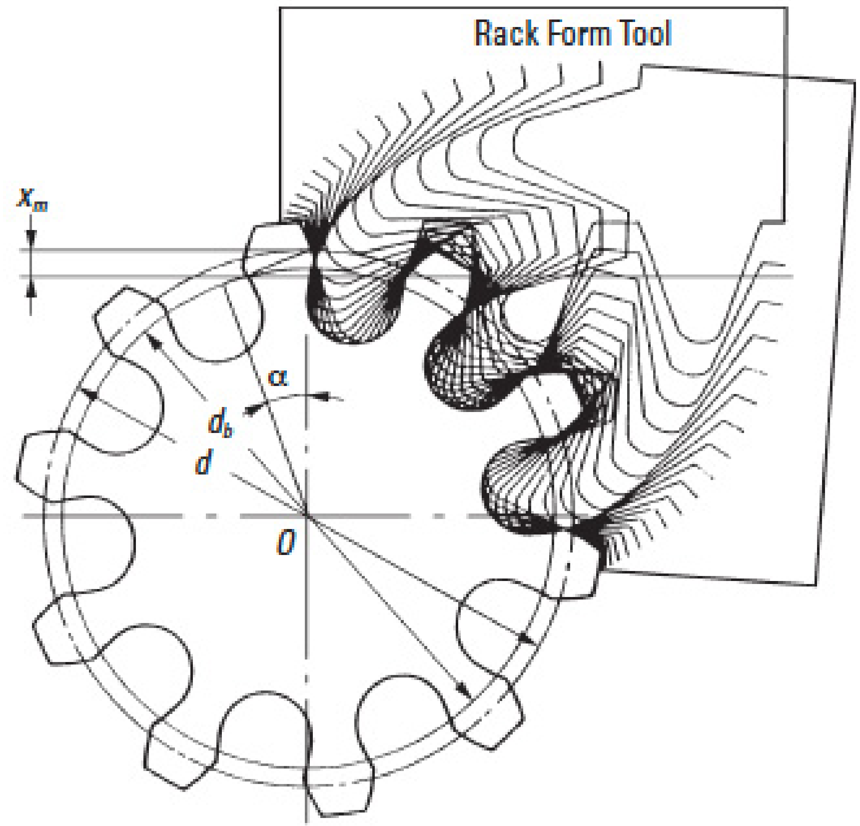


Fig. 4-7 The Generating of Negative Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = -0.5$)

Profil Kaydırmanın Etkisi

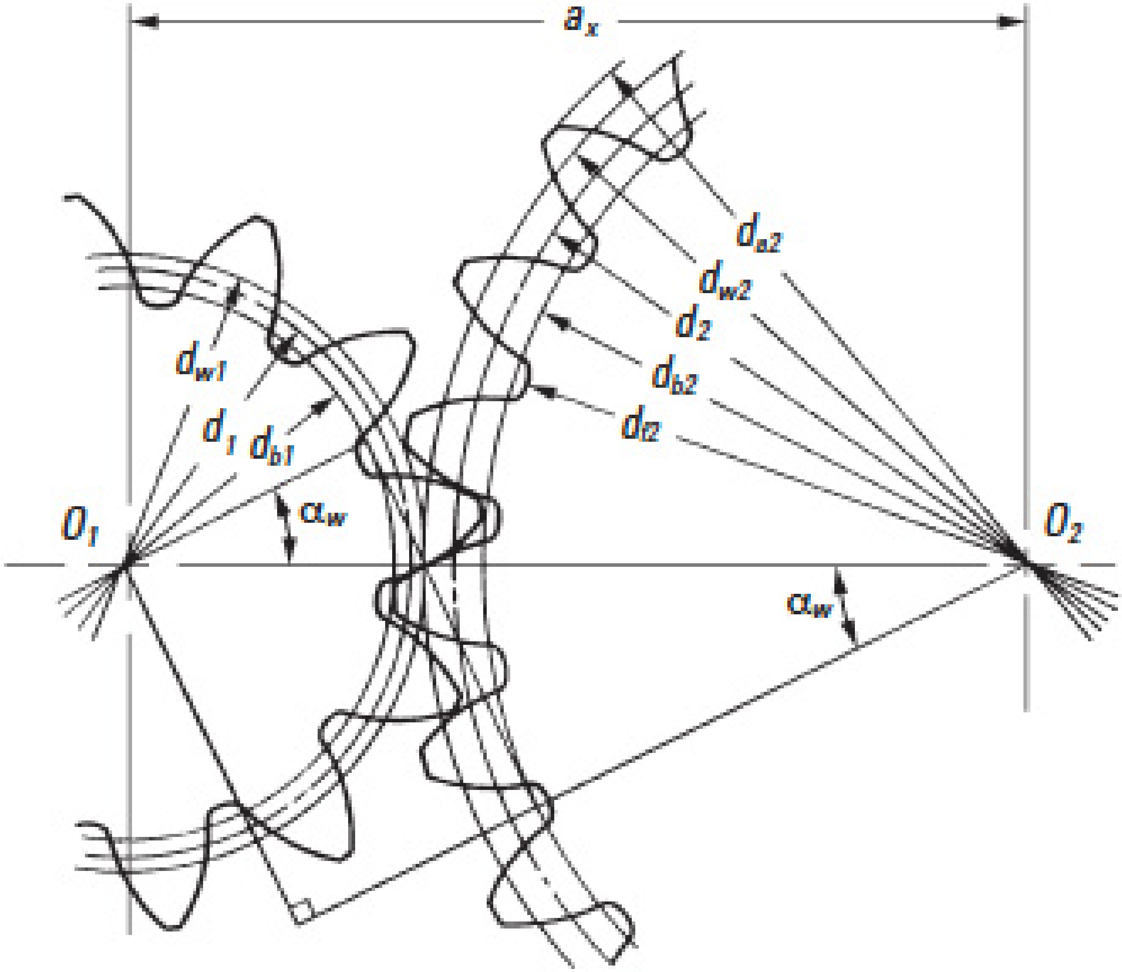


Fig. 4-8 The Meshing of Profile Shifted Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

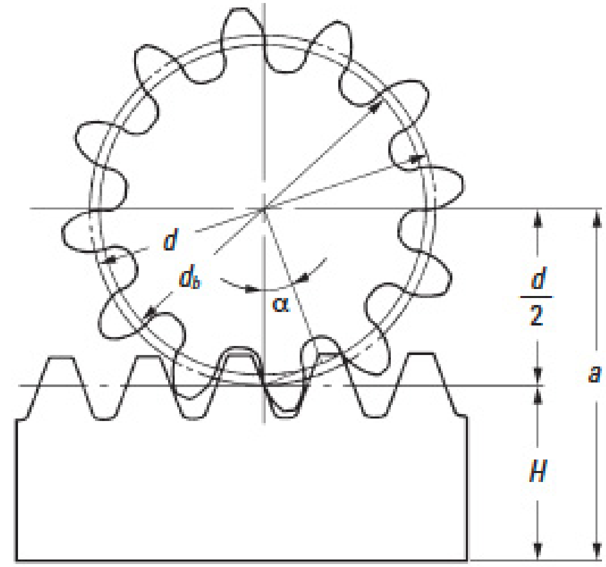


Fig. 4-9a The Meshing of Standard Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = 0$)

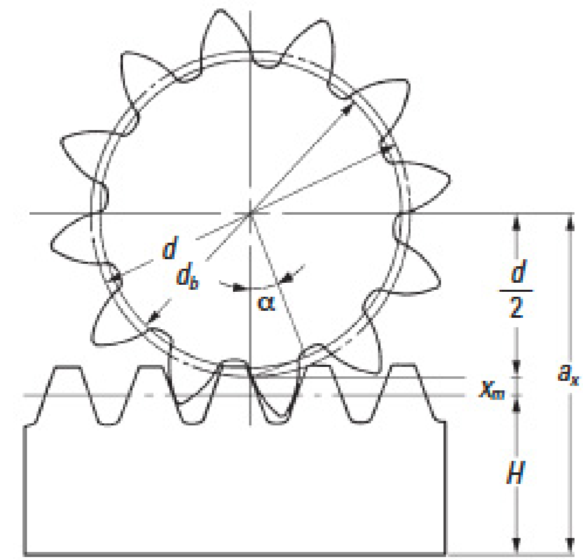


Fig. 4-9b The Meshing of Profile Shifted Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = +0.6$)

İç ve Dış dişli çark profilleri

the mesh of
an internal gear
and external gear

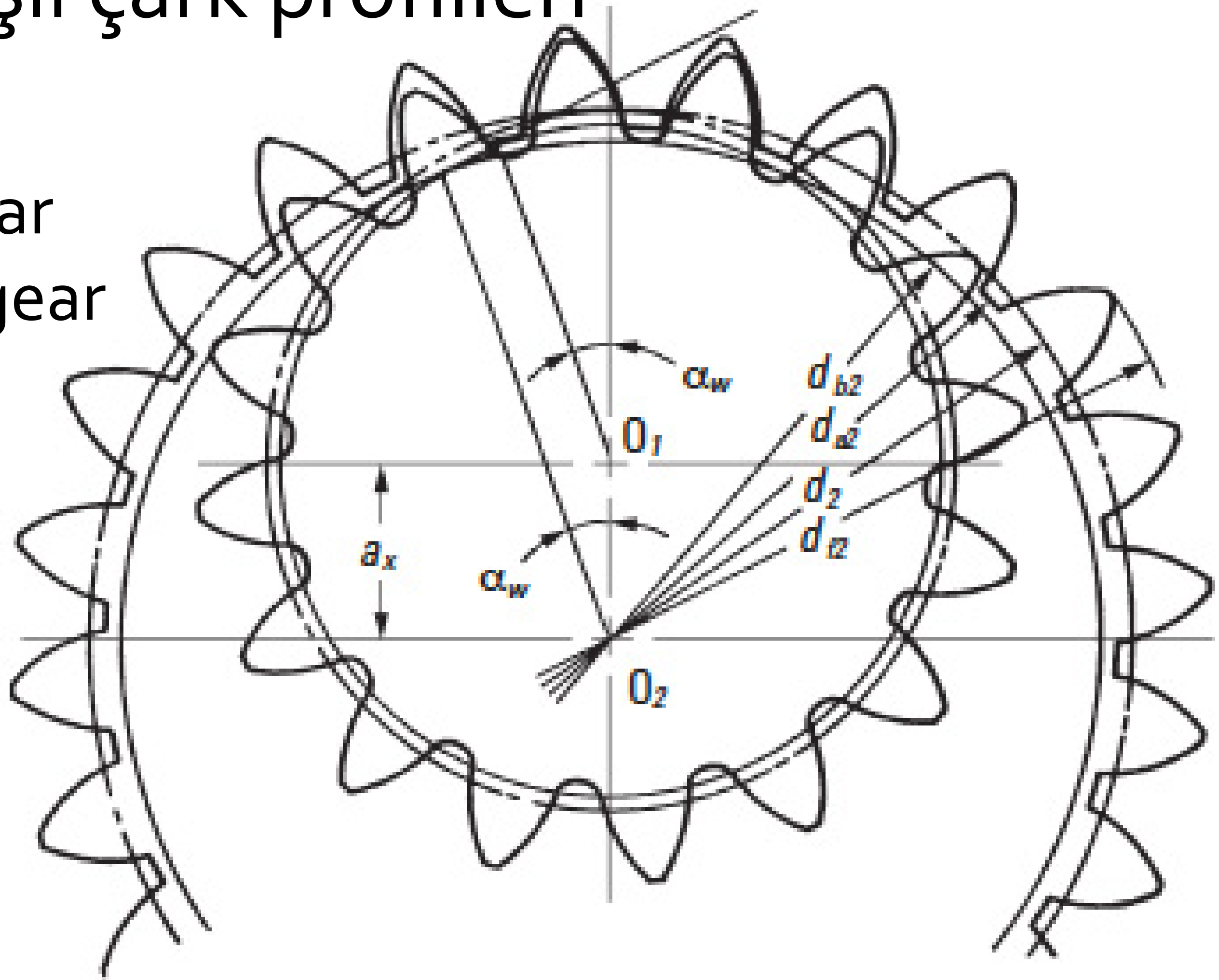


Fig. 5-1 The Meshing of Internal Gear and External Gear
($\alpha = 20°$, $z_1 = 16$, $z_2 = 24$, $x_1 = x_2 = 0.5$)

Profil Kaydırma

Profil kaydırma

ve

Alttan Kesme

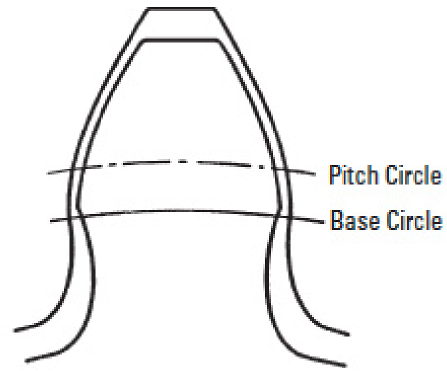
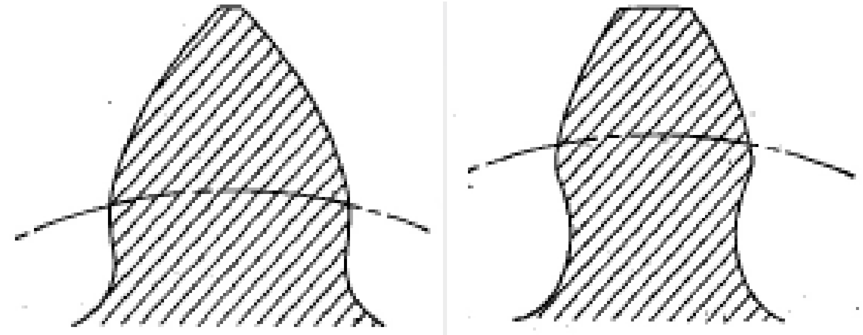
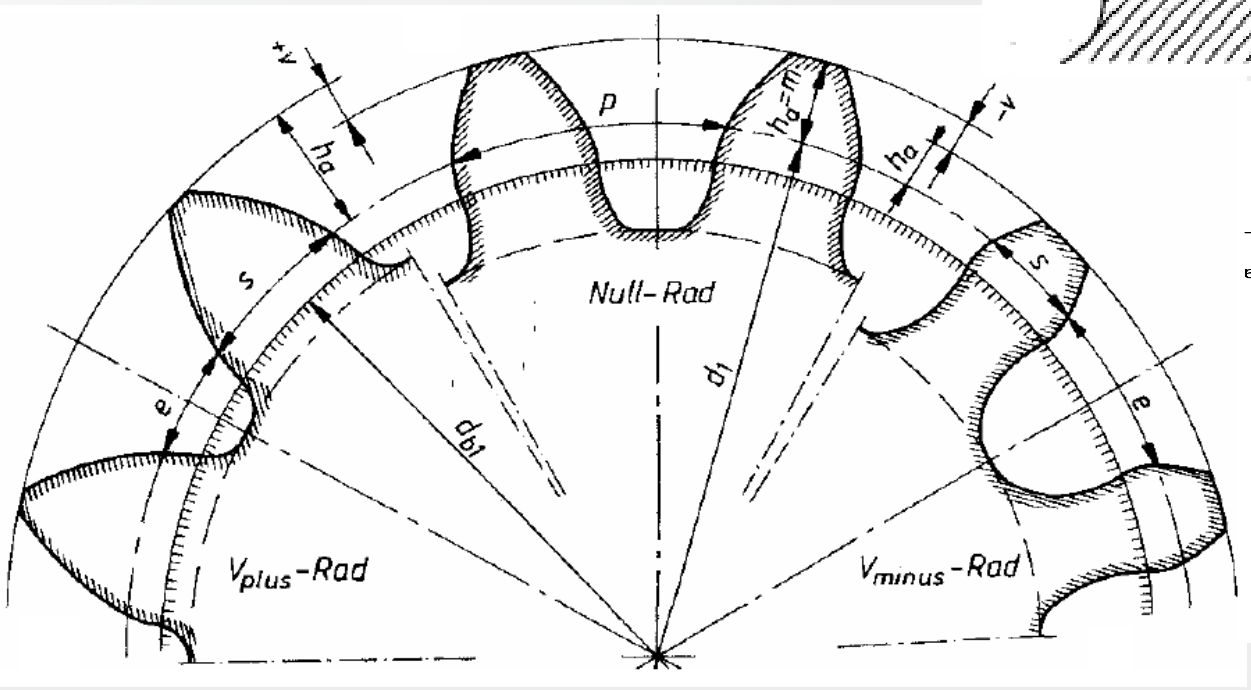
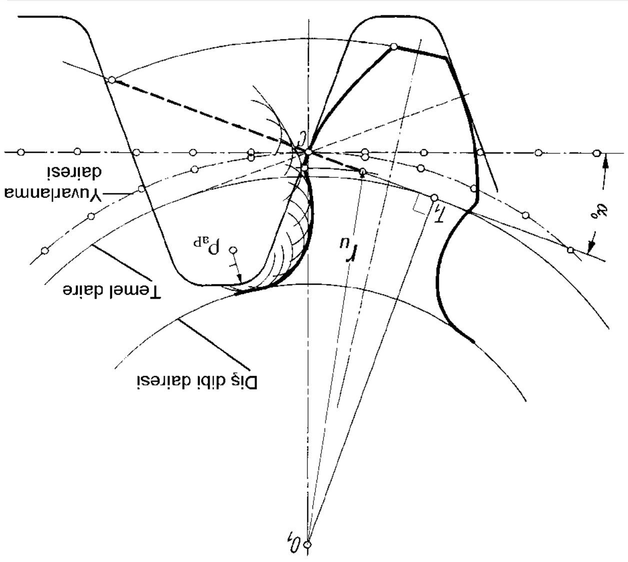


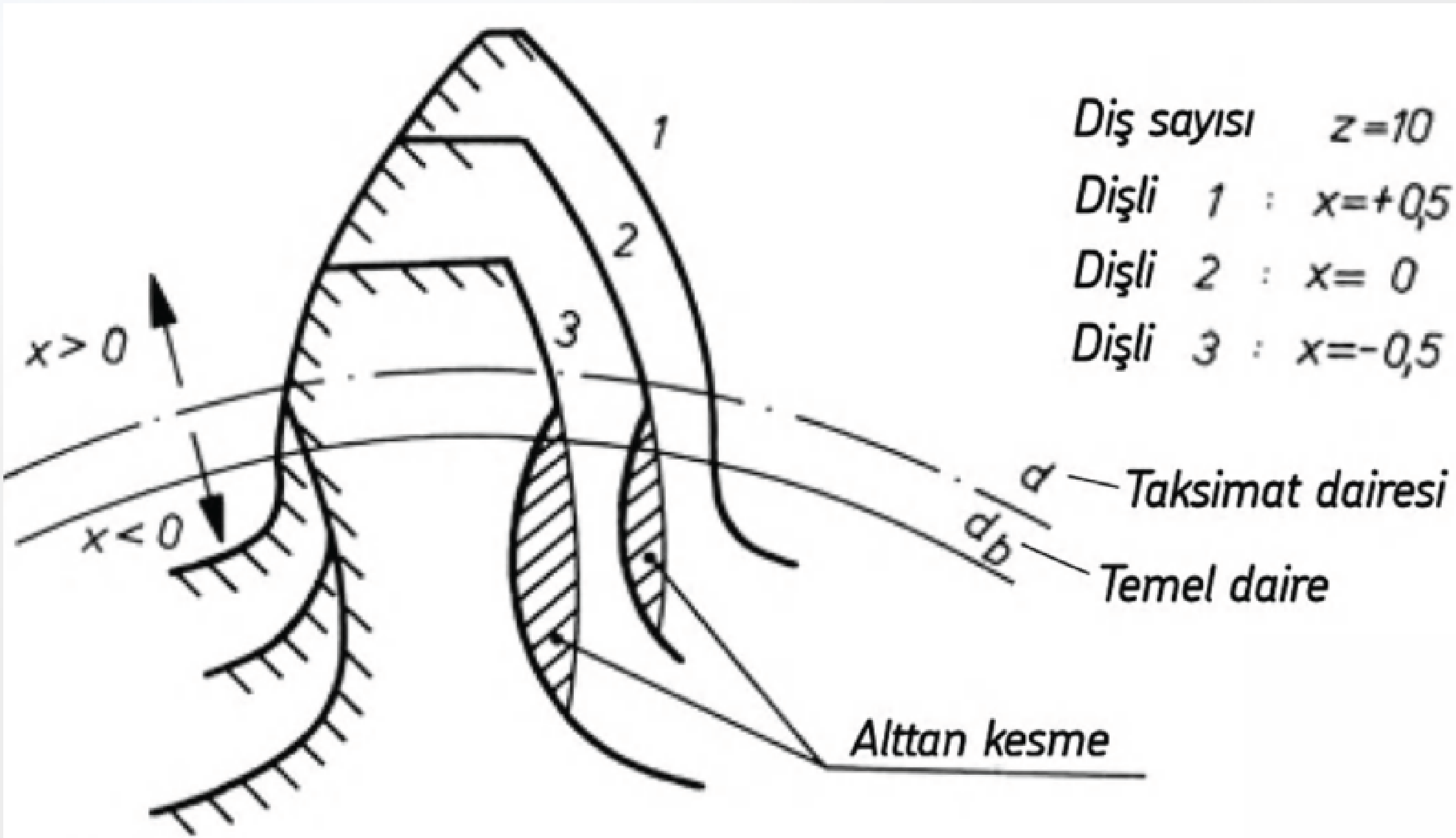
Fig. 4-5 Comparison of Enlarged and Undercut Standard Pinion
(13 Teeth, 20° Pressure Angle, Fine Pitch Standard)



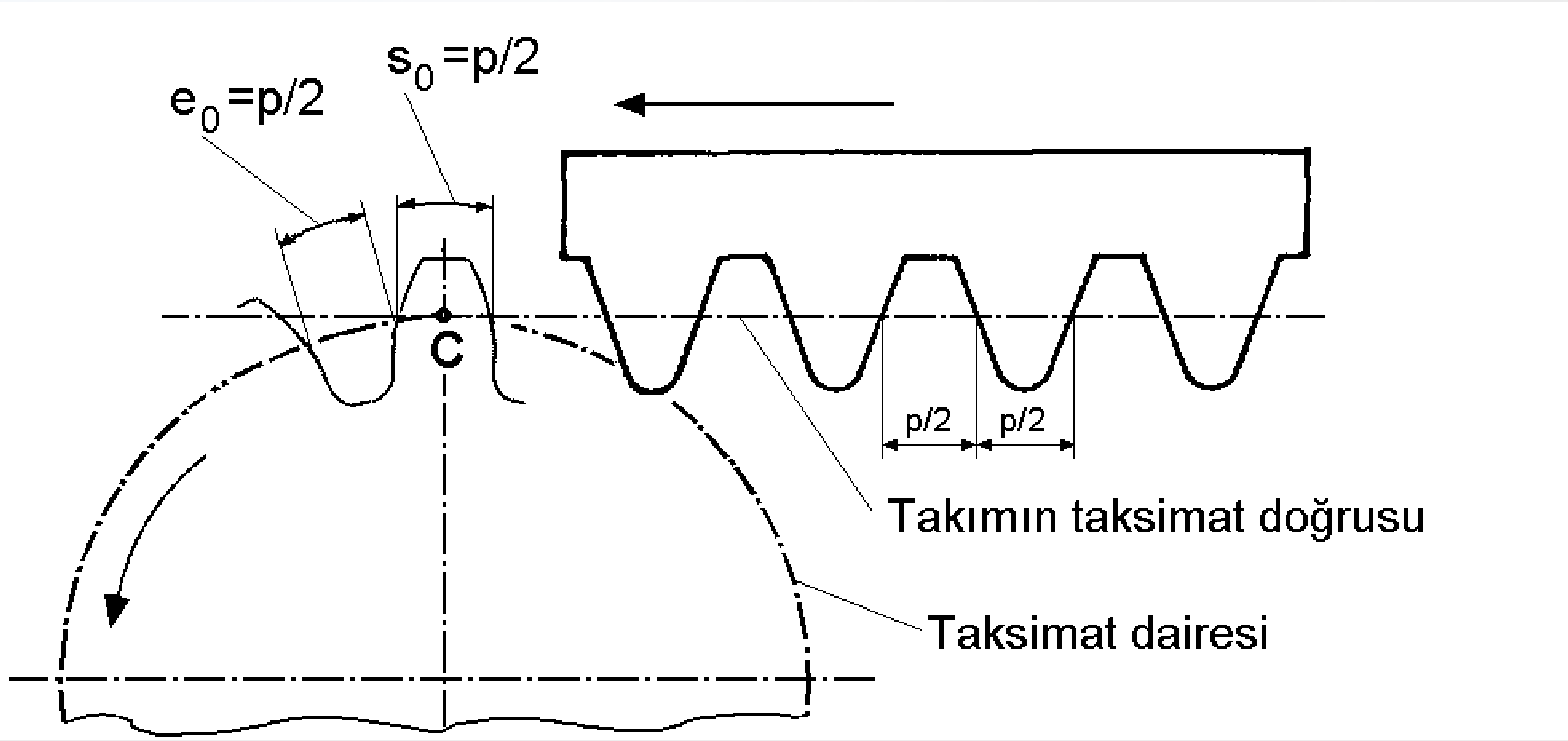




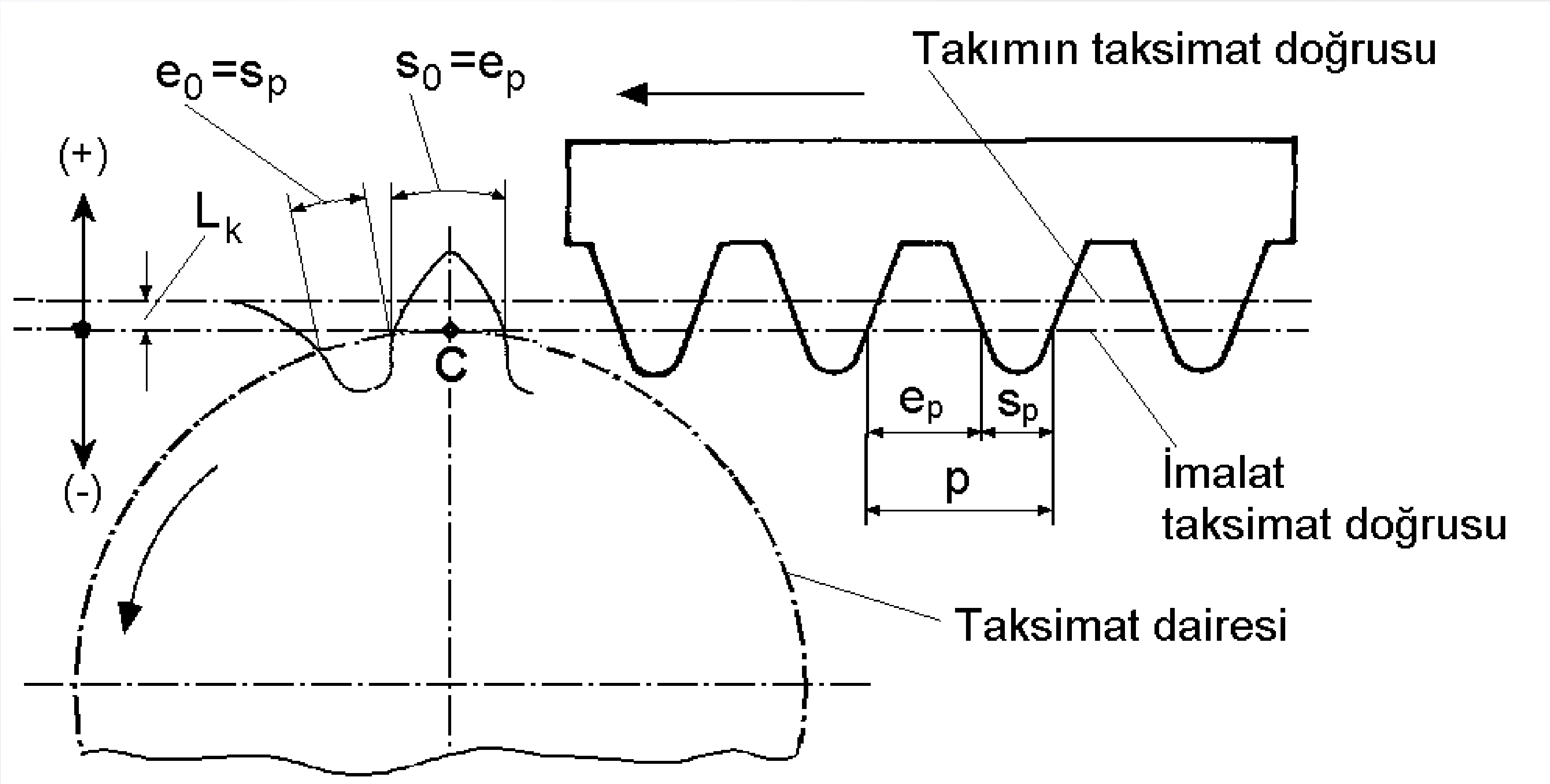
Profil kaydırmanın alttan kesmeye etkisi



Profil Kaydırma



Profil Kaydırma



Dişli çarklar

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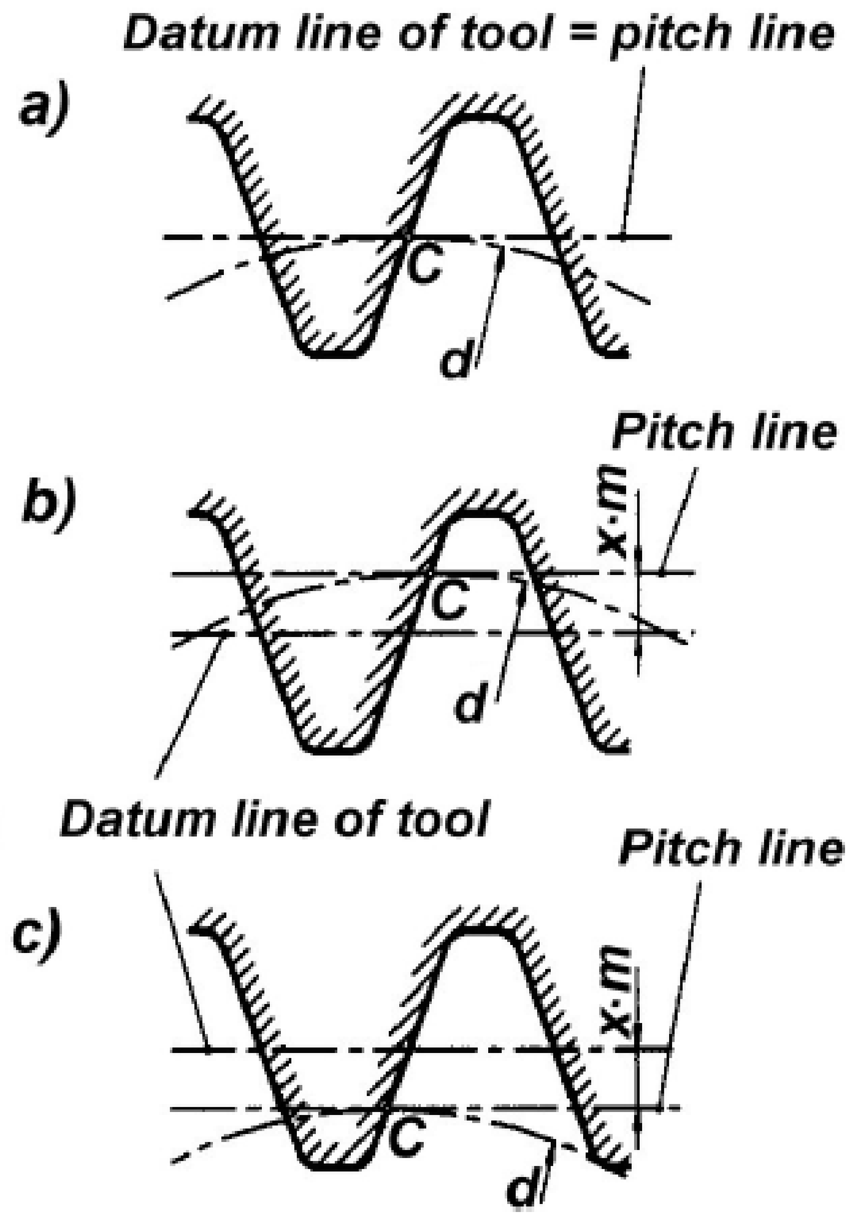
ADDENDUM MODIFICATION

Principle of the addendum modification:

a) Zero addendum modification x = 0

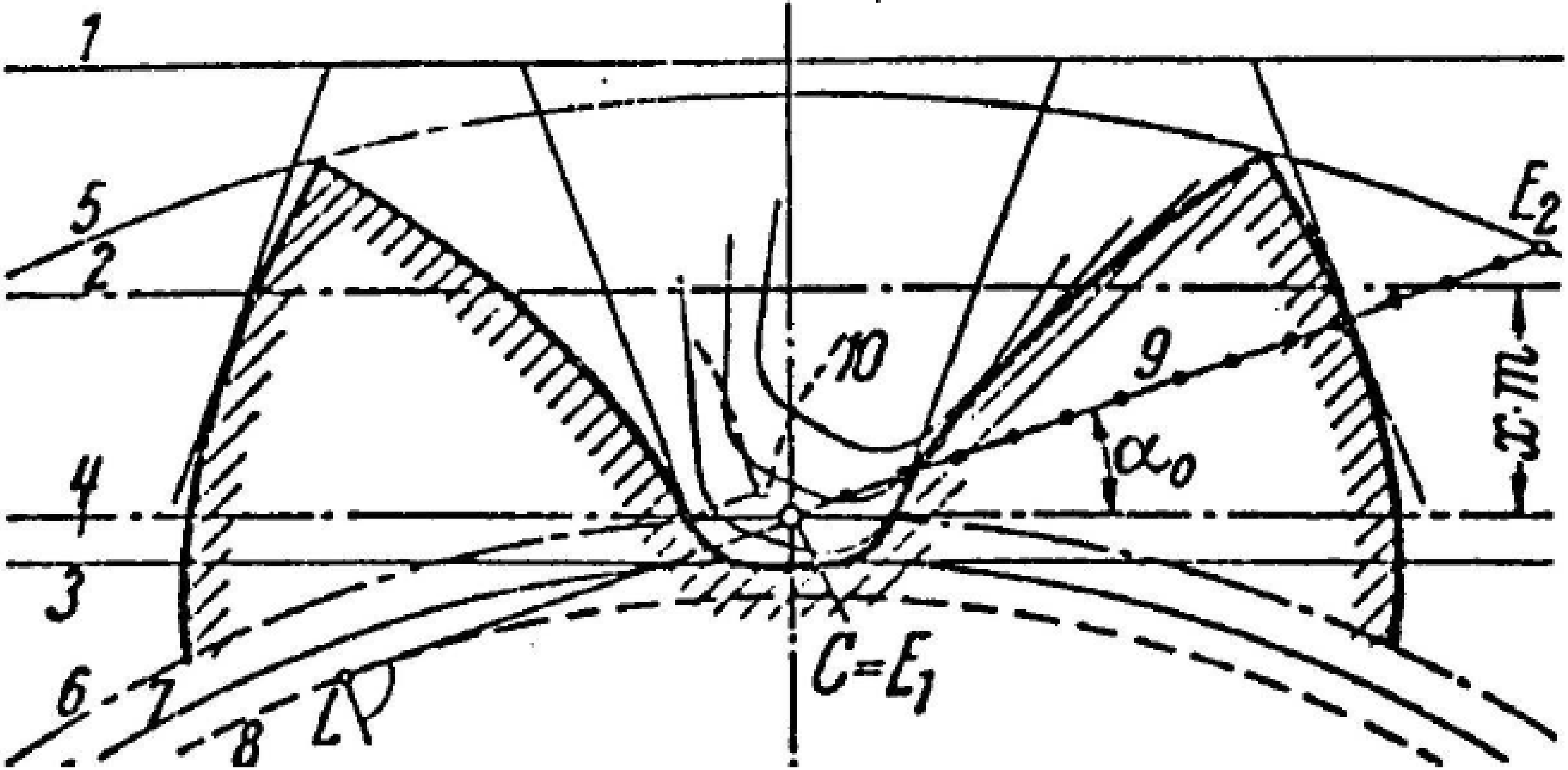
b) Negative addendum modification x < 0 (datum line of tool displaced toward root of the gear)

c) Positive addendum modification x > 0 (datum line of tool displaced toward tip of the gear)



ADDENDUM MODIFICATION

Positive addendum modification x=1; z_1=12



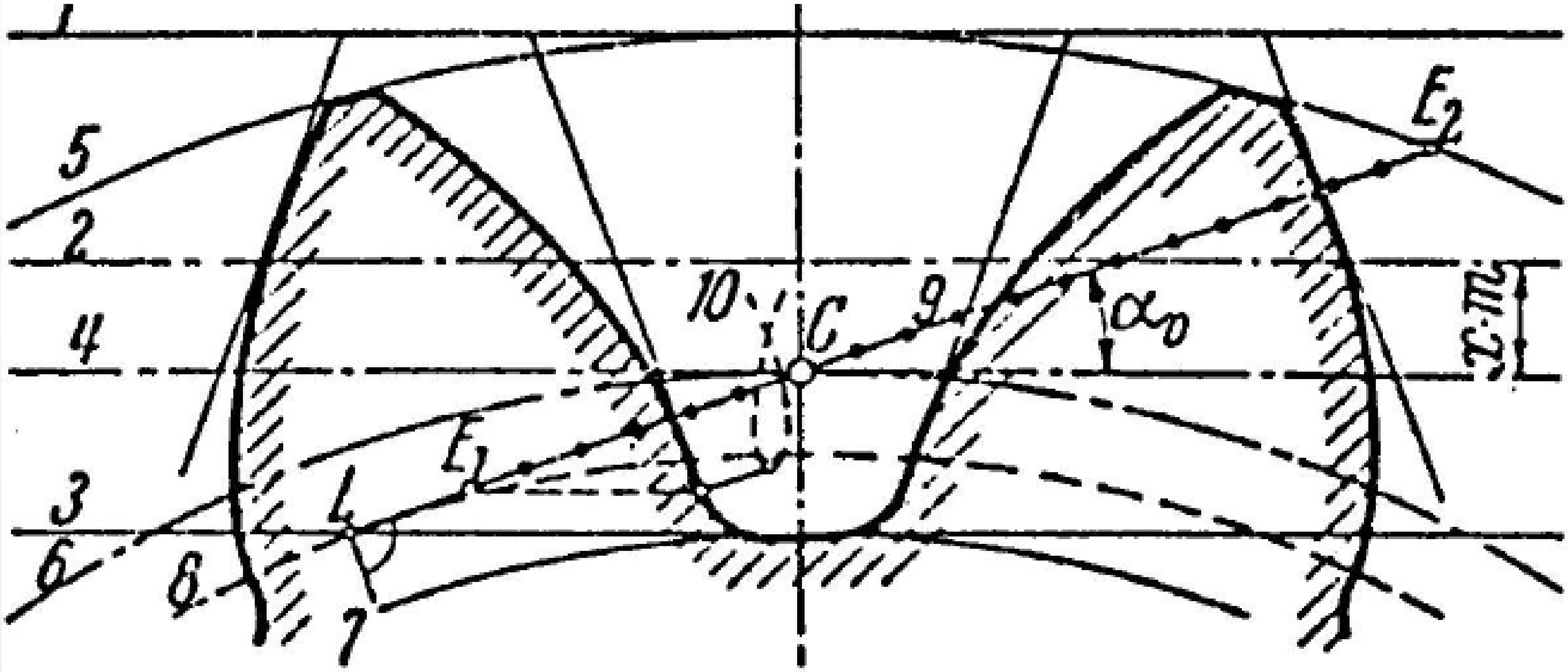
Rack:1 – dedendum line; 2 – pitch line; 3 – addendum line; 4 – pitch line

Gear: 5 – addendum circle; 6 – pitch circle; 7 – dedendum line; 8 – pitch circle

Engagement: 9 – path of contact; 10 – trajectory of the addendum

ADDENDUM MODIFICATION

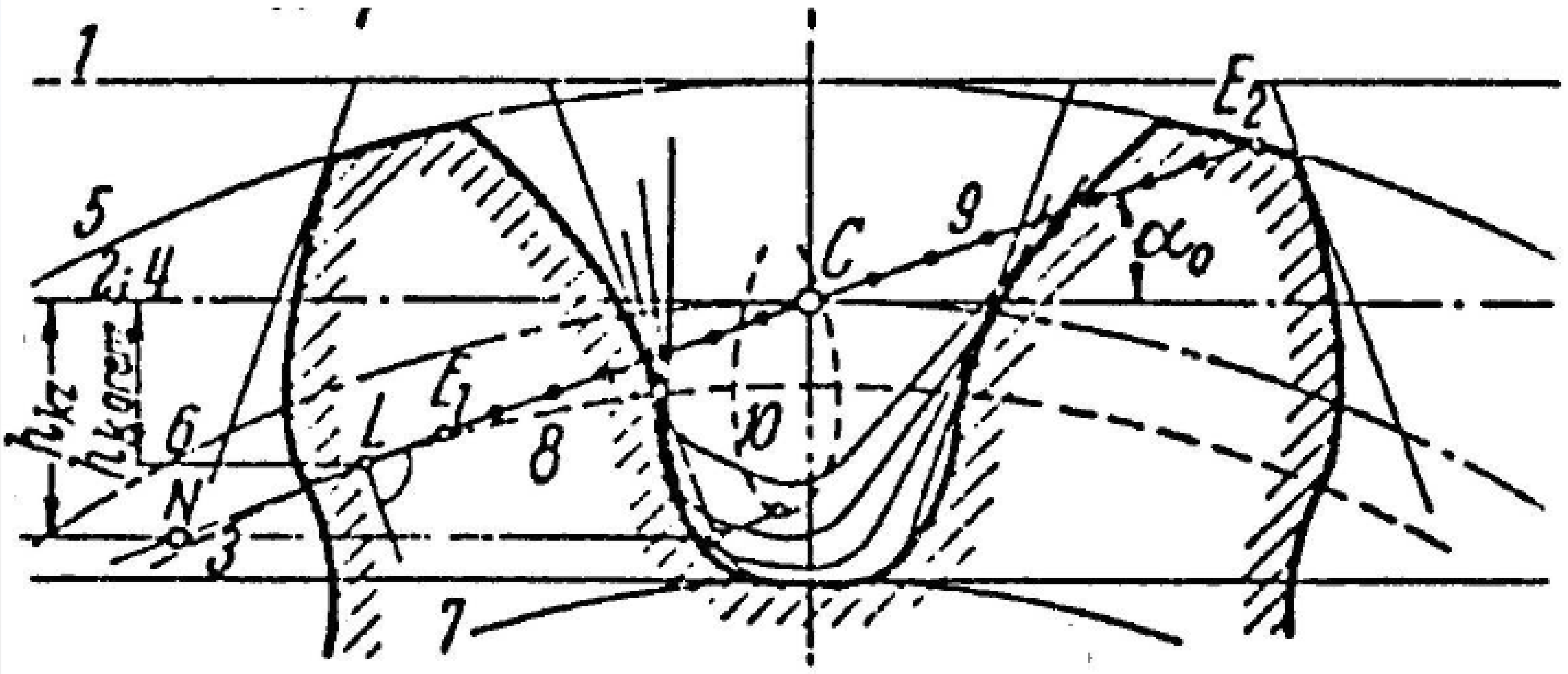
Positive addendum modification $x=0.5$; $z_1=12$



Positive addendum modification causes an increase in the sliding velocities at the tips of the gear teeth and a decrease in the width of teeth at addendum circle.

ADDENDUM MODIFICATION

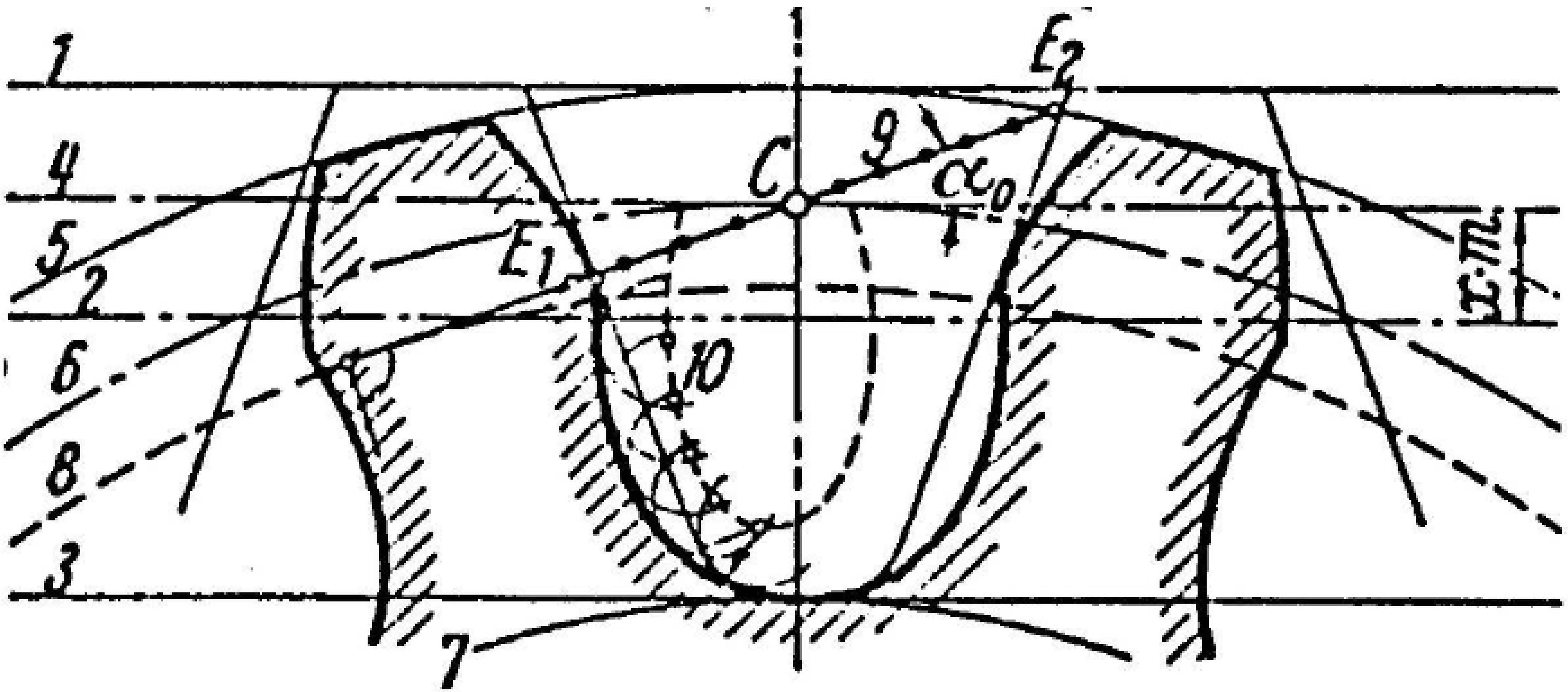
No addendum modification $x=0$; $z_1=12$



Rack: 1 – dedendum line; 2 – pitch line; 3 – addendum line; 4 – pitch line
Gear: 5 – addendum circle; 6 – pitch circle; 7 – dedendum line; 8 – pitch circle
Engagement: 9 – path of contact; 10 – trajectory of the addendum

ADDENDUM MODIFICATION

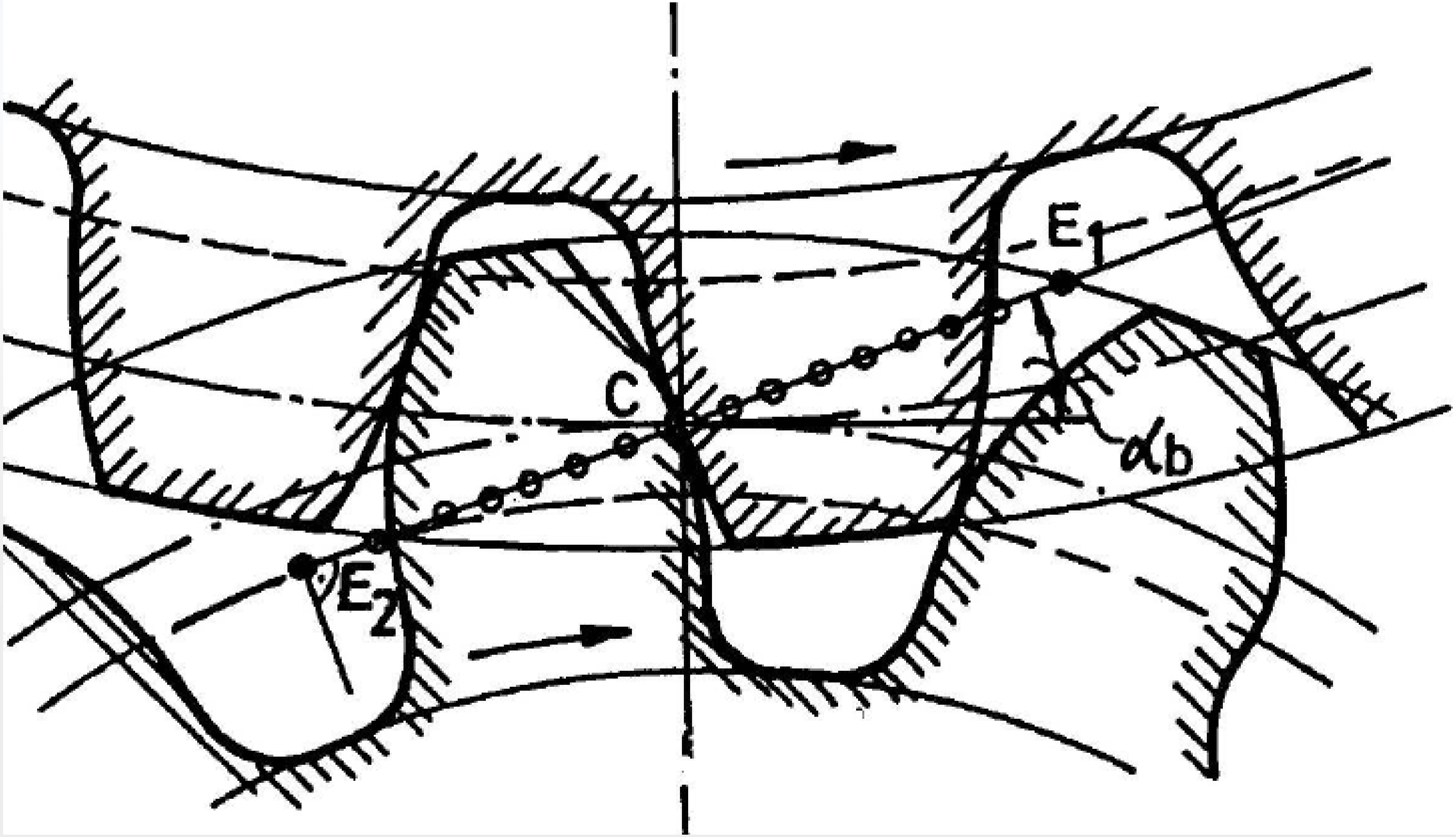
Negative addendum modification x=-0.5; z_1=12



Negative addendum modification causes a decrease in the sliding velocities at the tips of the gear teeth and an increase in the width of teeth at addendum circle.

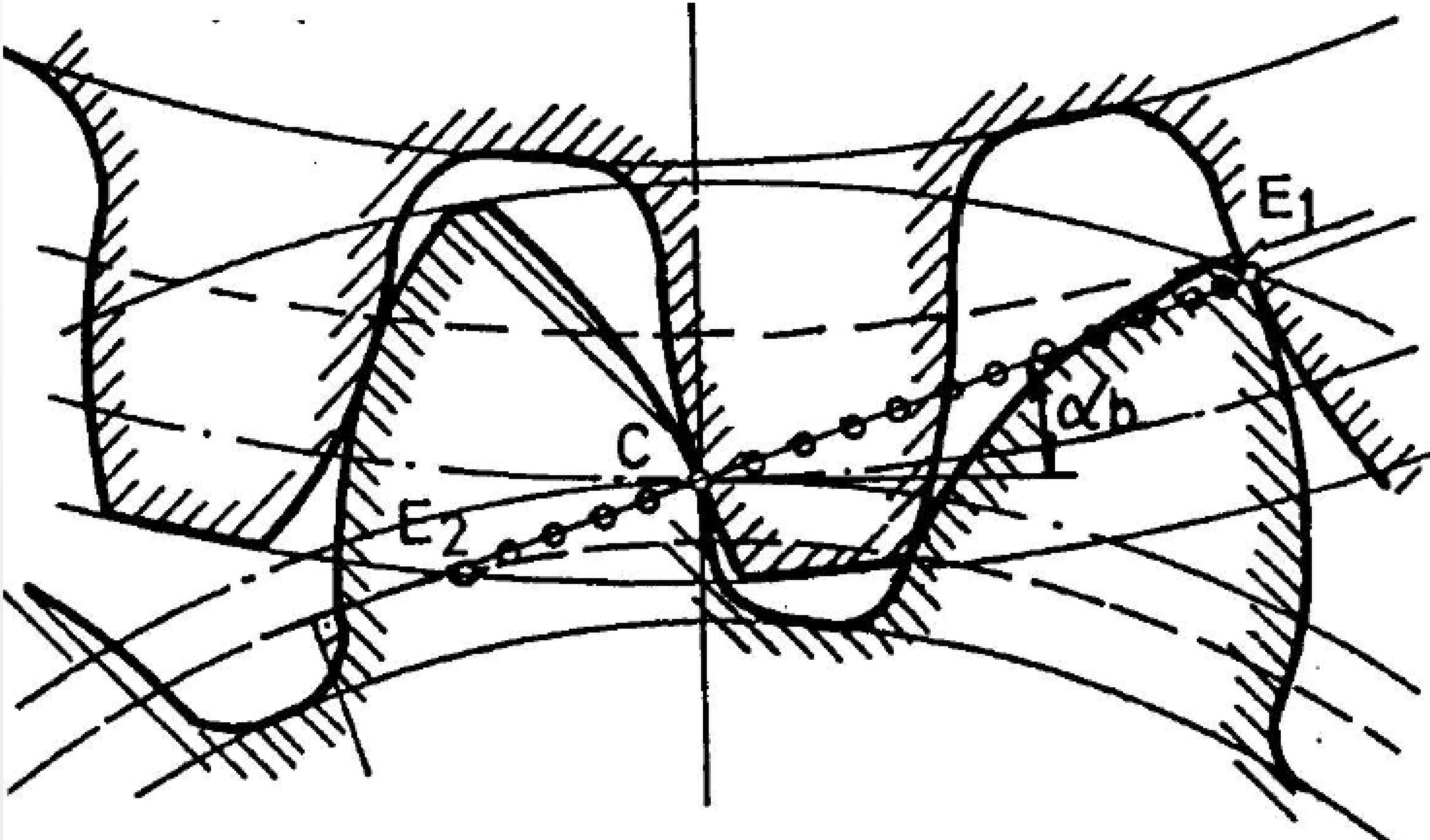
ADDENDUM MODIFICATION

Mating gears: $x_1 = x_2 = 0$; $\alpha_b = \alpha_0 = 20°$; $\varepsilon = 1.28$



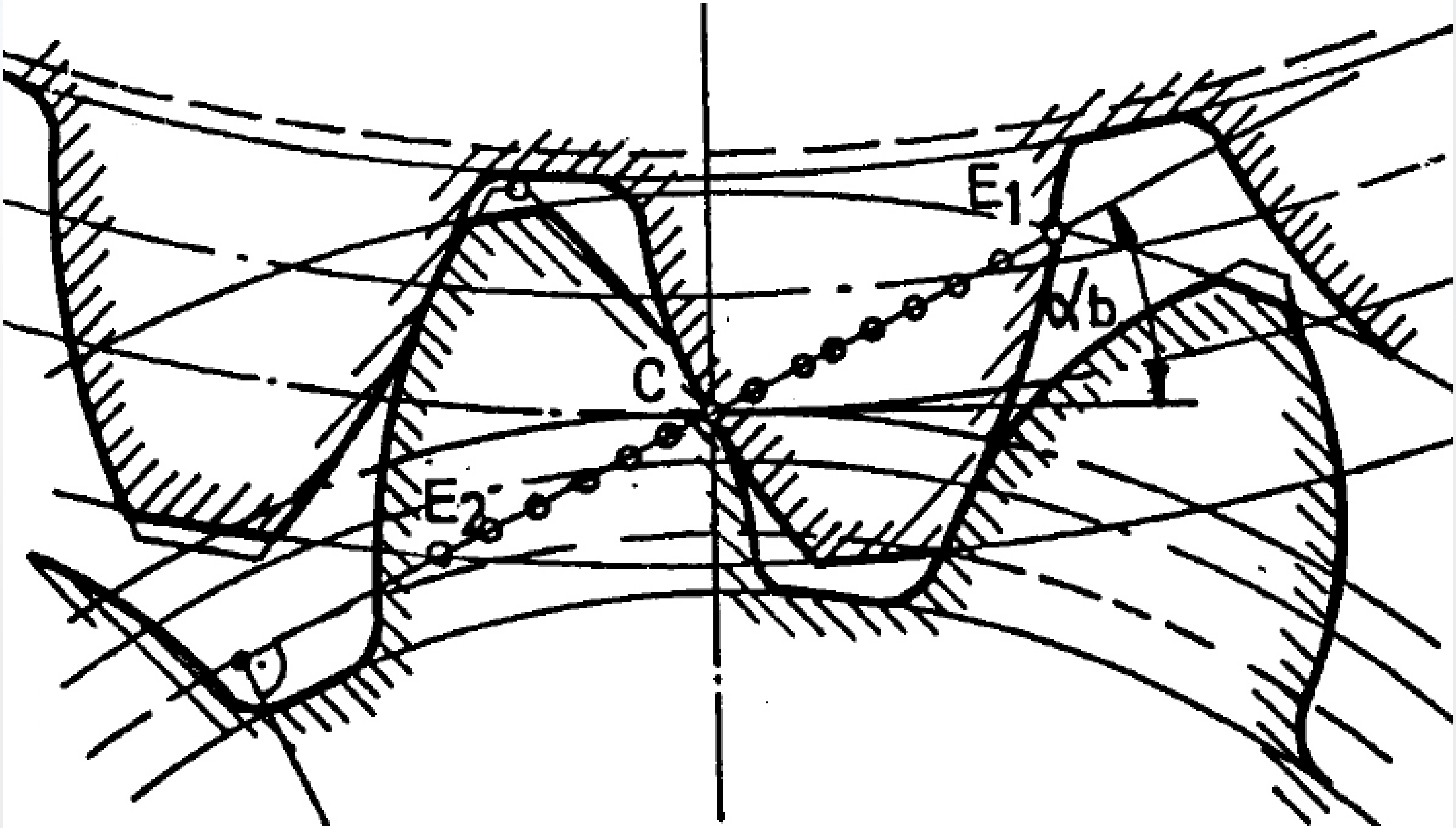
ADDENDUM MODIFICATION

Mating gears: $x_1 = -x_2 = 0.5$; $\alpha_b = \alpha_0 = 20°$; $\varepsilon = 1.43$



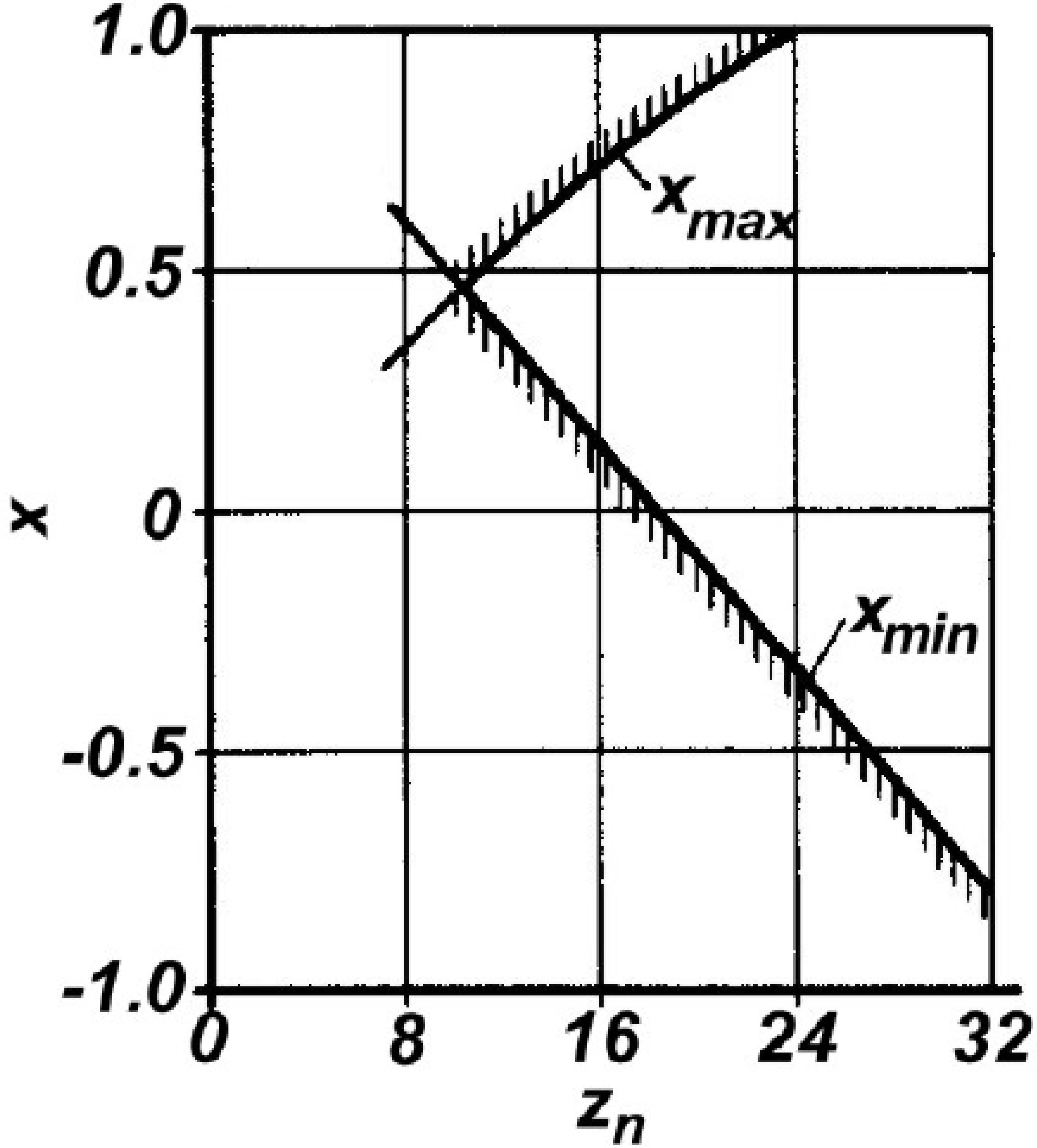
ADDENDUM MODIFICATION

Mating gears: $x_1 = x_2 = 0.5$; $\alpha_b = 26.15°$; $\alpha_0 = 20°$; $\varepsilon = 1.19$

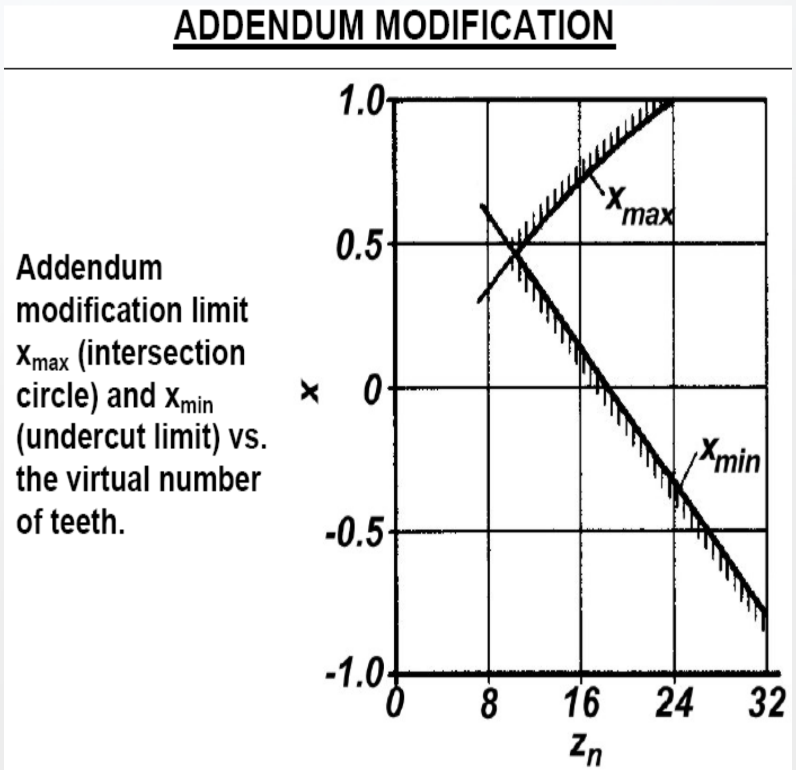


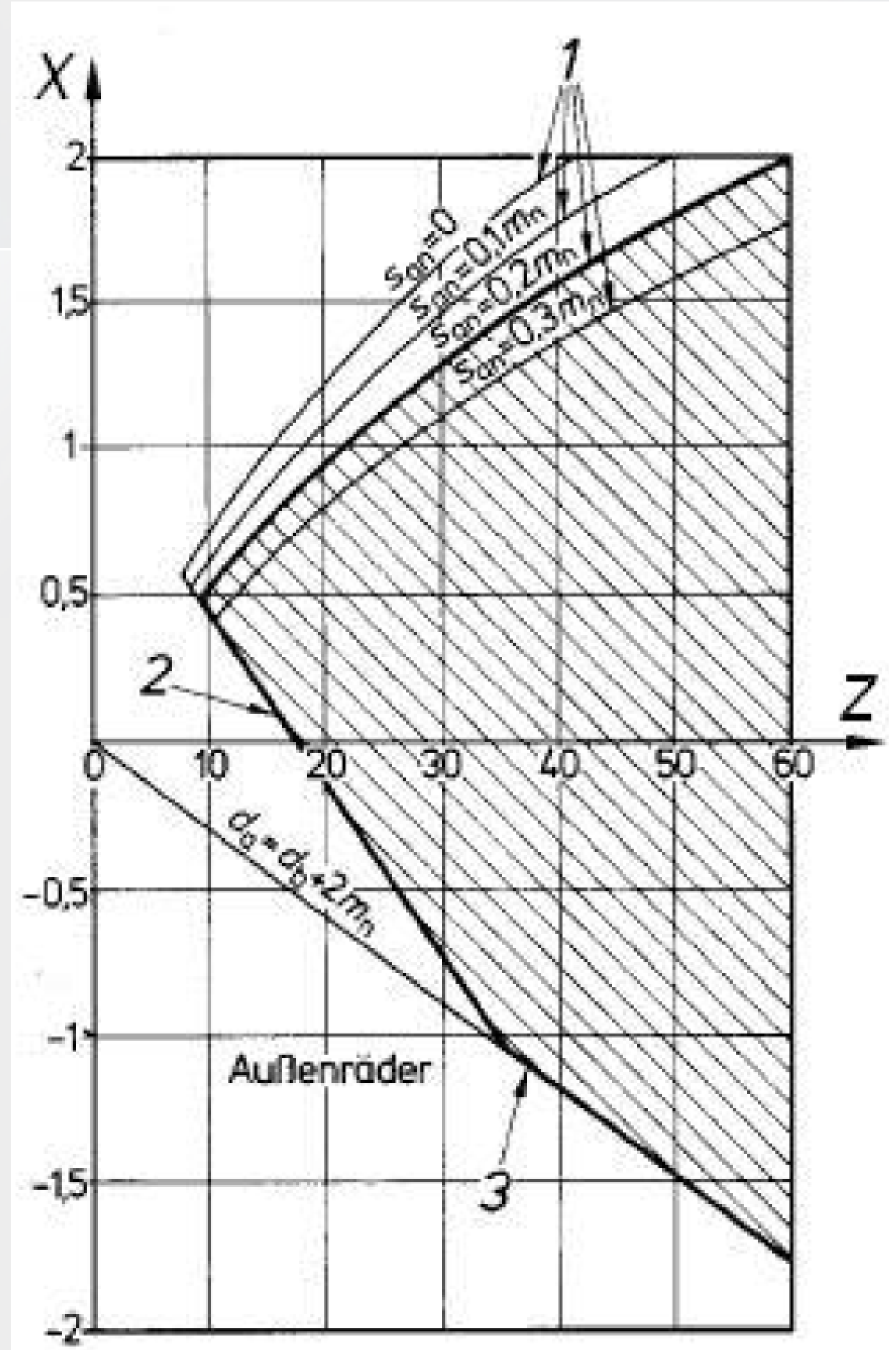
ADDENDUM MODIFICATION

Addendum modification limit x_{max} (intersection circle) and x_{min} (undercut limit) vs. the virtual number of teeth.



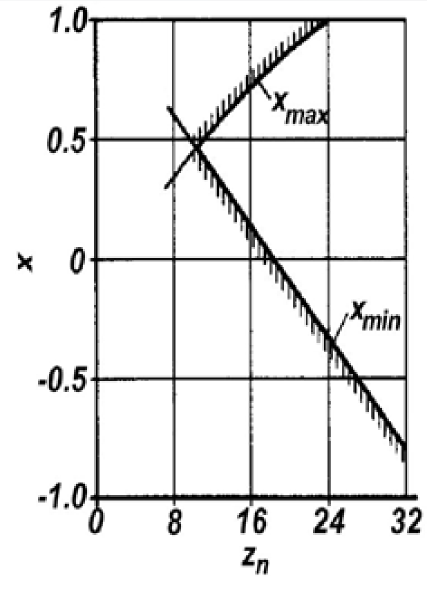
Profil Kaydırma sınırları

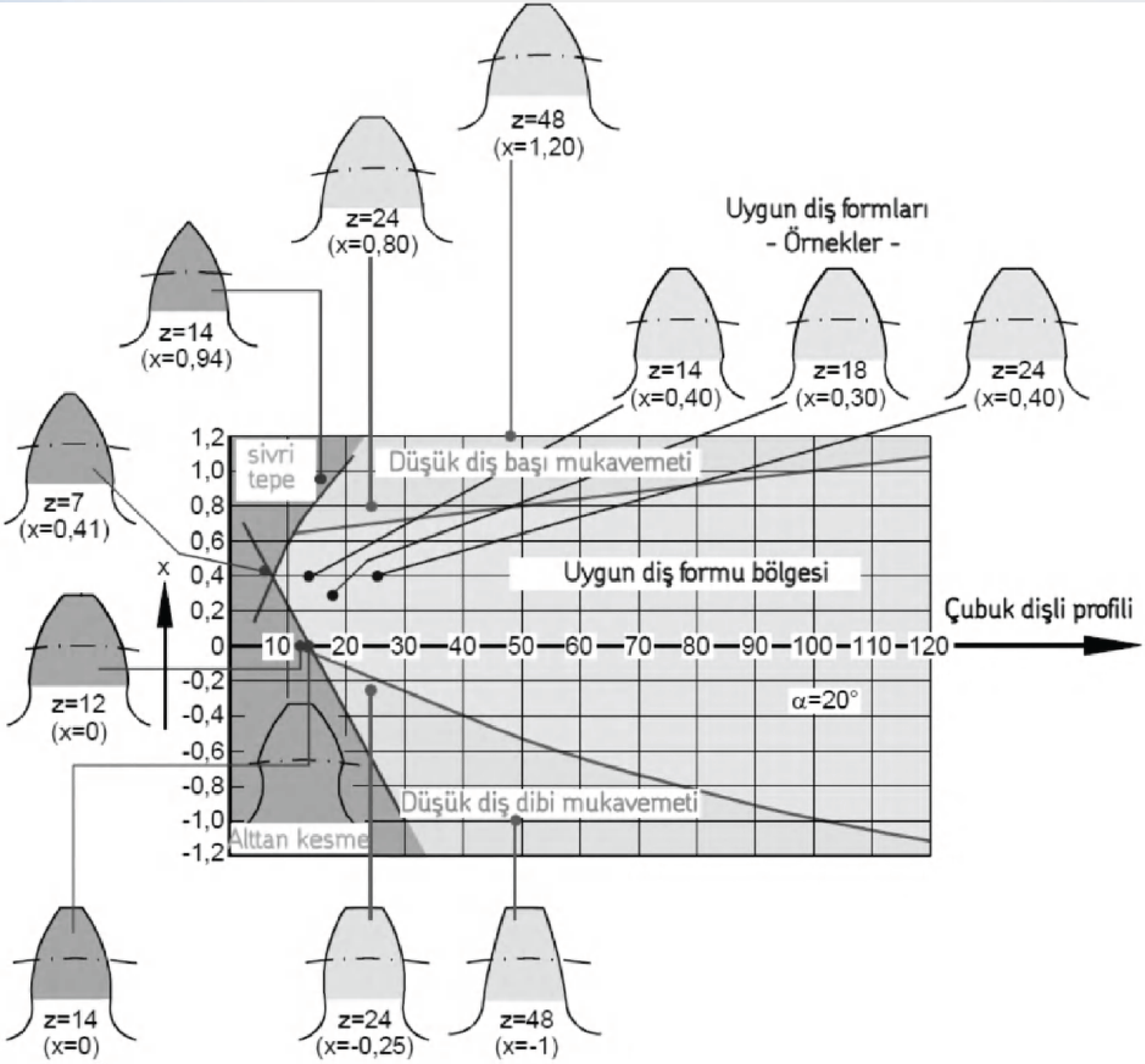




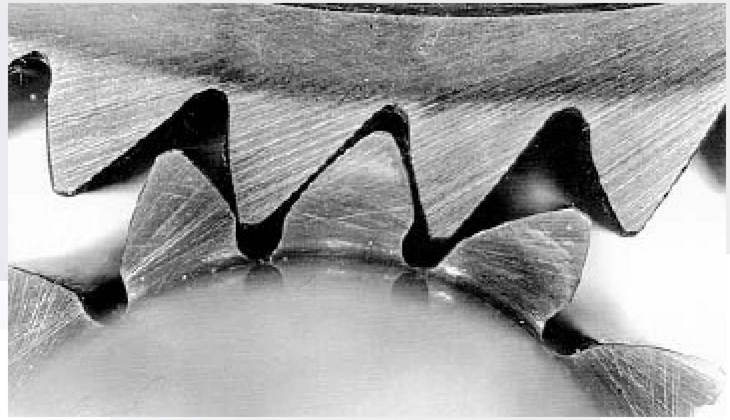
Profil Kaydırma

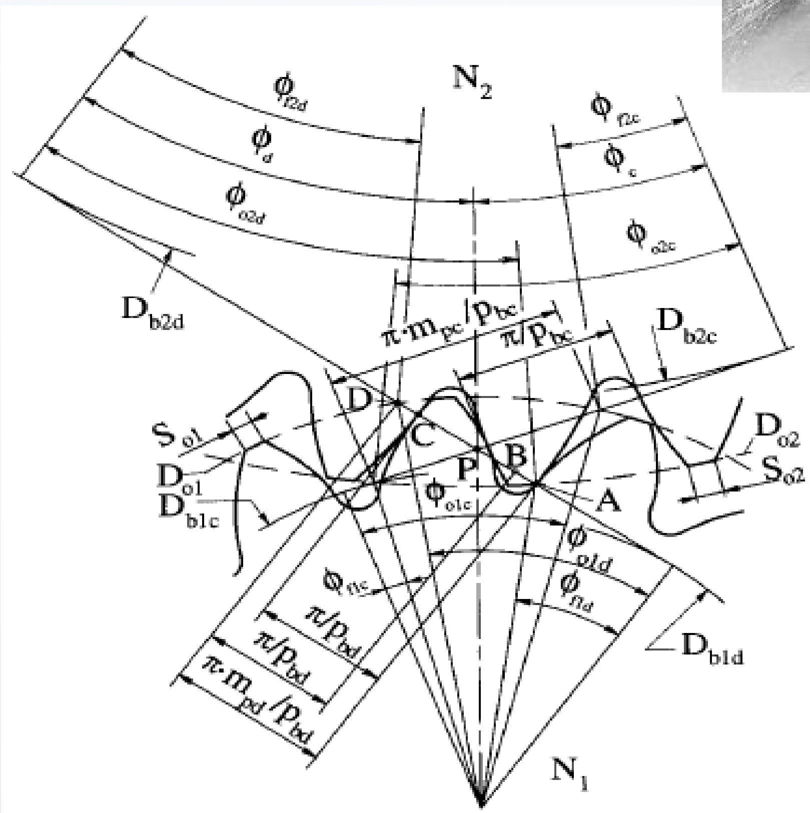
Alt ve Üst sınırları





Asimetrik dişli ve çubuk bıçak





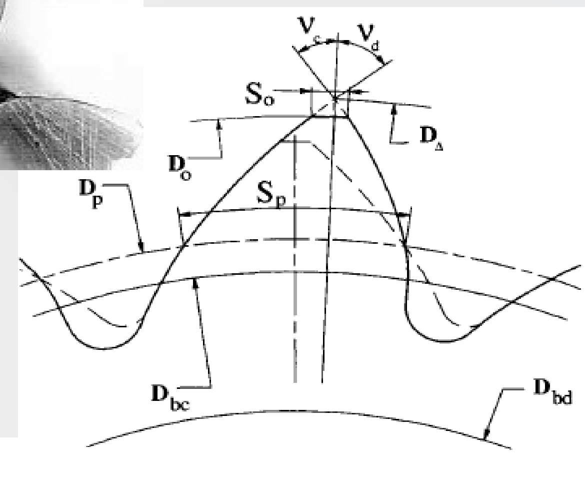


Fig. 1. Formation of the asymmetric involute tooth.

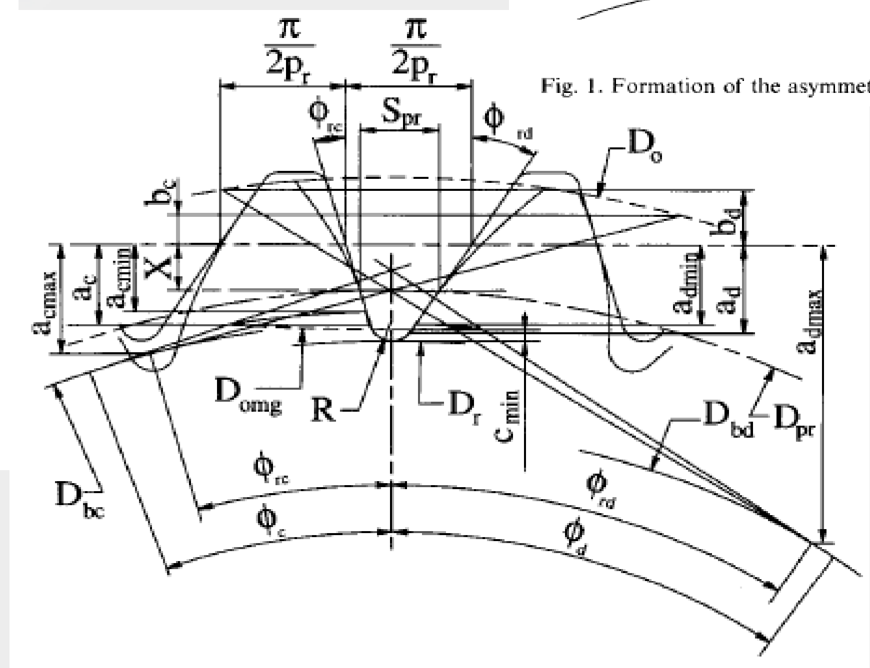
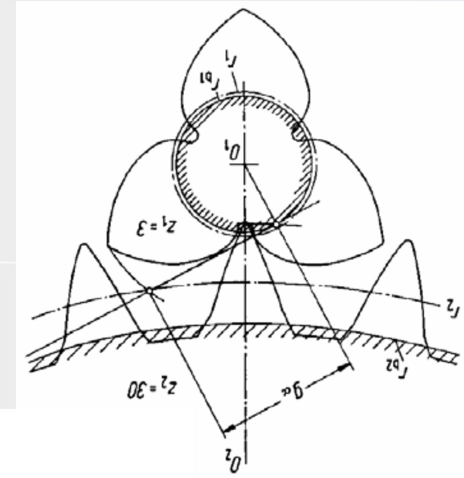
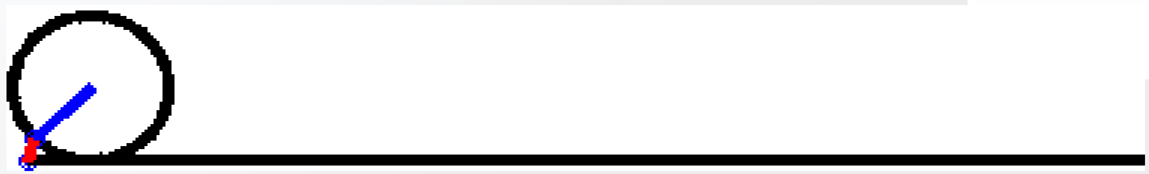


Fig. 4. The asymmetric generating rack.

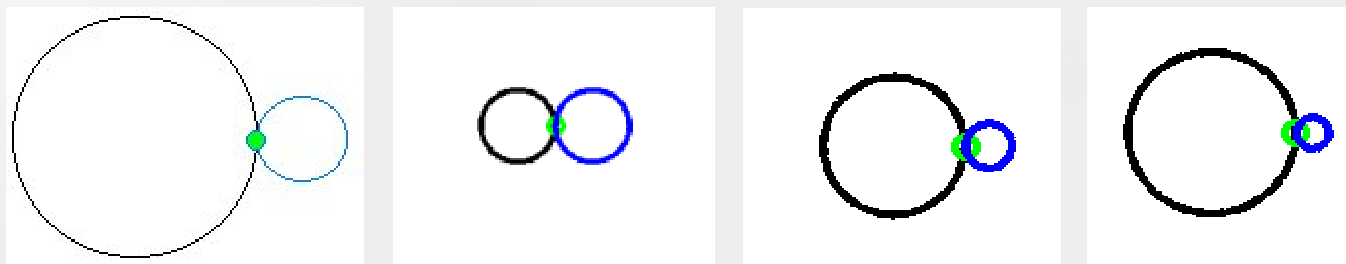
Sikloid diş profili



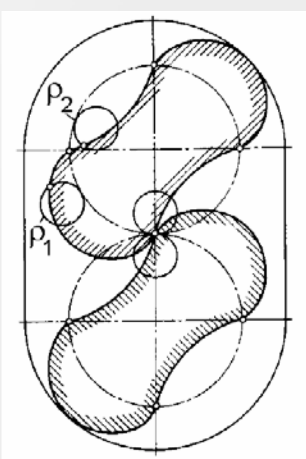
Ortosikloid

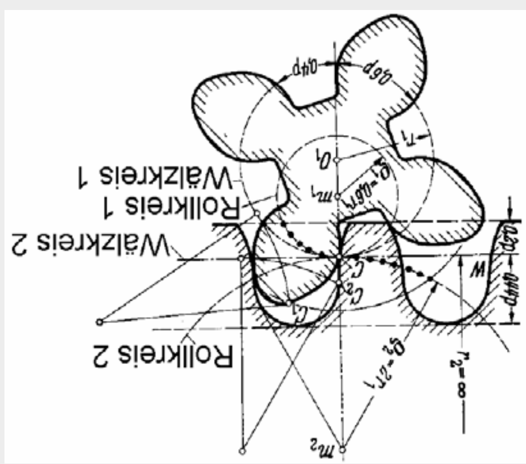


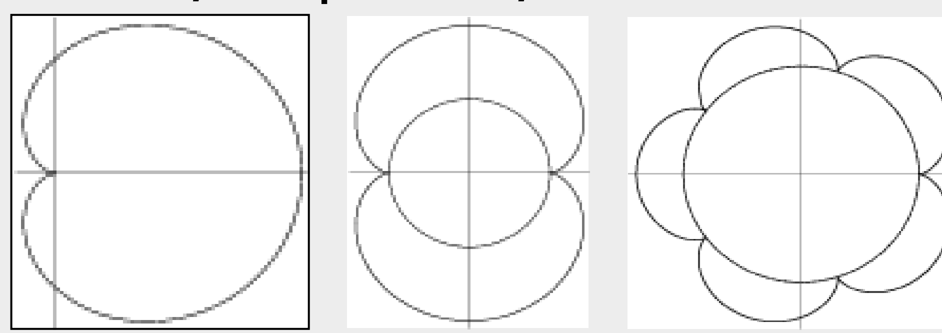
Episikloid



Cardioid, Nephroid, Ranunculoid

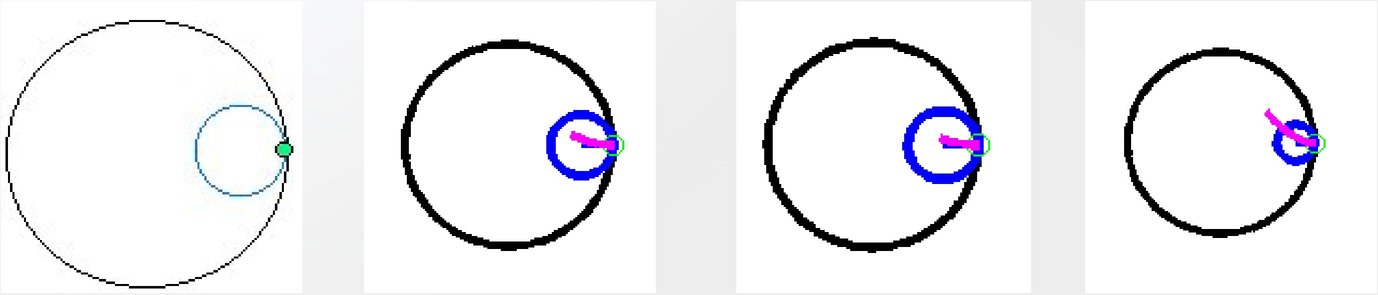


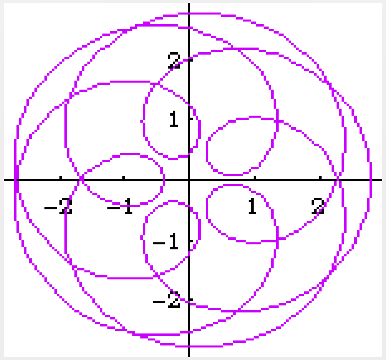




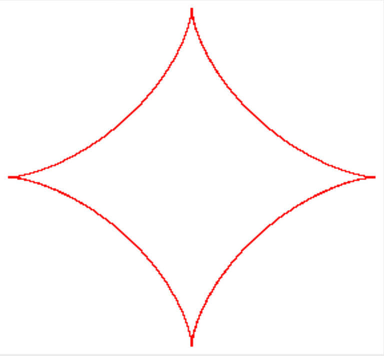
Sikloid diş profili

Hiposikloid

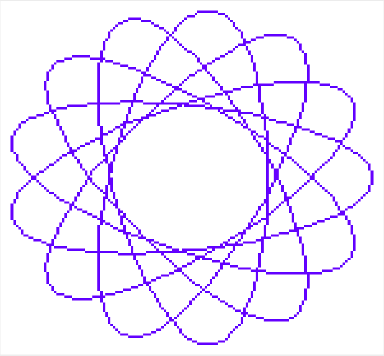




Epitrochoid

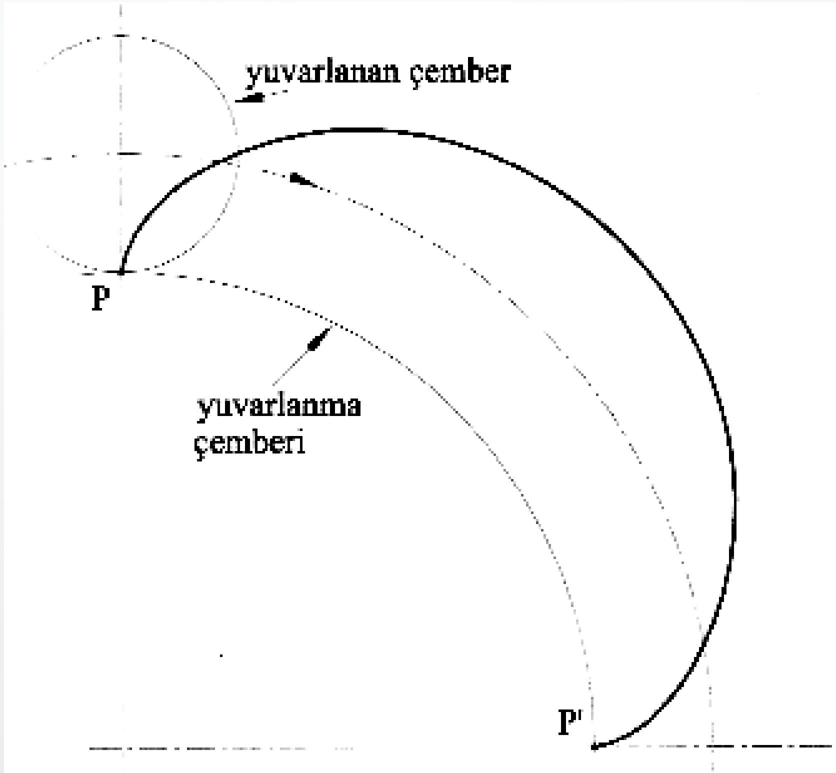


Hypocycloid

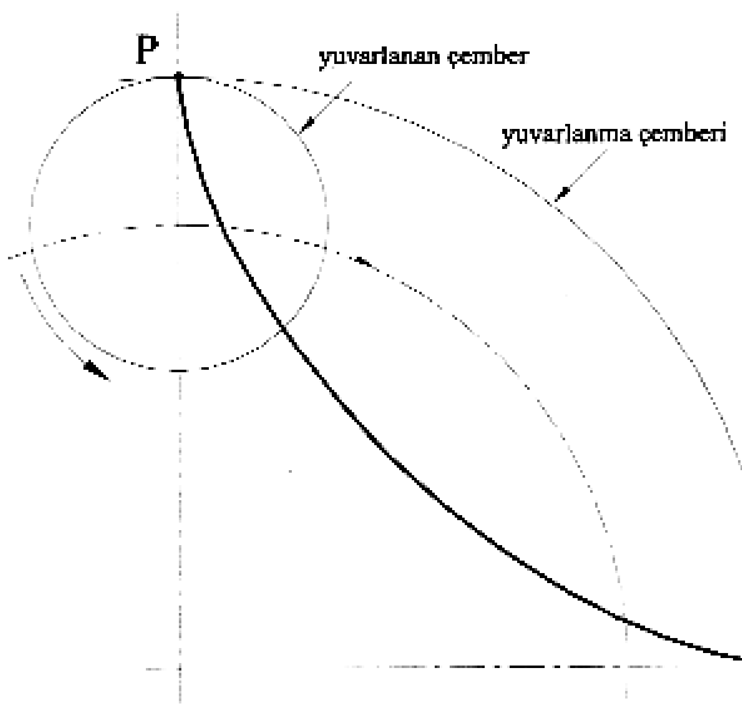


Hypotrochoid

Episikloid ve Hiposikloid diş profili

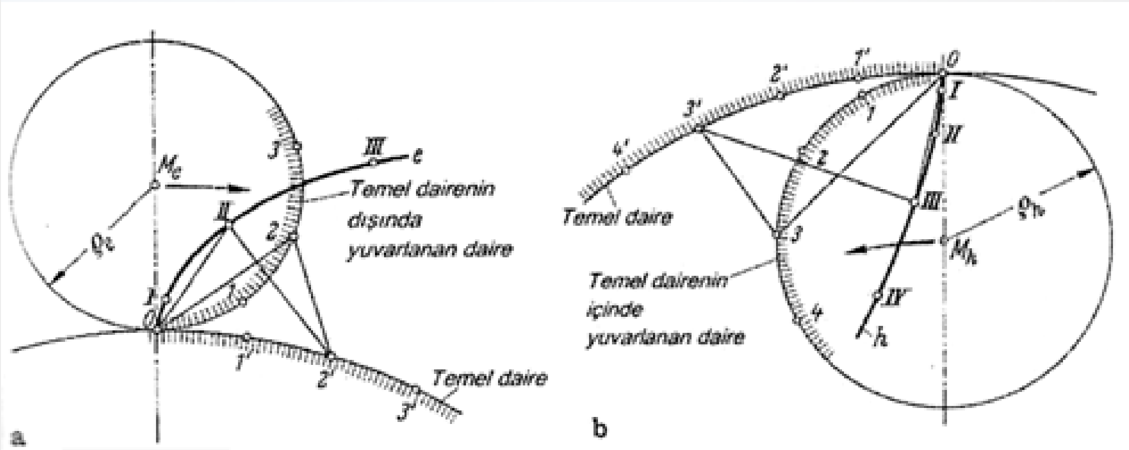


Episikloid

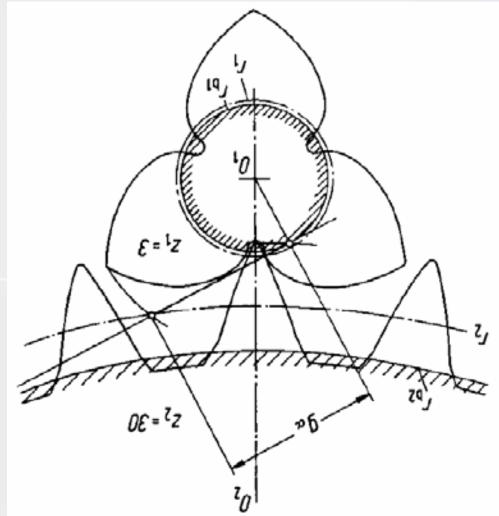


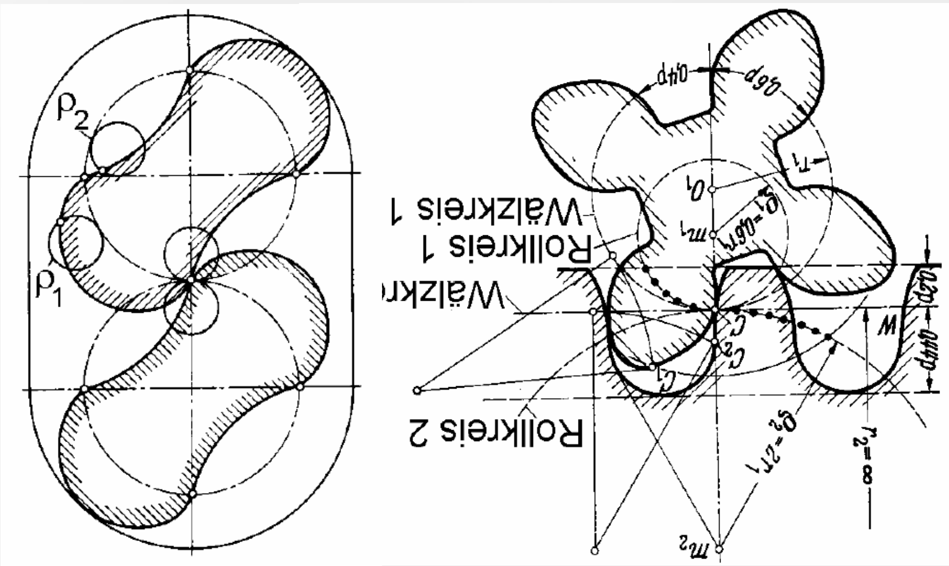
Hiposikloid

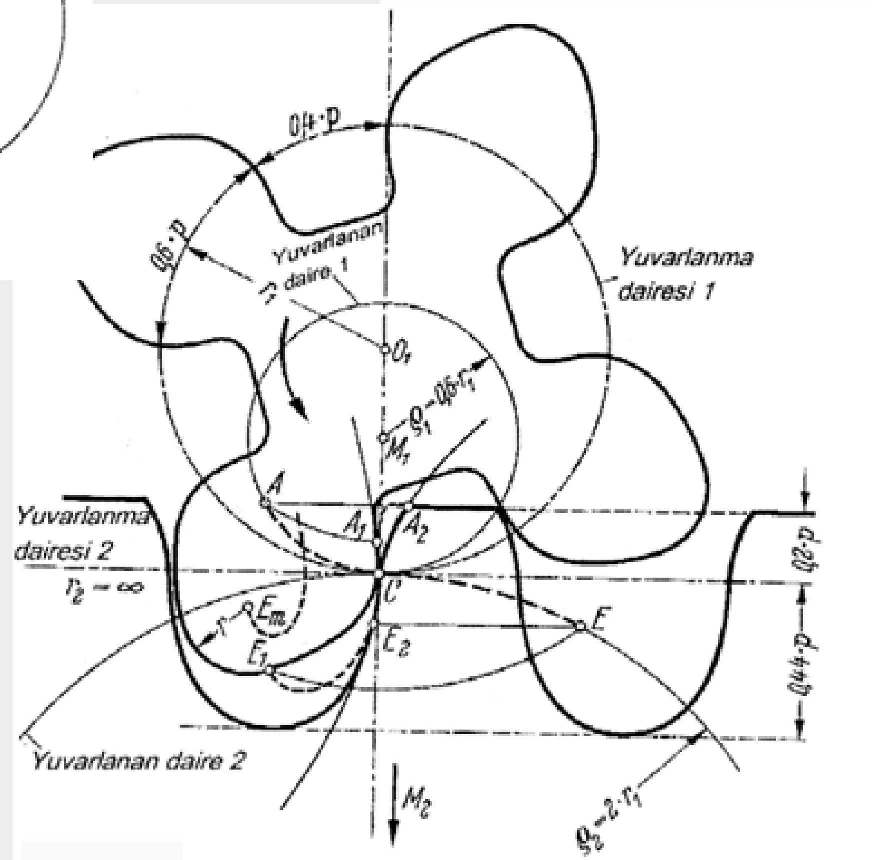
Episikloid, Hiposikloid



Sikloid eğrilerin oluşması a) Episikloid (*e*) b) Hiposikloid (*h*)



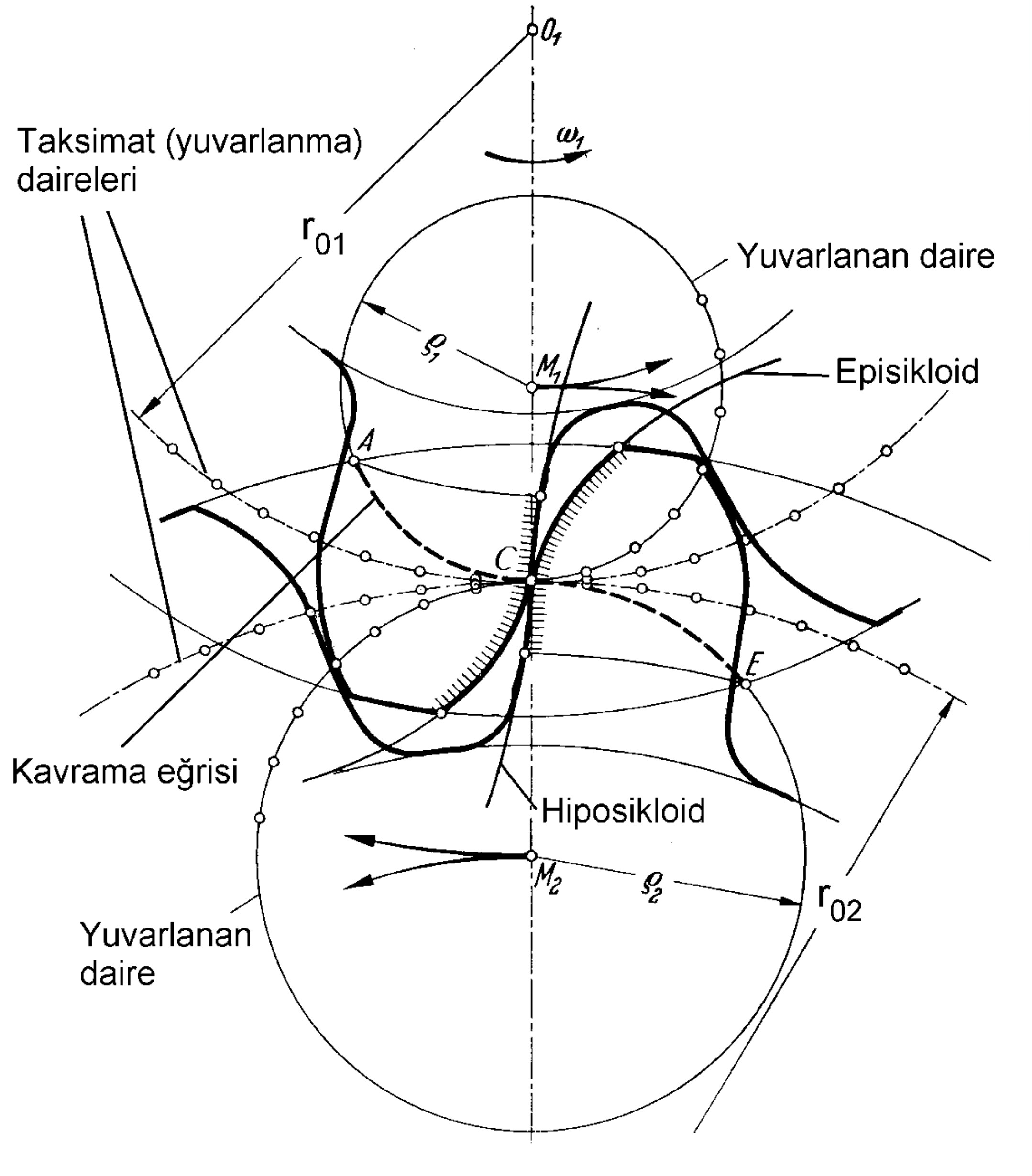




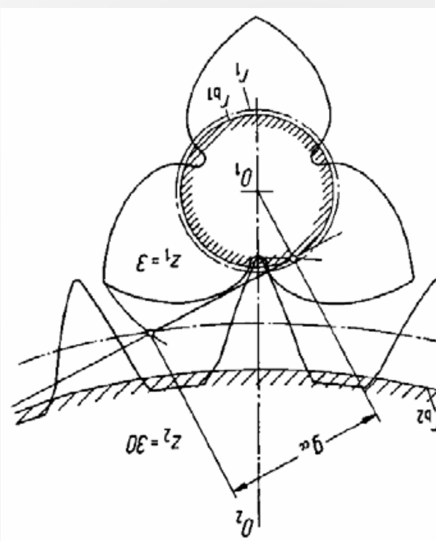
Kremayer dişliye ait sikloid dişli ($z_1 = 4$, $z_2 = \infty$)

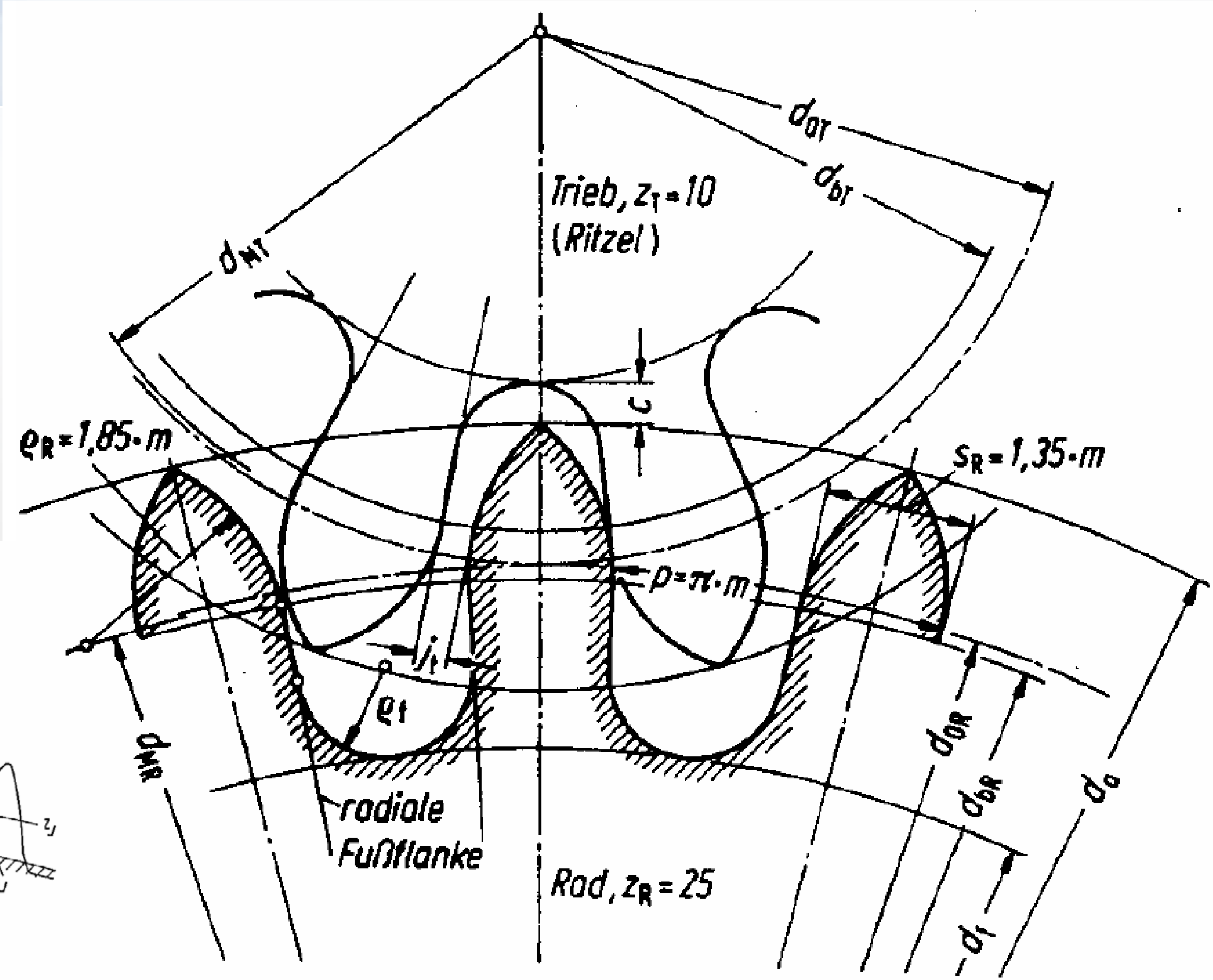
Sikloid

Sikloid, bir dairenin kendisinden büyük bir başka dairenin üzerinde yuvarlanması sırasında yuvarlanan dairenin üzerindeki bir noktanın çizdiği eğridir.

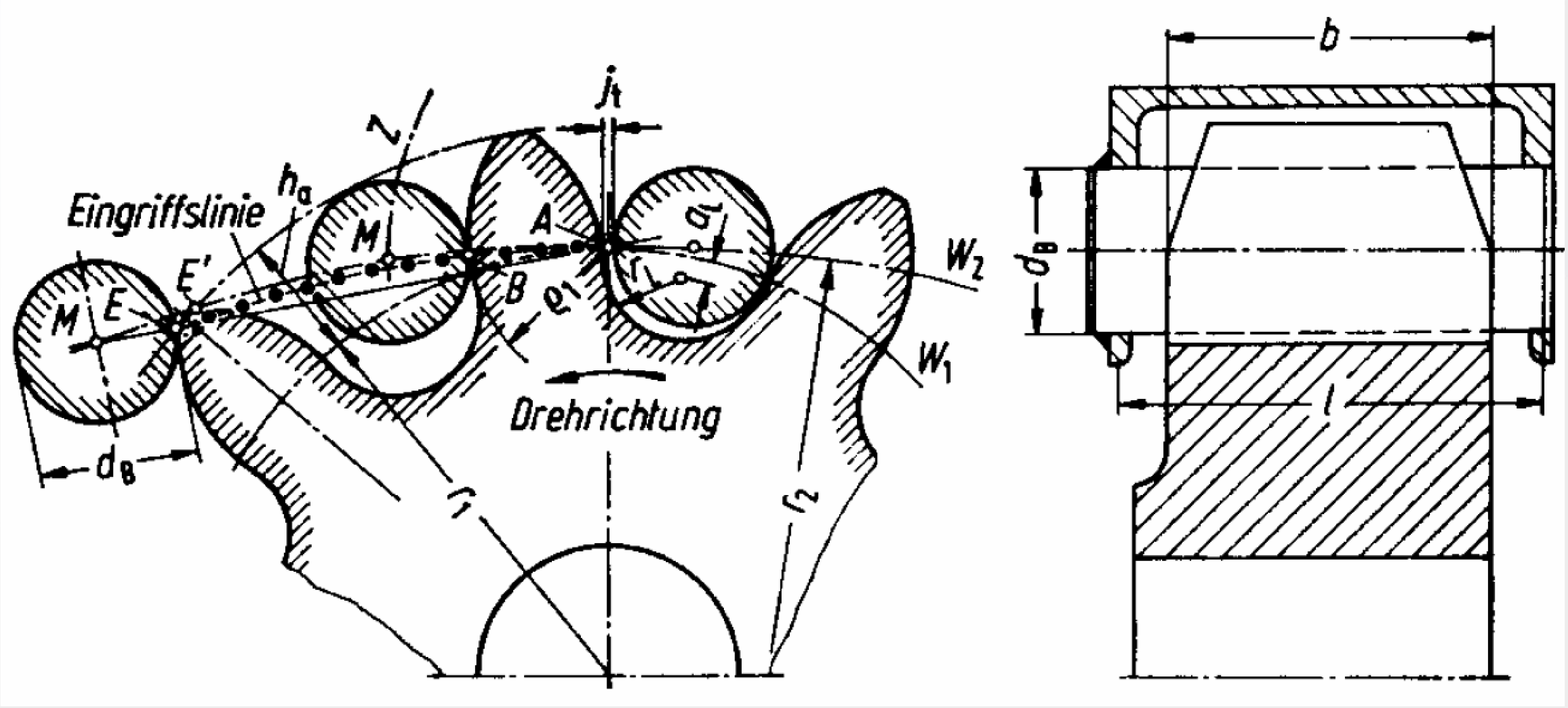


Sikloid

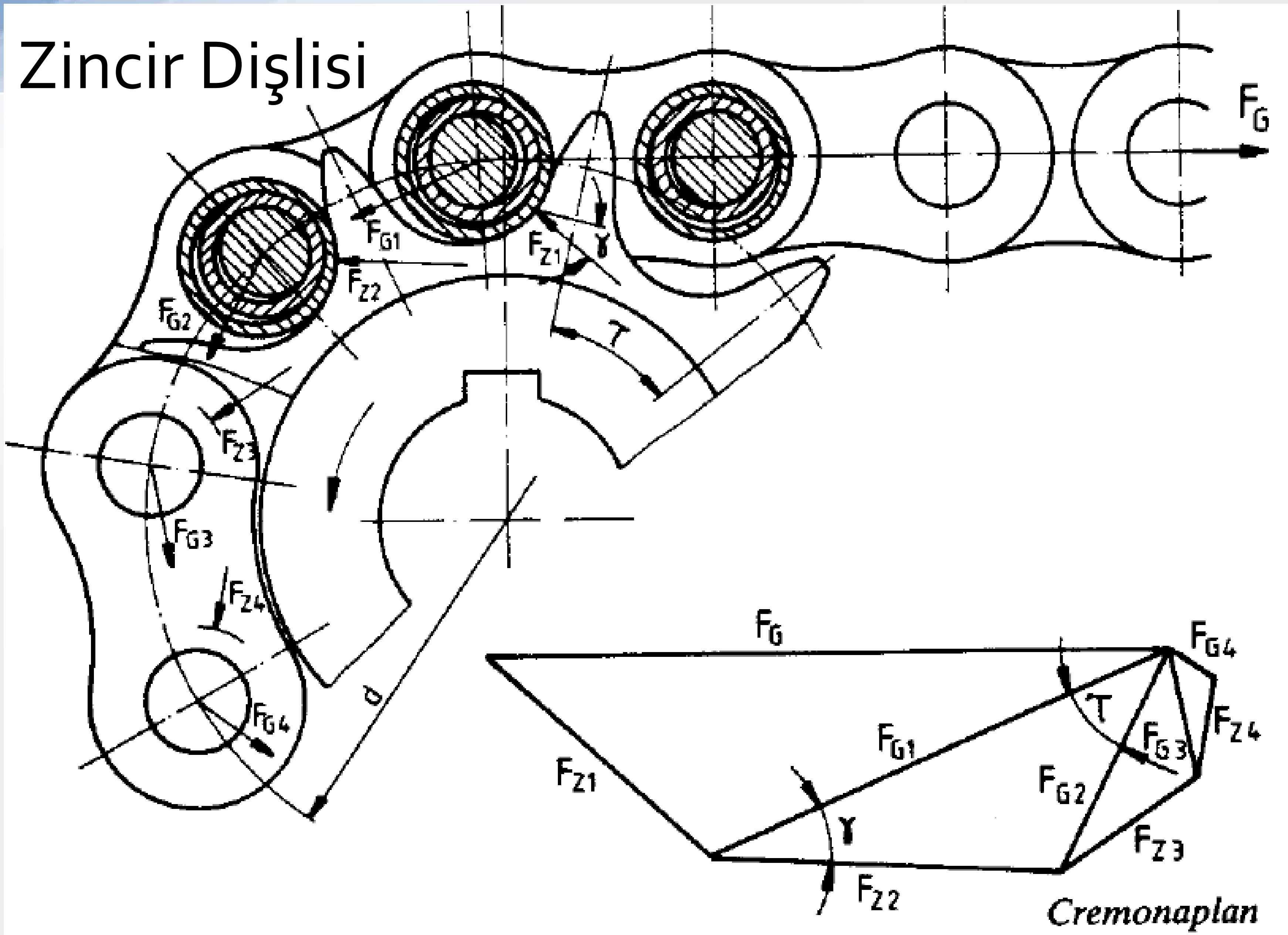



d0T
dbT
Trieb, zT = 10
(Ritzel)
dMT
c
ϱR = 1,85 · m
sR = 1,35 · m
p = π · m
jt
ϱt
dMR
radiale
Fußflanke
Rad, zR = 25
d0R
dbR
da
df

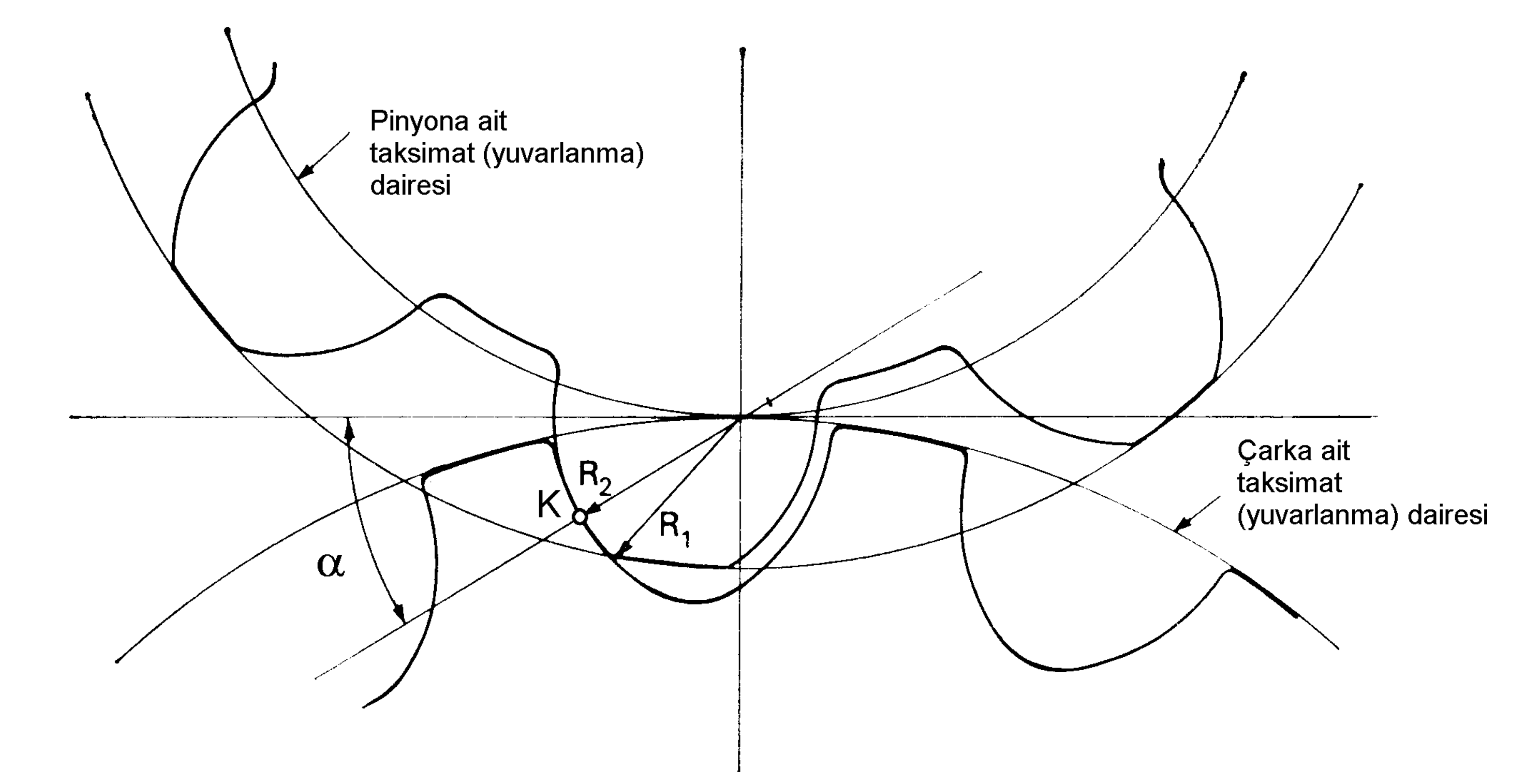
Zincir Dişlisi



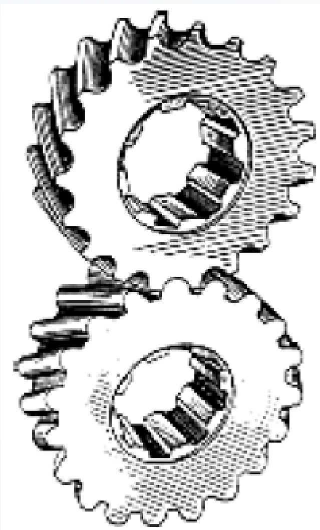
Zincir Dişlisi

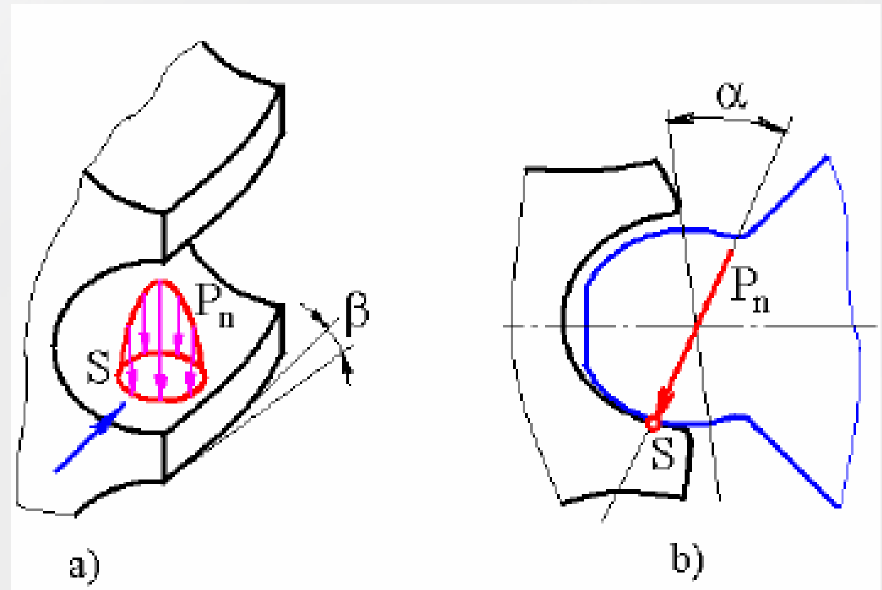


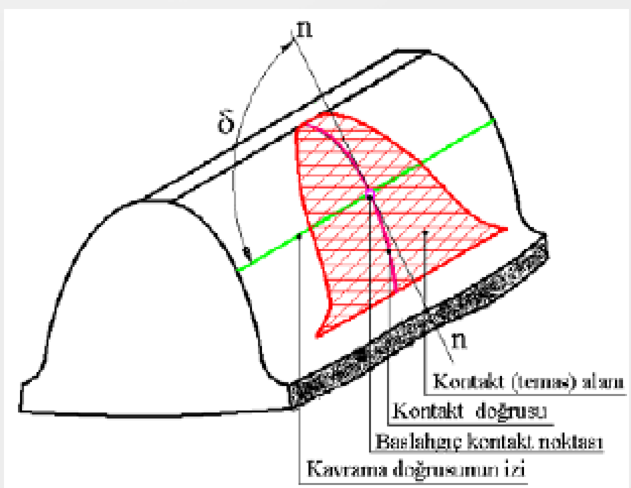
Wildhaber-Novikov diş geometrisine sahip dişliler

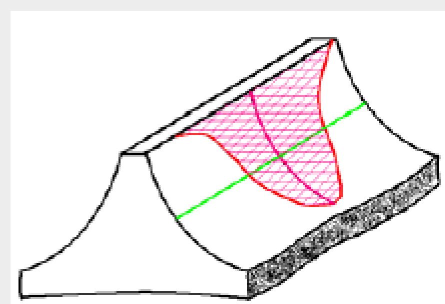


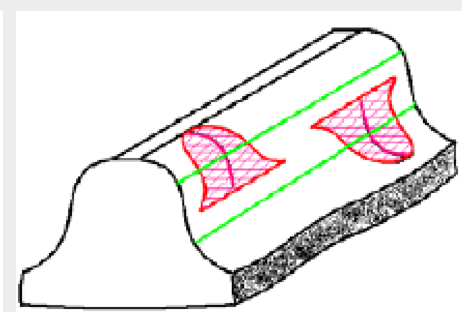
Novikov dişlerinde kavrama





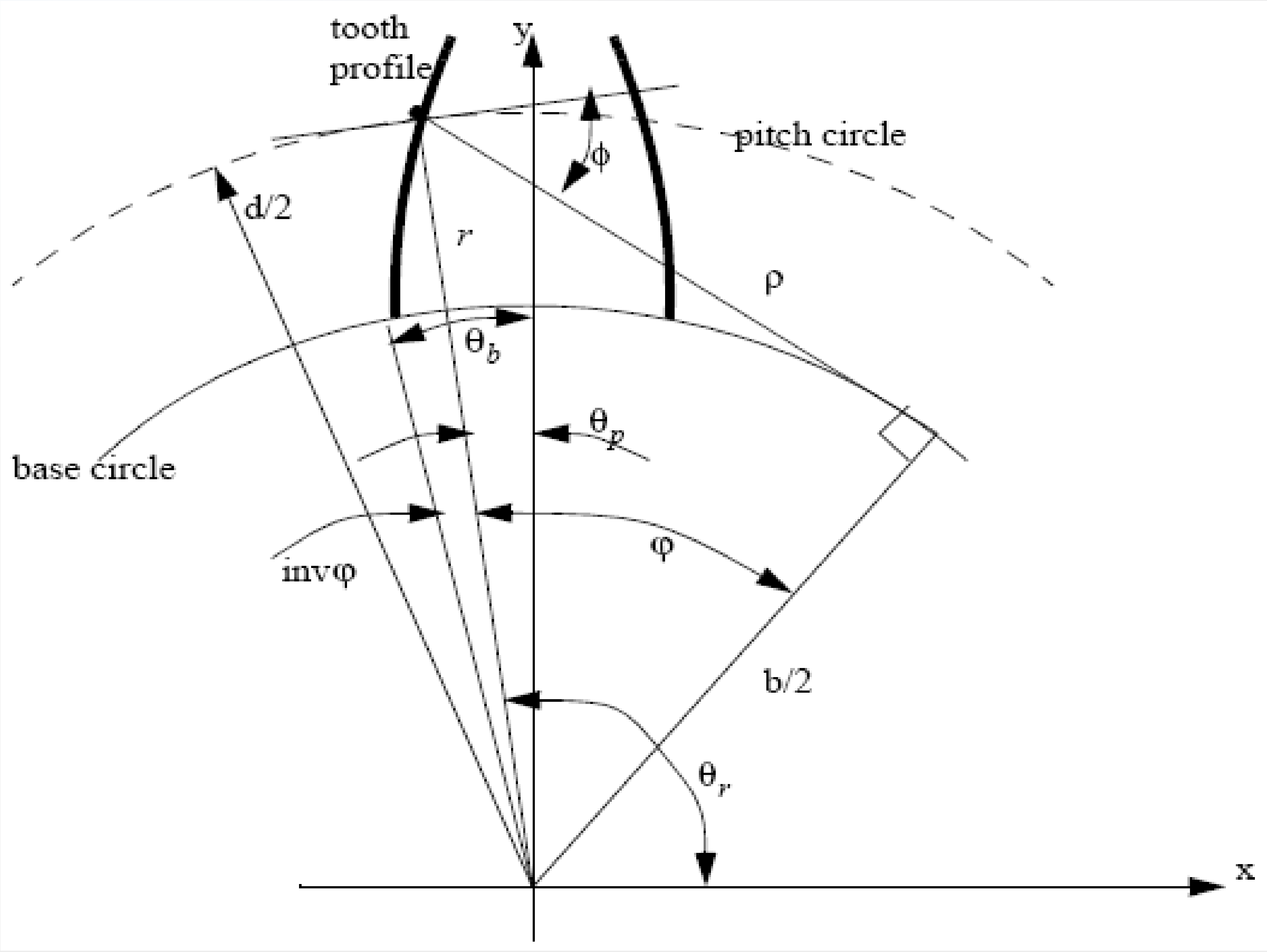






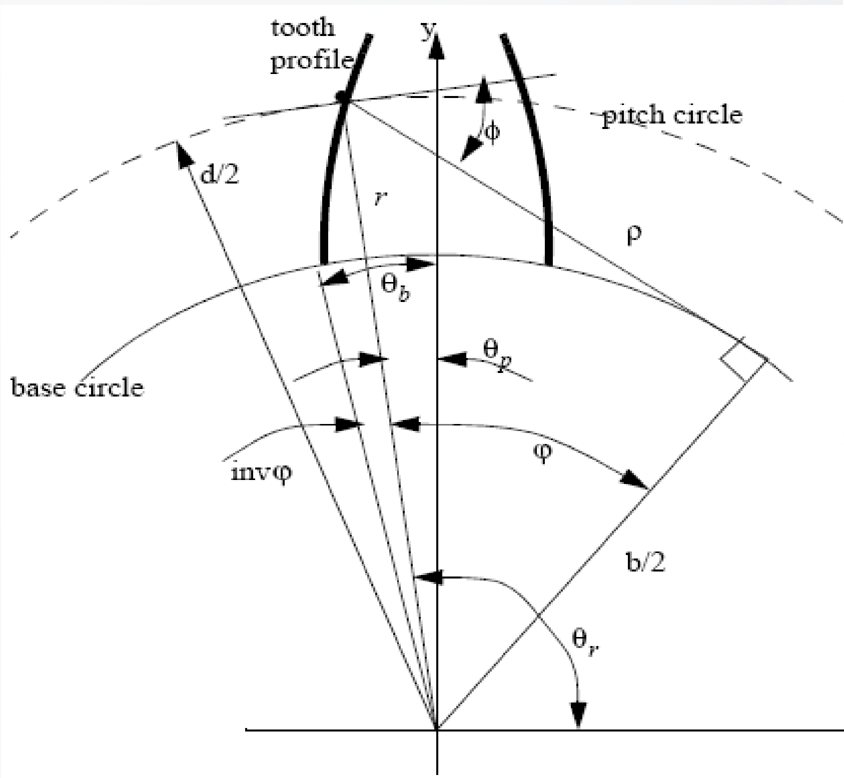
Evolvent Profilli Dişli Çark

evolvent



Evolvent Profilli Dişli Çark

evolvent



For the gear we must consider the pitch,

$$\boxed{r = \frac{d}{2}} \quad \text{when} \quad \varphi = \phi$$

where,

ϕ = the pressure angle

d = diameter of pitch circle

b = diameter of given base circle

Therefore,

$$\boxed{\frac{b}{2} = r\cos(\phi)}$$

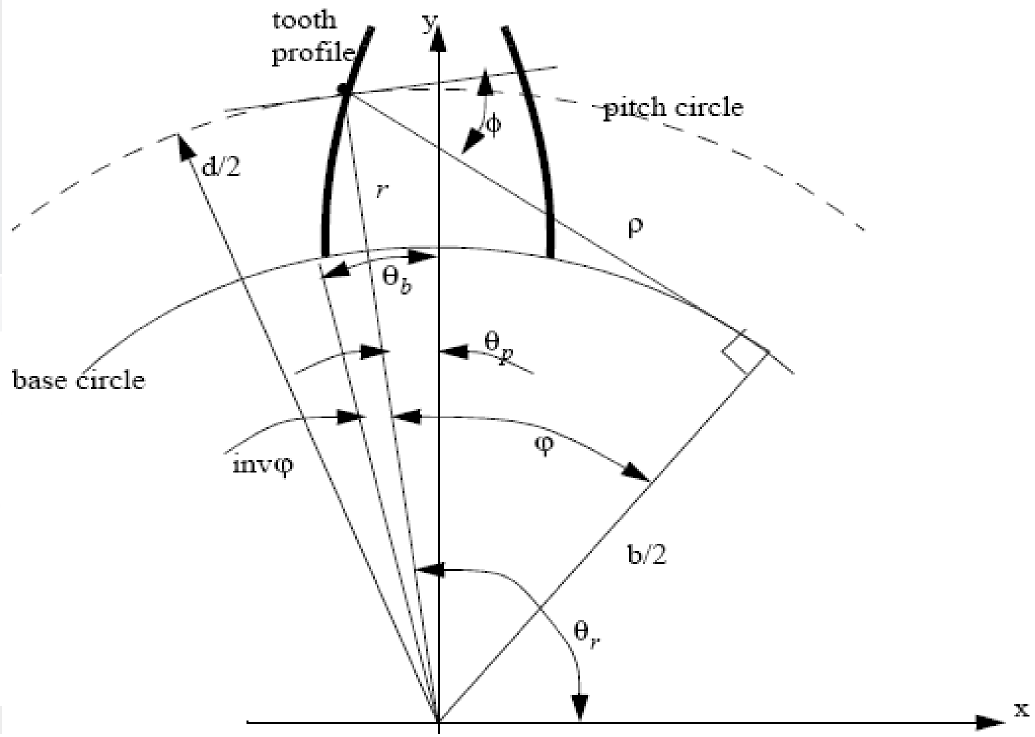
Evolvent Profilli Dişli Çark

ϕ = the pressure angle

d = diameter of pitch circle

b = diameter of given base circle

$$\boxed{\frac{b}{2} = r\cos(\phi)}$$



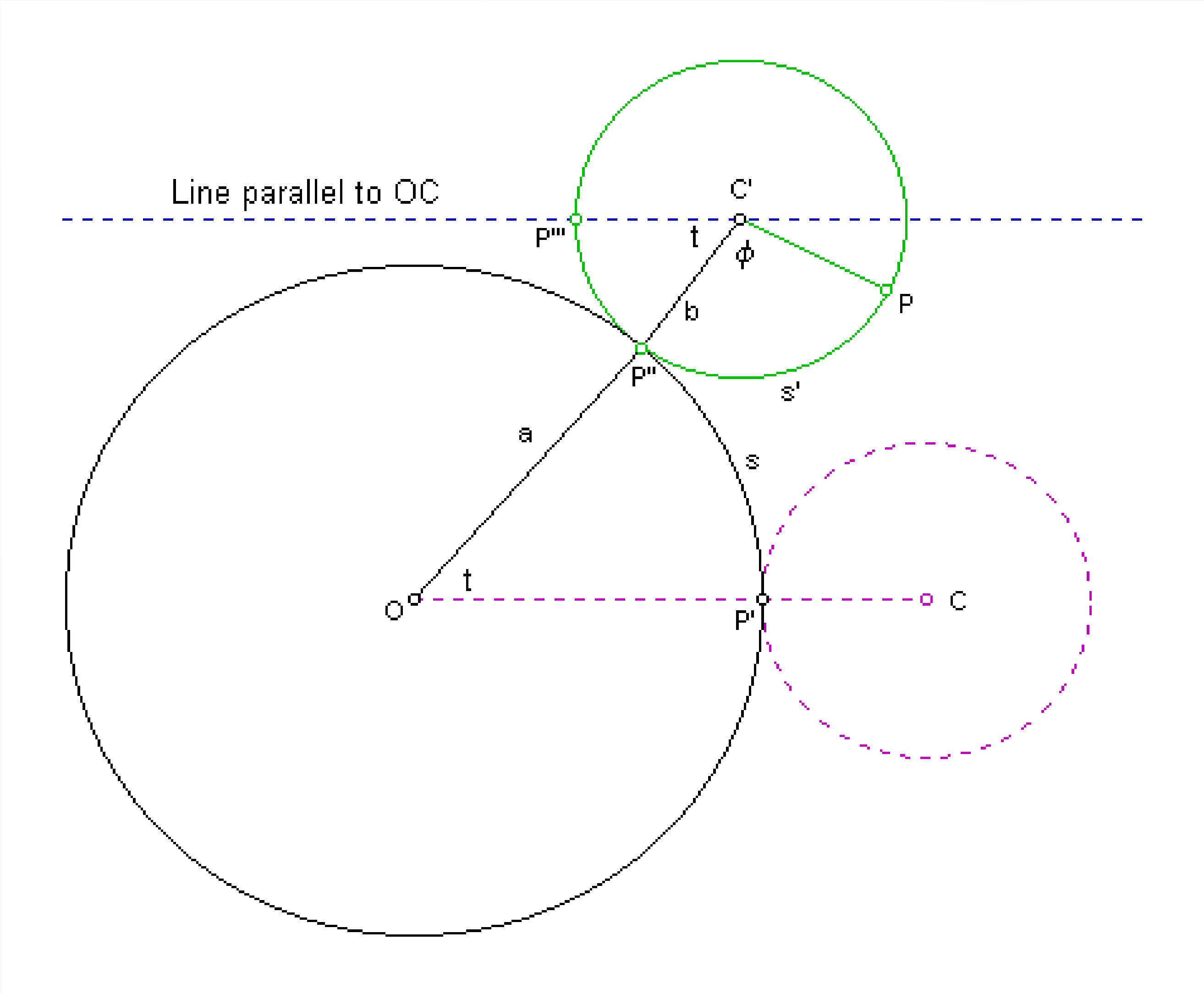
$$p = \left(\frac{\pi d}{N}\right) = 2(2r\theta_p) = 2(d\theta_p) \qquad \therefore \theta_p = \frac{\pi}{2N}$$

$$\theta_b = \theta_p + \mathrm{inv}\phi = \frac{\pi}{2N} + \mathrm{inv}\phi$$

$$\theta_r = \frac{\pi}{2} + \theta_b - \mathrm{inv}\varphi = \frac{\pi}{2} + \frac{\pi}{2N} + \mathrm{inv}\phi - \mathrm{inv}\varphi = \boxed{\frac{\pi}{2}\left(1 + \frac{1}{N}\right) + \mathrm{inv}\phi - \mathrm{inv}\varphi}$$

$$x = r\cos(\theta_r) \qquad y = r\sin(\theta_r) \qquad \varphi \geq 0deg$$

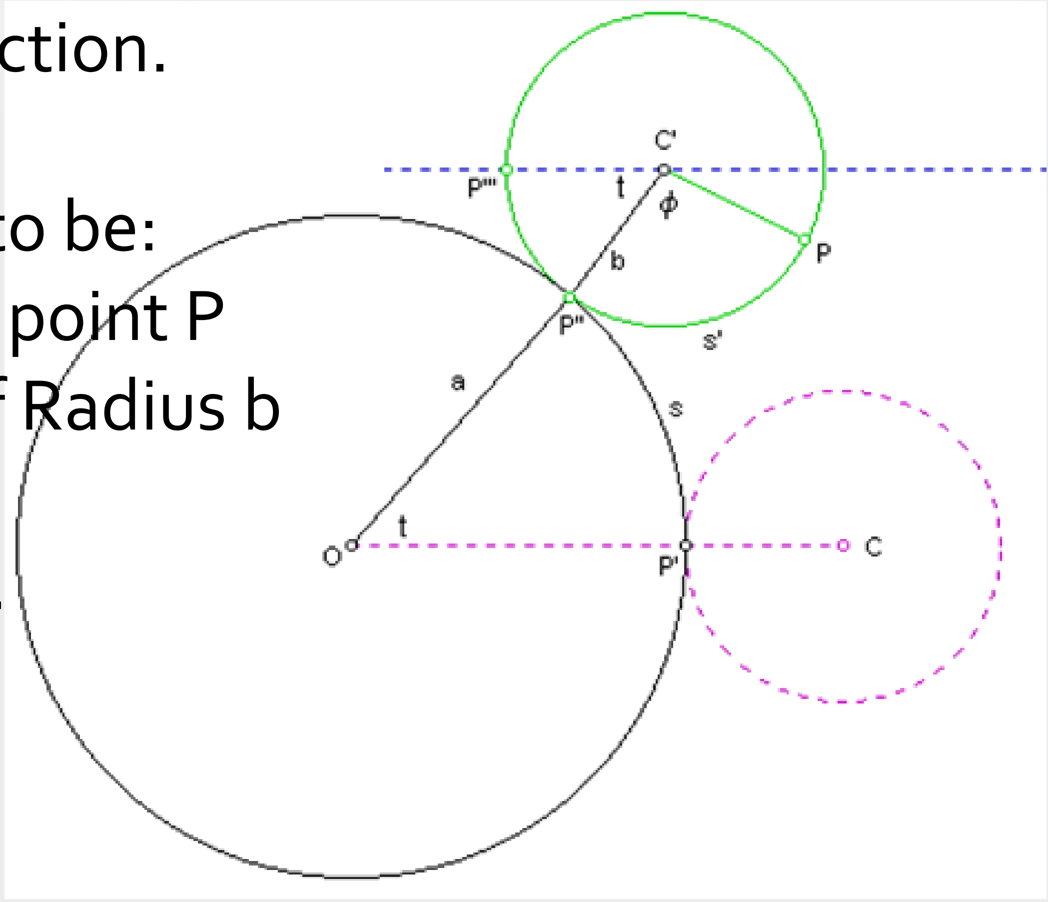
Epicycloid



Epicycloid

The epicycloid was named by Ole Roemer in 1674.
He discovered that cog-wheels with epicycloidal teeth turned with minimum friction.

An epicycloid is defined to be:
The path traced out by a point P on the edge of a Circle of Radius b rolling on the outside of a fixed Circle of Radius a.



Epicycloid

Point **P** moves about circle of radius **b**:

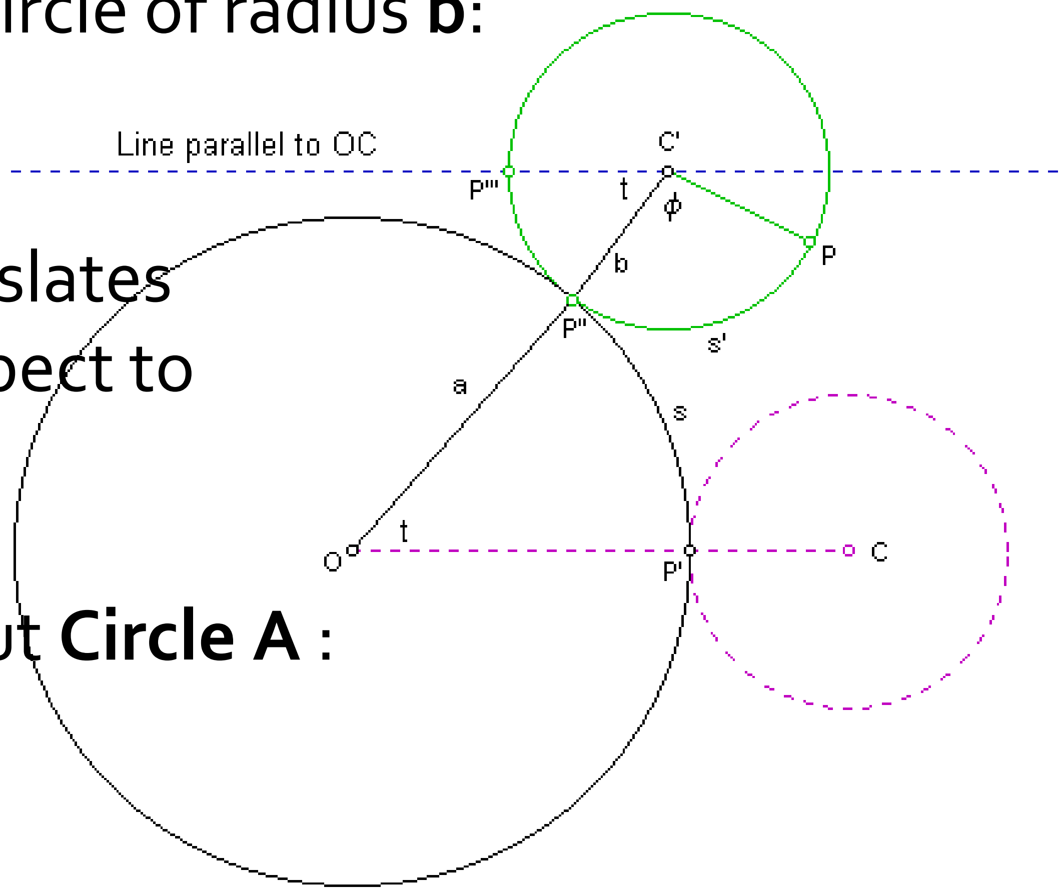
$\mathbf{x = b \cos \Phi}$

$\mathbf{y = b \sin \Phi}$

The axis **P''C'** also translates at the angle **t** with respect to its original place **P'C** as the **Circle B** travels counterclockwise about **Circle A** :

$\mathbf{x = b \cos (\Phi + t)}$

$\mathbf{y = b \sin (\Phi + t)}$



Epicycloid

The arc length traced by point **P** is equal to the distance **Circle B** has traveled about **Circle A**:

s = a * t

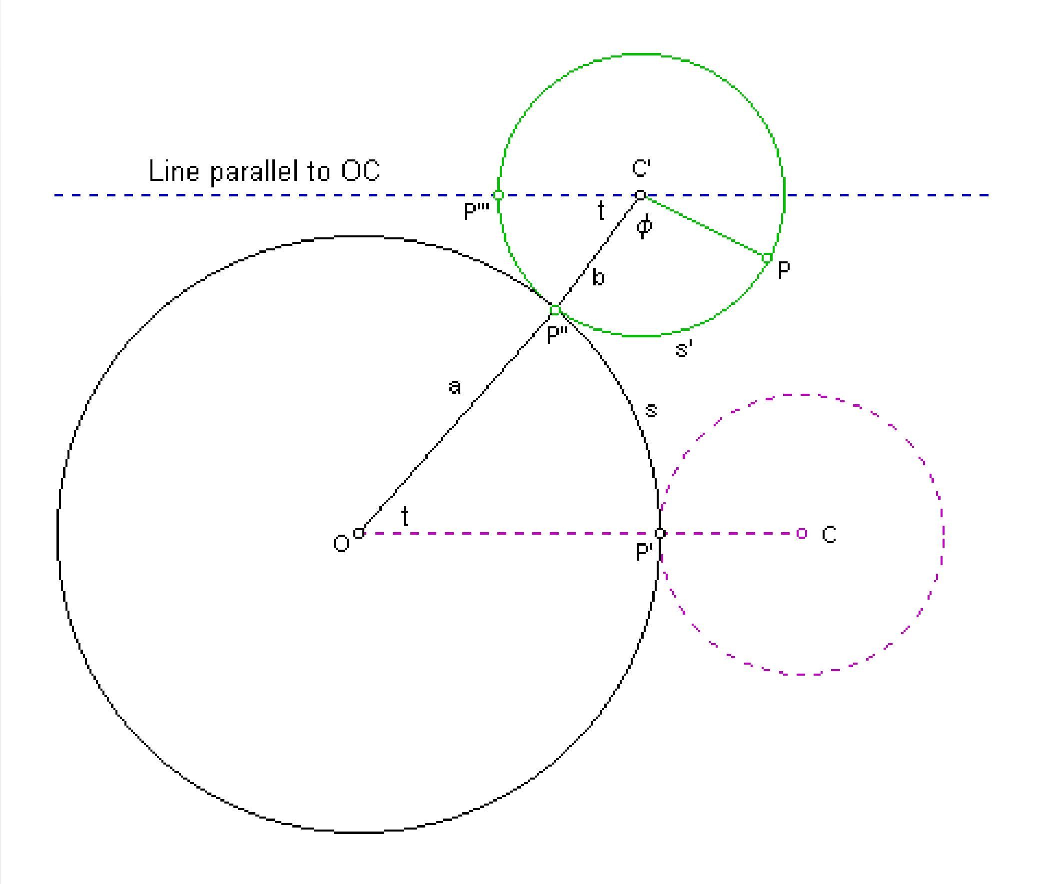
s' = b * Φ

and,

s = s'

a * t = b * Φ

f = (a/b)t



Epicycloid

so, replacing **f** in the equations:

$$x = b \cos [(a/b)t + t] = b \cos \{[(a + b)/b]\, t\}$$

$$y = b \sin [(a/b)t + t] = b \sin \{[(a + b)/b]\, t\}$$

In addition the center of **Circle B** (the point, **C**) moves counterclockwise with respect to **O** by angle **t,** with a radius of **(a + b)**.

This is added to the above translation:

$$x = (a + b) \cos t + b \cos \{[(a + b)/b]\, t\}$$

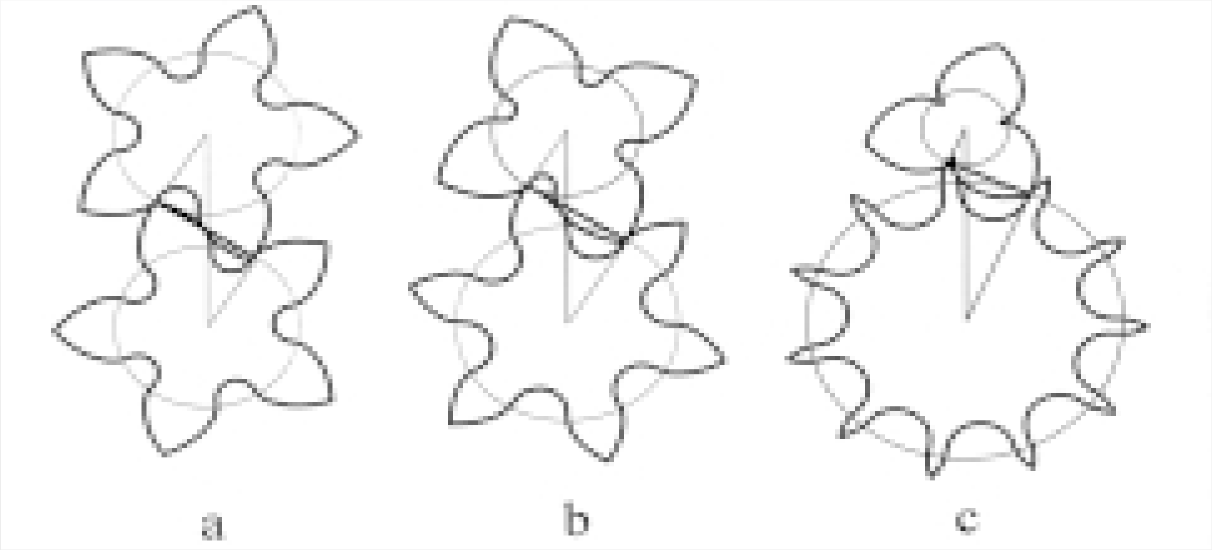
$$y = (a + b) \sin t + b \sin \{[(a + b)/b]\, t\}$$

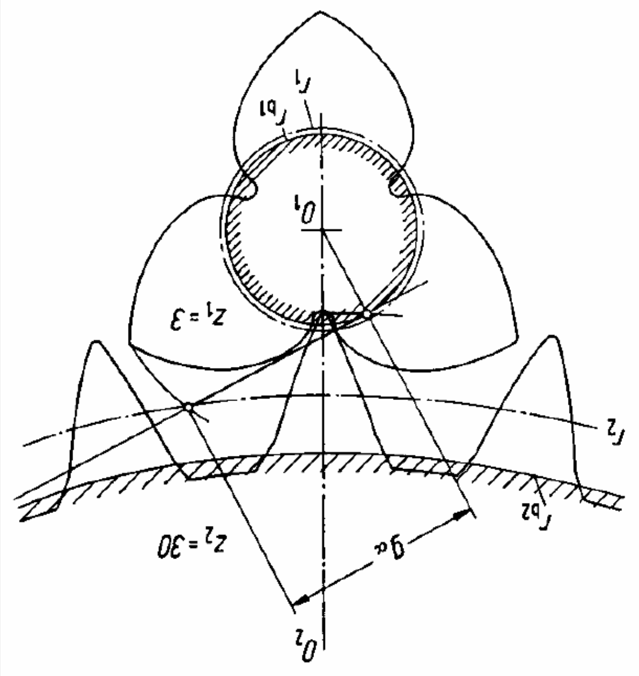
Minimum diş sayısı ile dişli çifti

a) $z_1 = 5,\quad z_2 = 5\quad \alpha_w = 33.1\quad \varepsilon_\alpha = 1.04$

b) $z_1 = 4\quad z_2 = 6\quad \alpha_w = 32.6\quad \varepsilon_\alpha = 1.02$

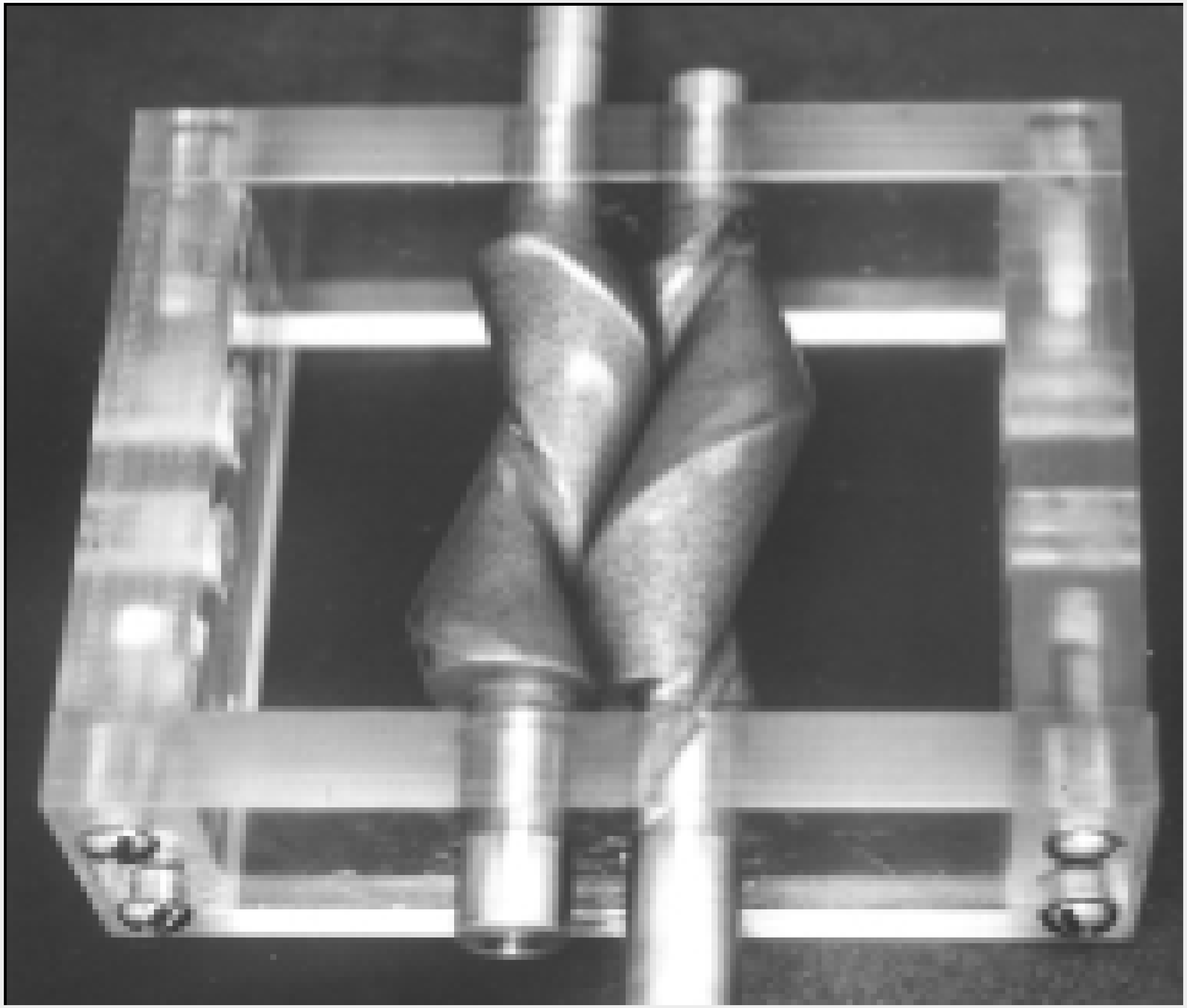
c) $z_1 = 3\quad z_2 = 11\quad \alpha_w = 24.3\quad \varepsilon_\alpha = 1.01$





Bir dişe sahip helisel dişli

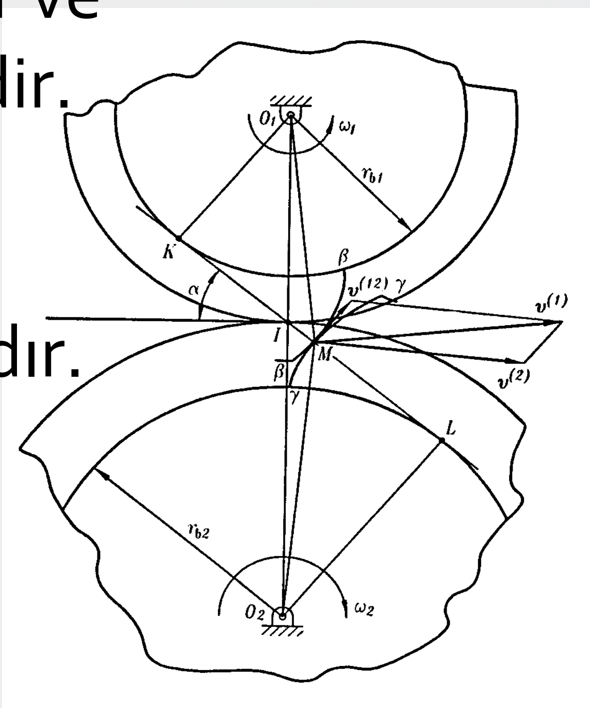
(Helical gears with one tooth)



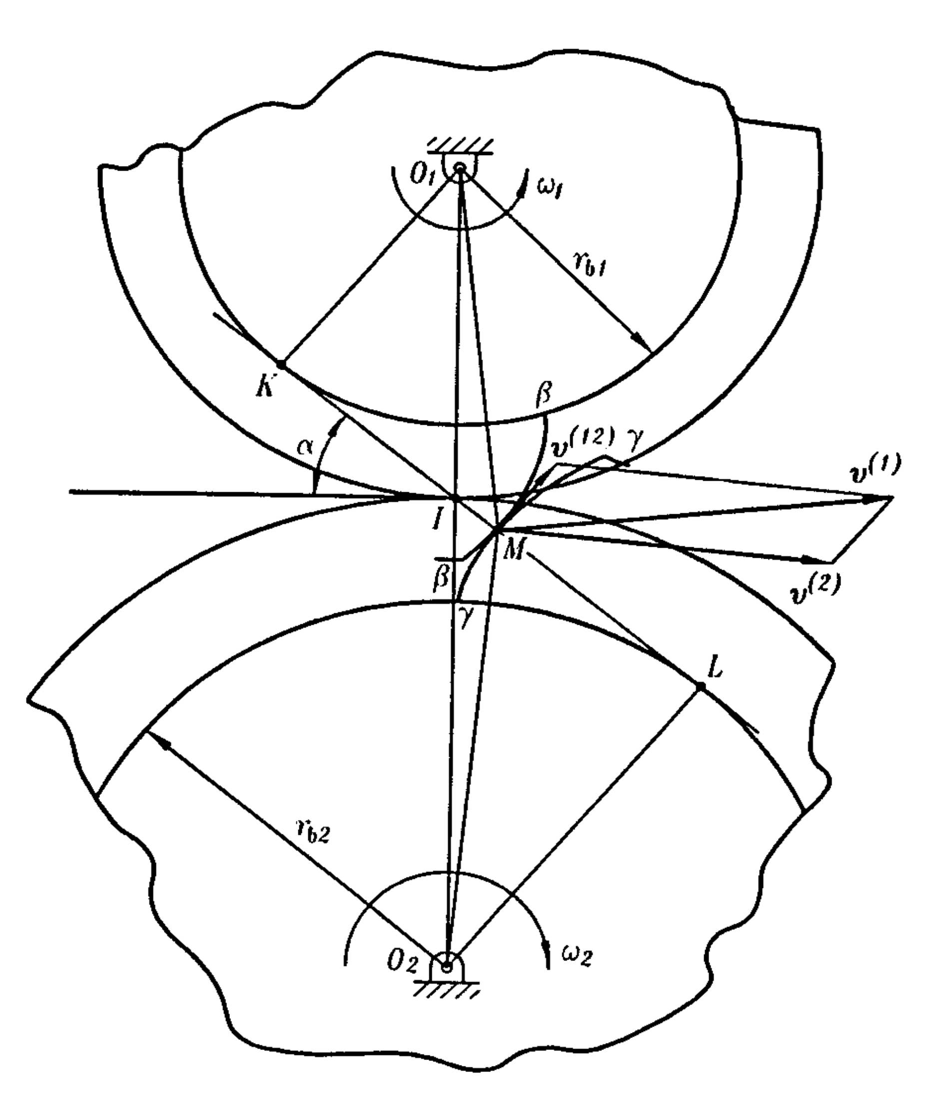
Dişli Ana Kanunu

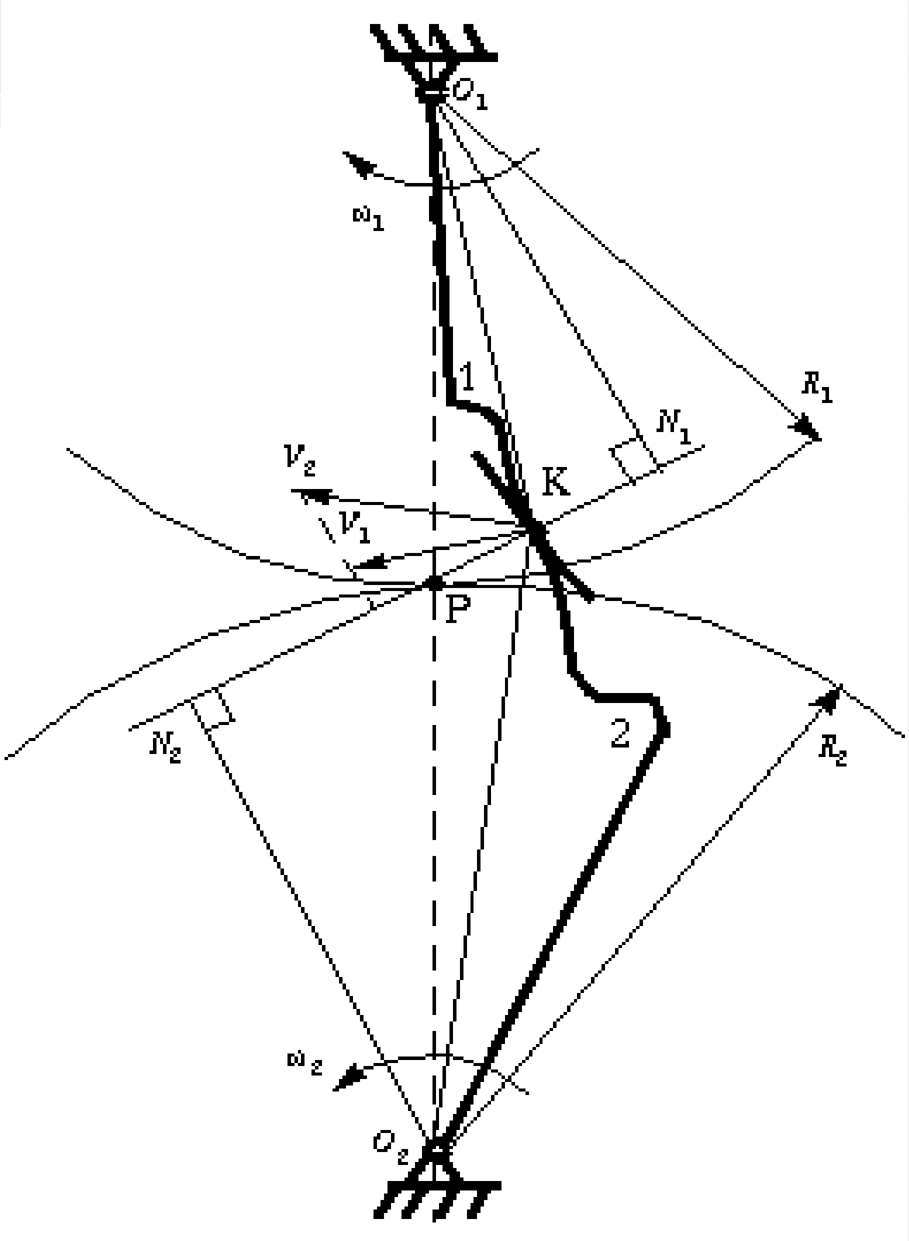
İki yuvarlanma dairesine (sürtünmeli çark mekanizması) kinematik olarak eşdeğer ve birbirleri ile eş çalışan iki dişin yan yüzeyi (diş refilleri) bir kayma-yuvarlanma kinematik çifti teşkil ederler ve izafi hareketleri bir ani dönme hareketidir.

Bu harekette ani dönme merkezi yuvarlanma dairelerinin değme noktasıdır.

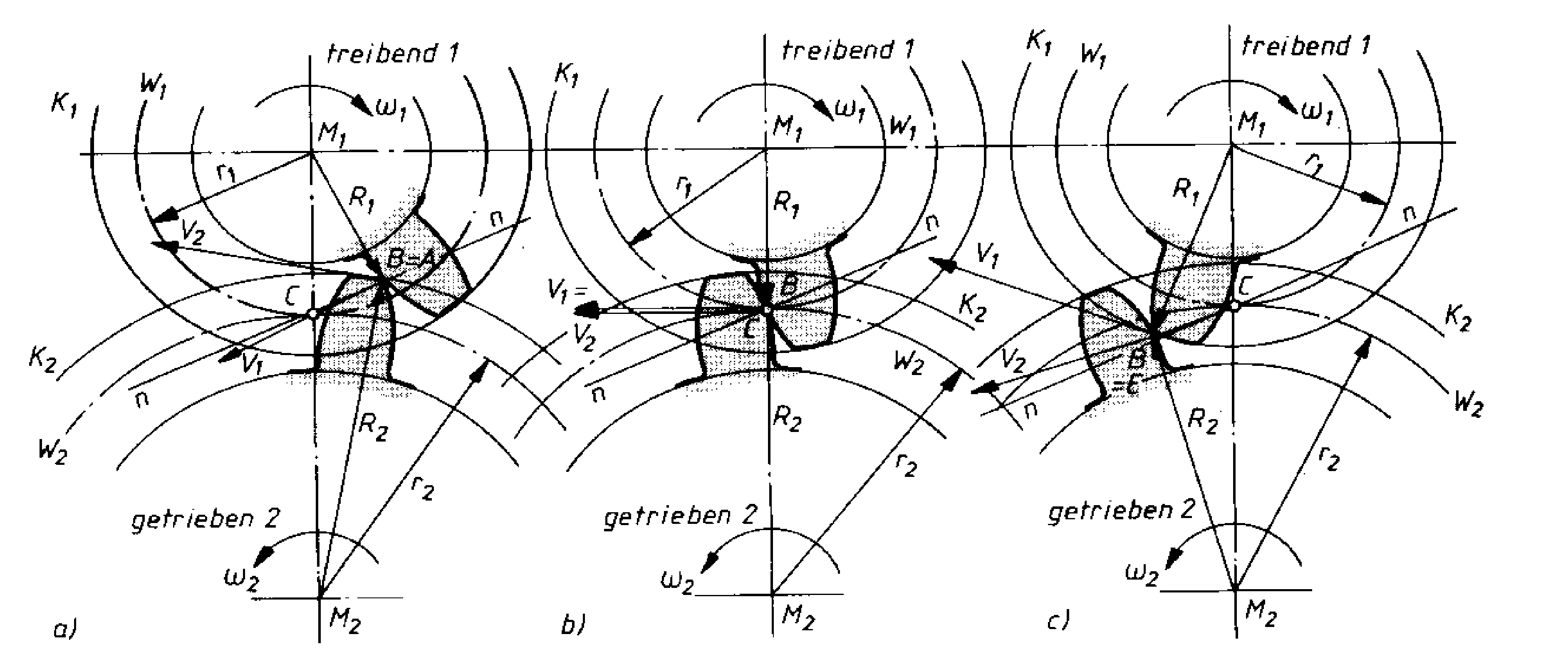


Dişli Ana Kanunu

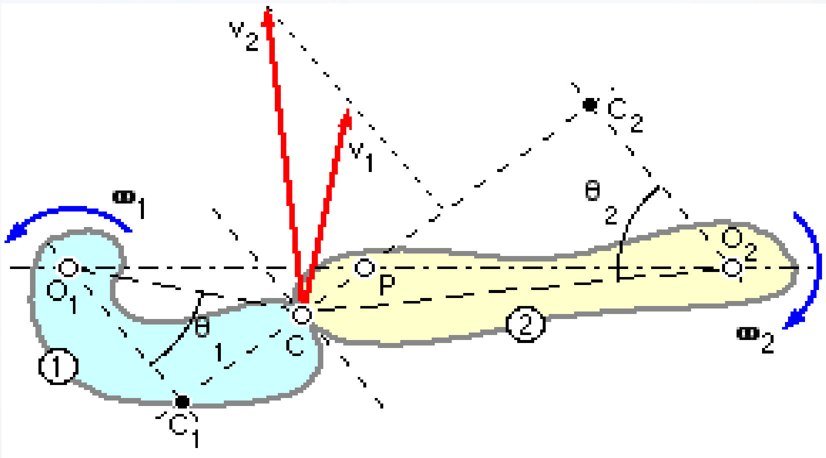


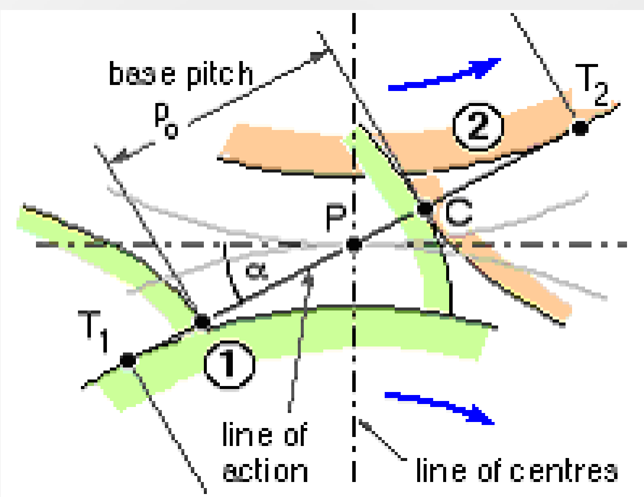


Dişli Ana Kanunu

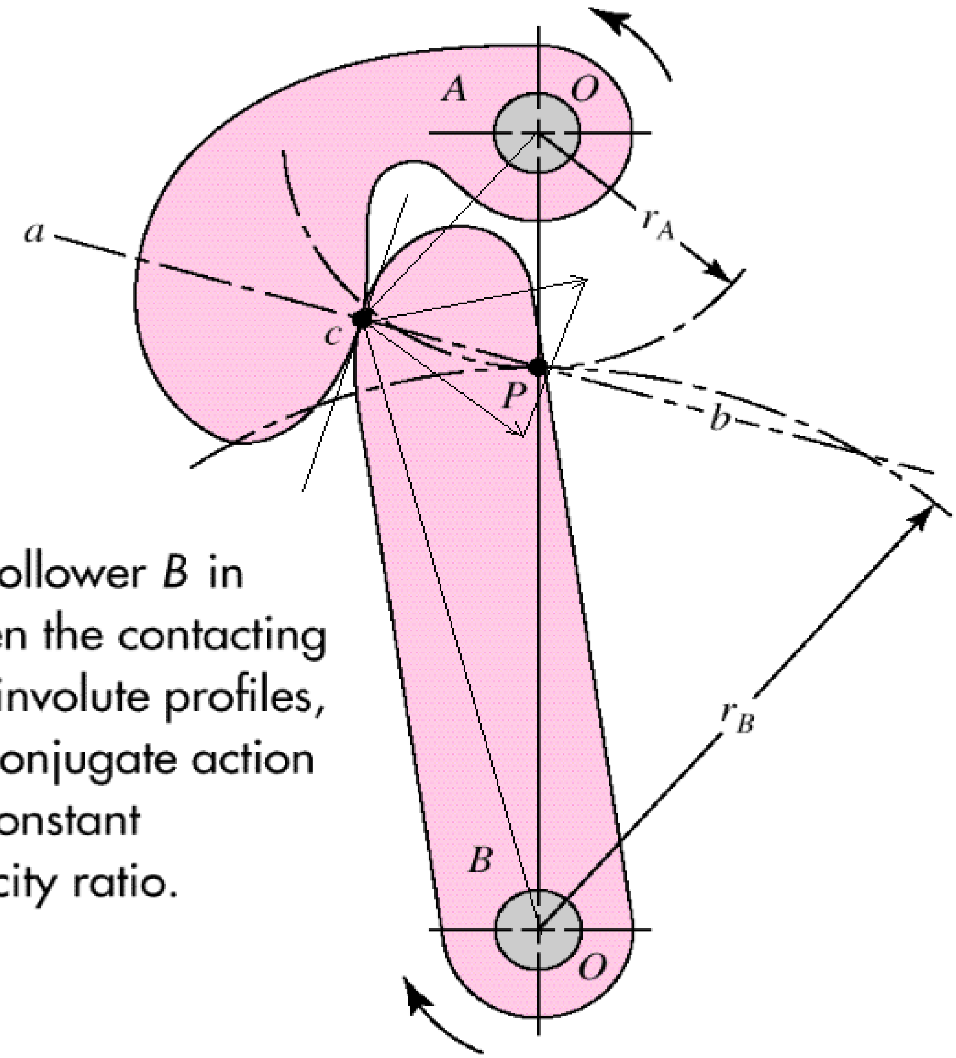


Dişli çifti mekanizması

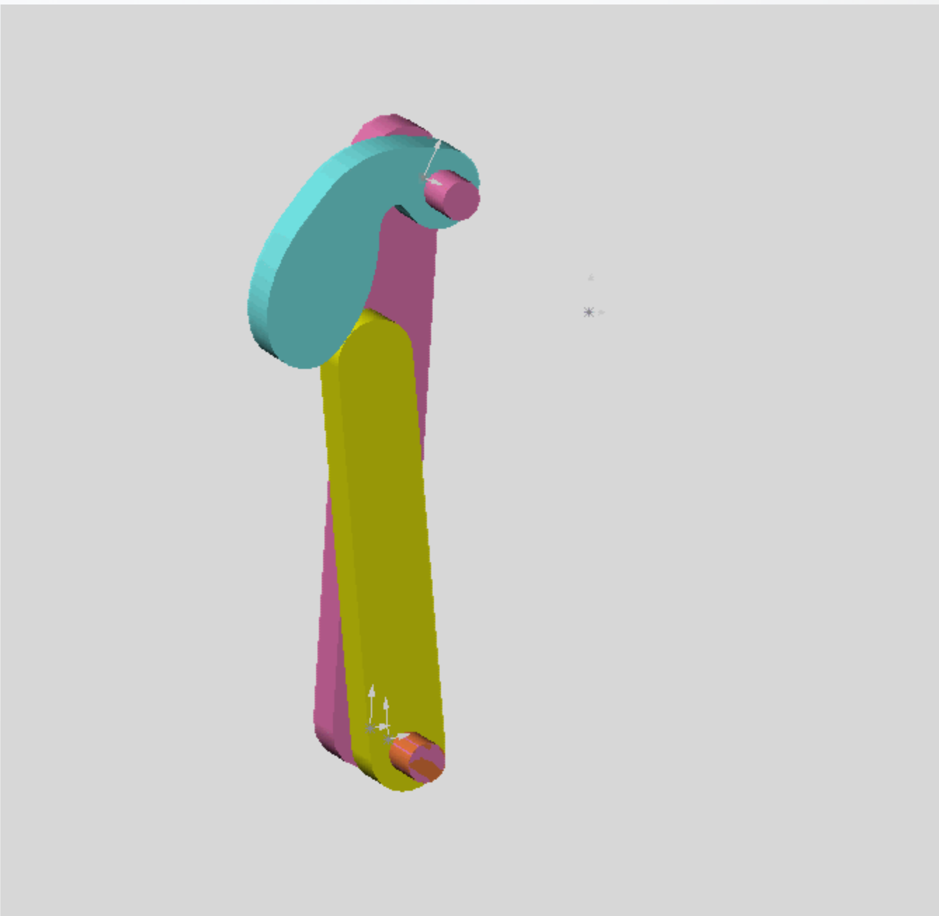


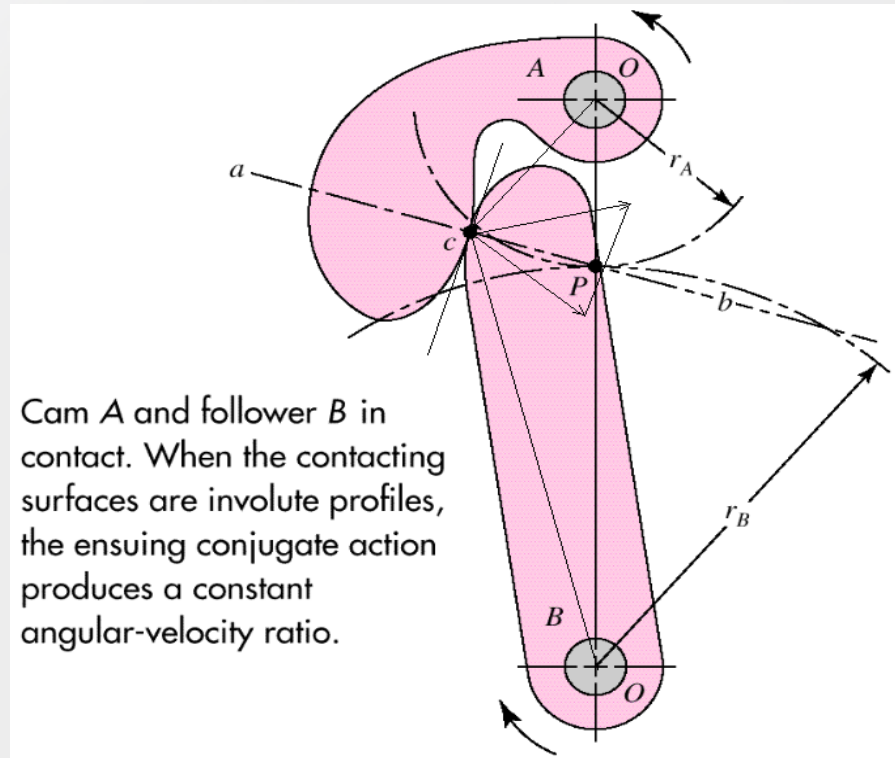


Cam *A* and follower *B* in contact. When the contacting surfaces are involute profiles, the ensuing conjugate action produces a constant angular-velocity ratio.



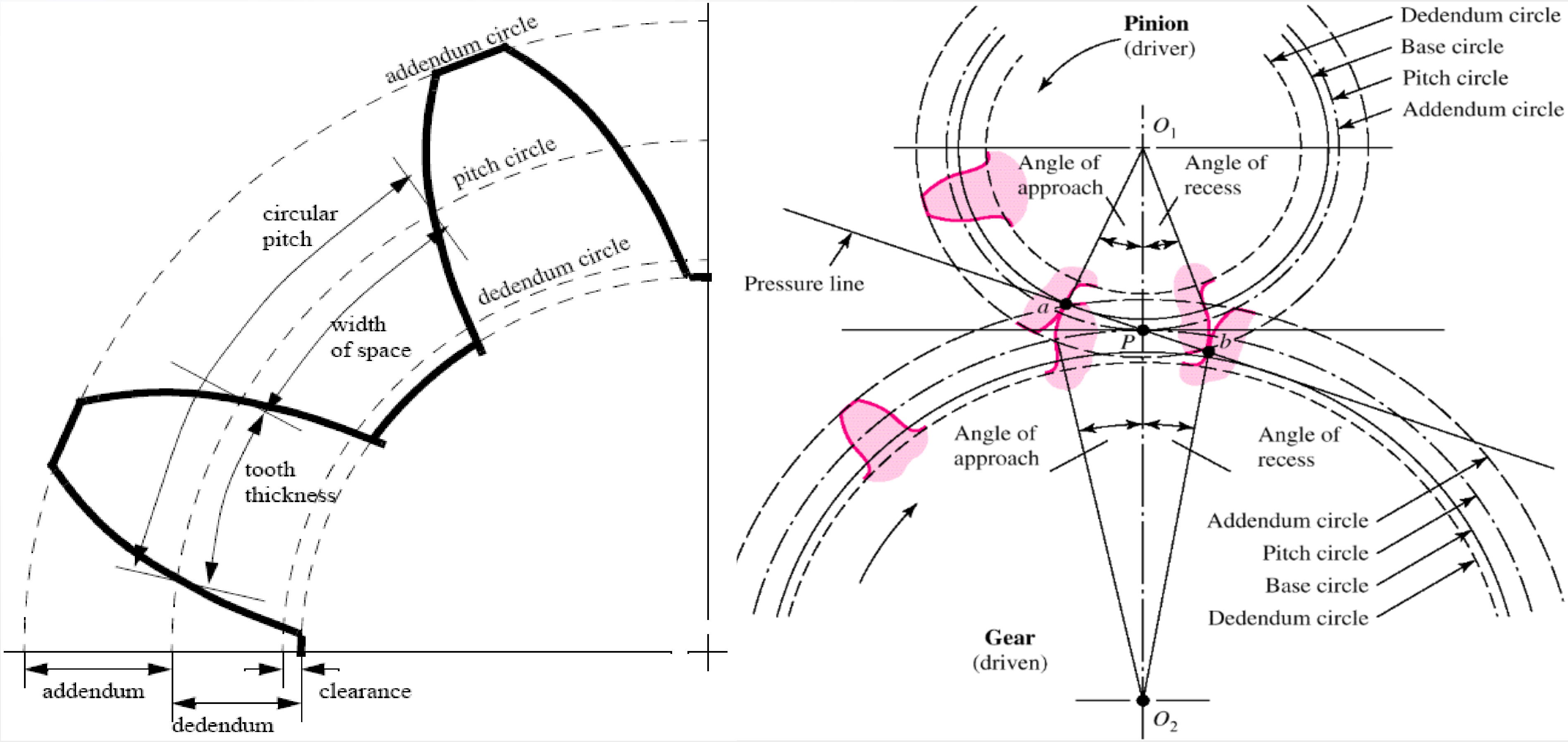
Dişli çifti mekanizması



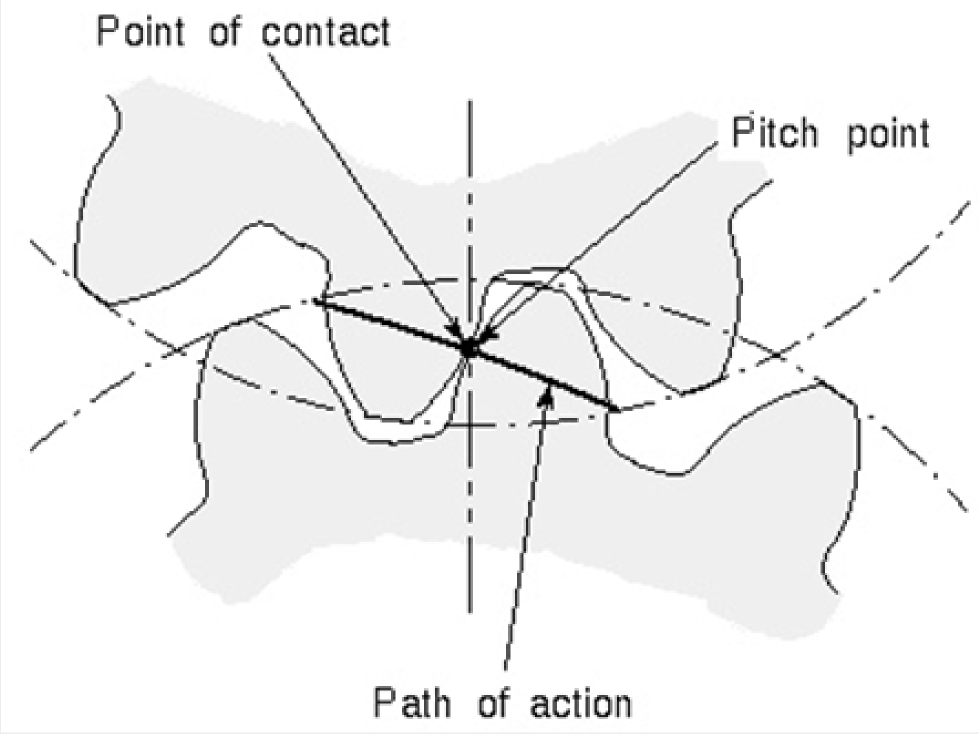


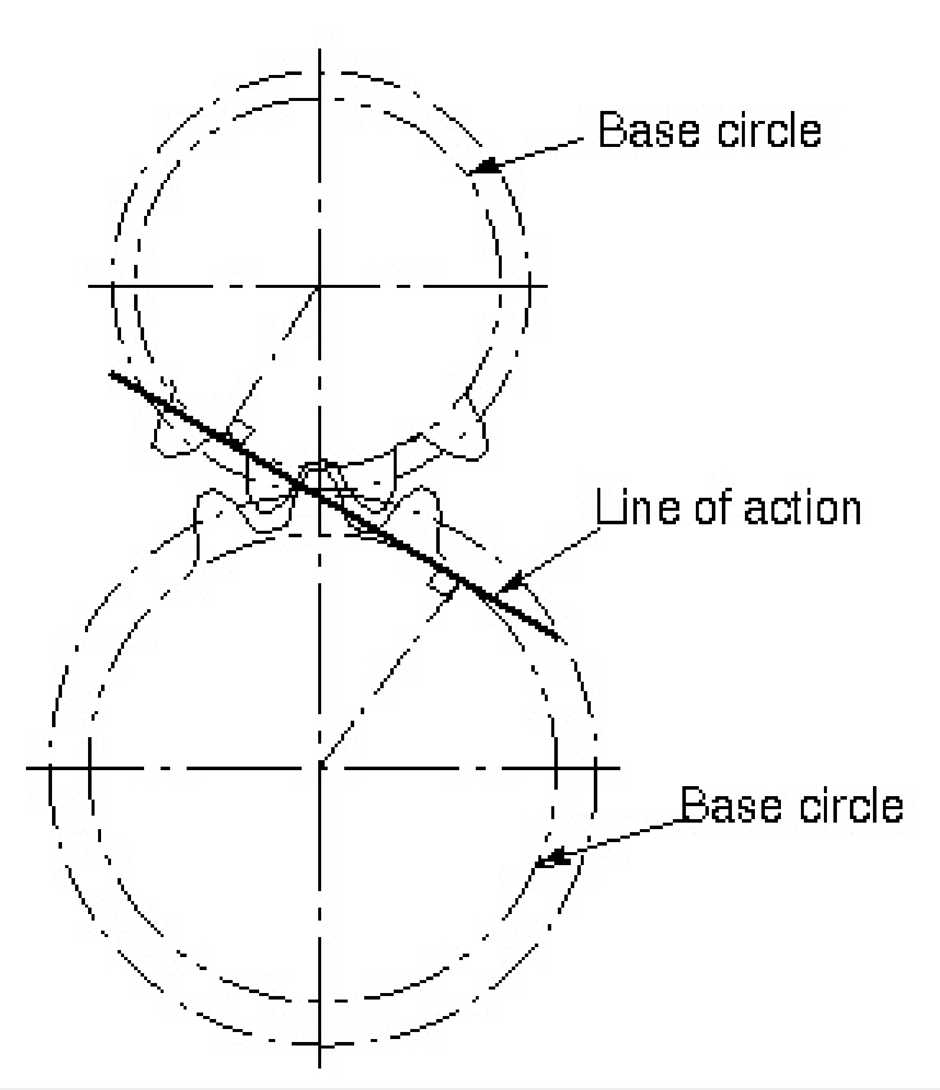
Cam *A* and follower *B* in contact. When the contacting surfaces are involute profiles, the ensuing conjugate action produces a constant angular-velocity ratio.

Dişli çifti mekanizması

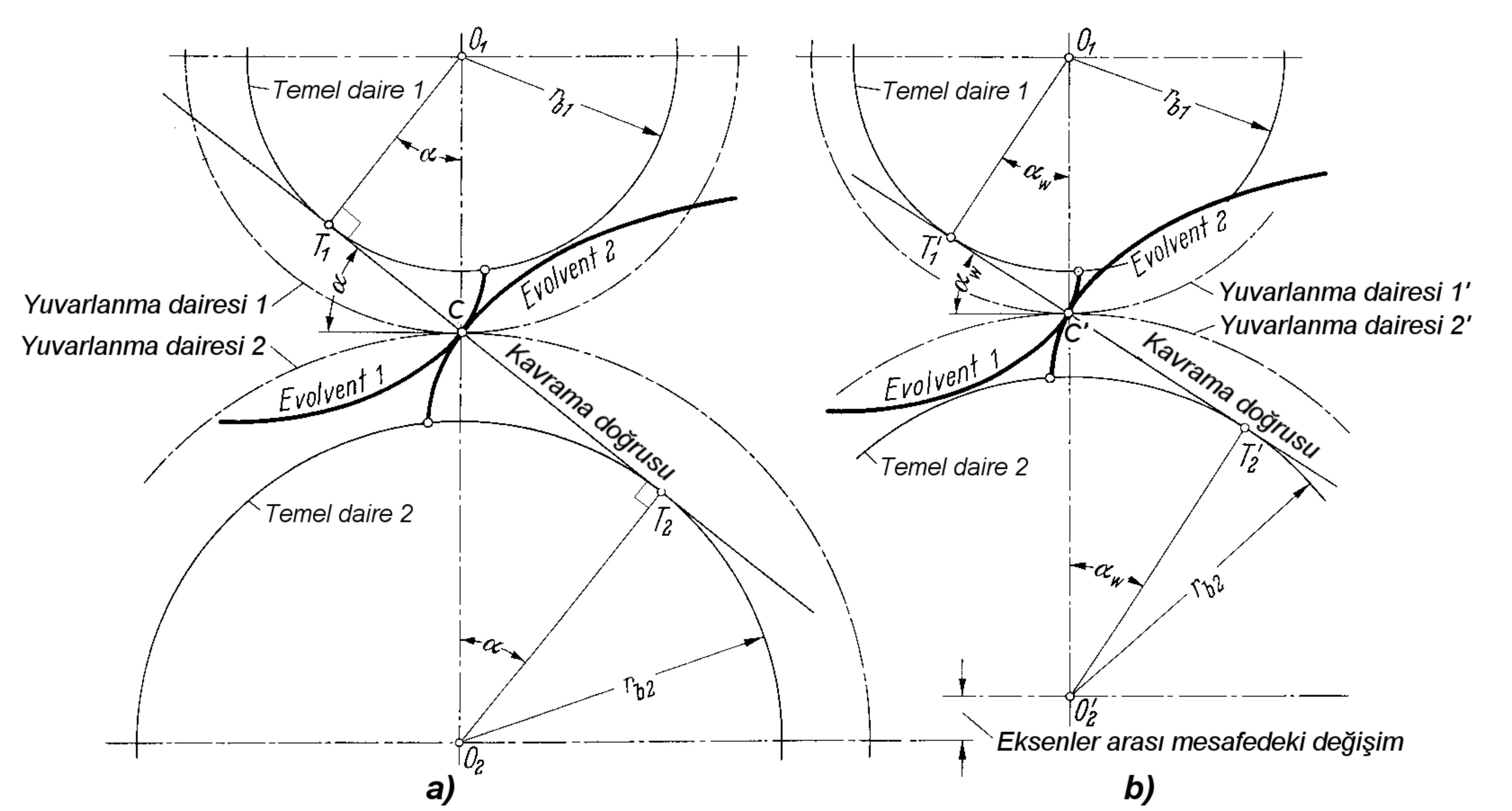


Dişli çifti mekanizması

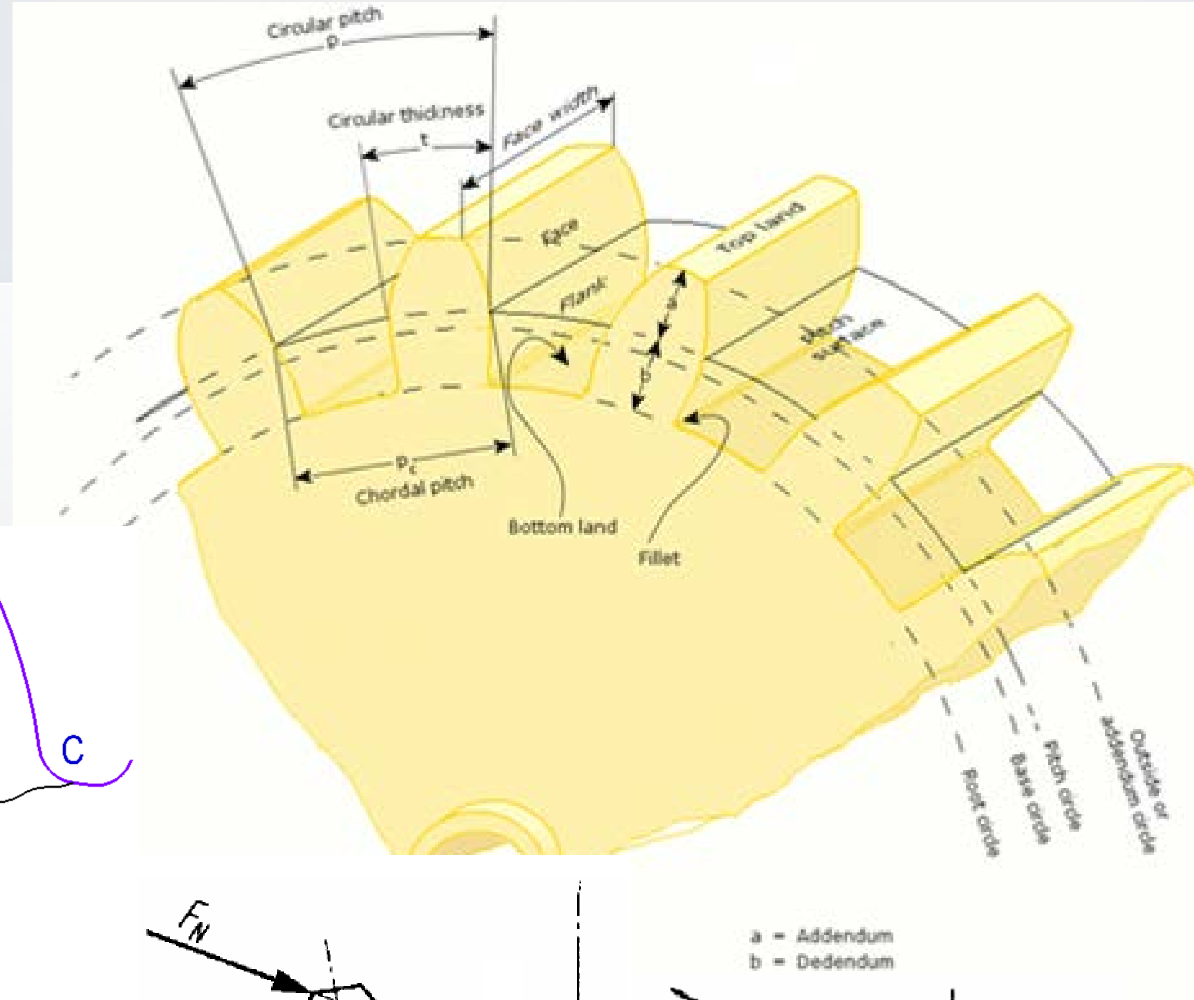


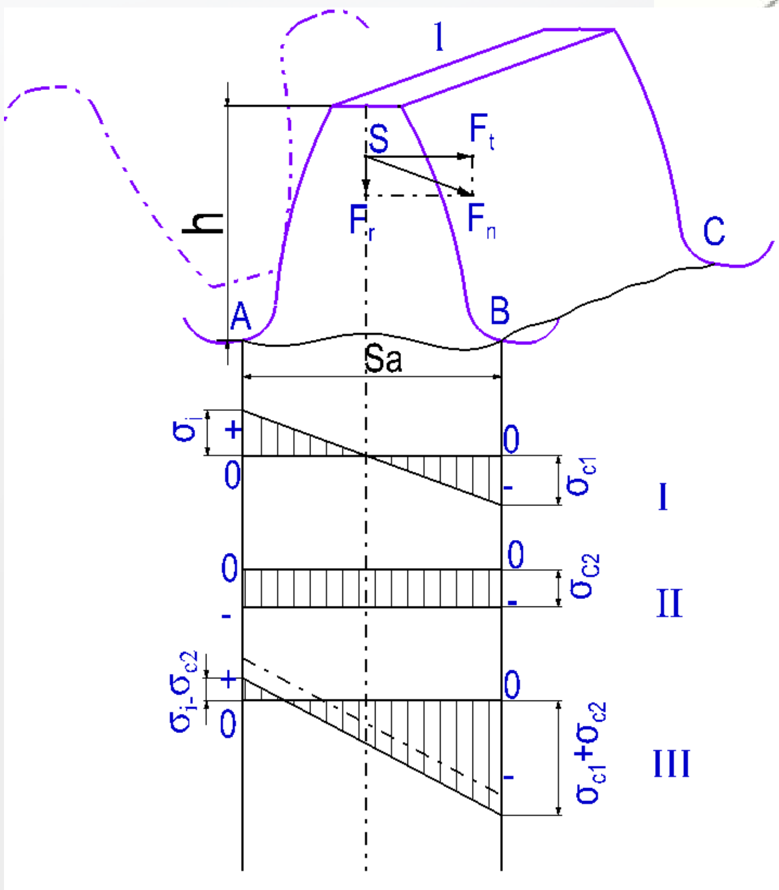


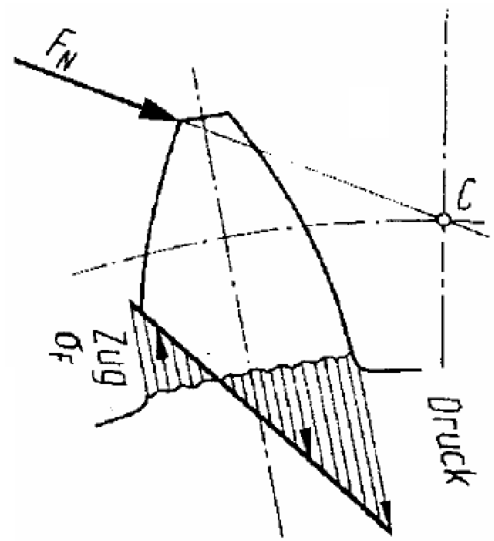
Evolvent profilli dişlilerde eksen aralığındaki bir değişme eş çalışabilme özelliğini bozmaz

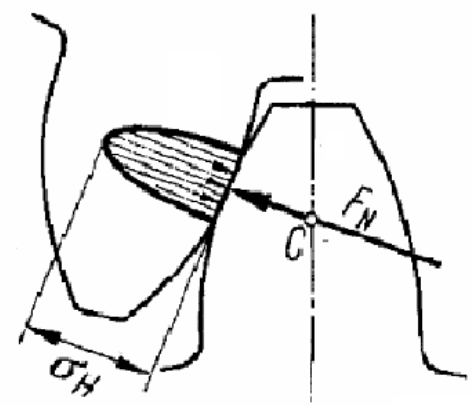


Dişli boyutları

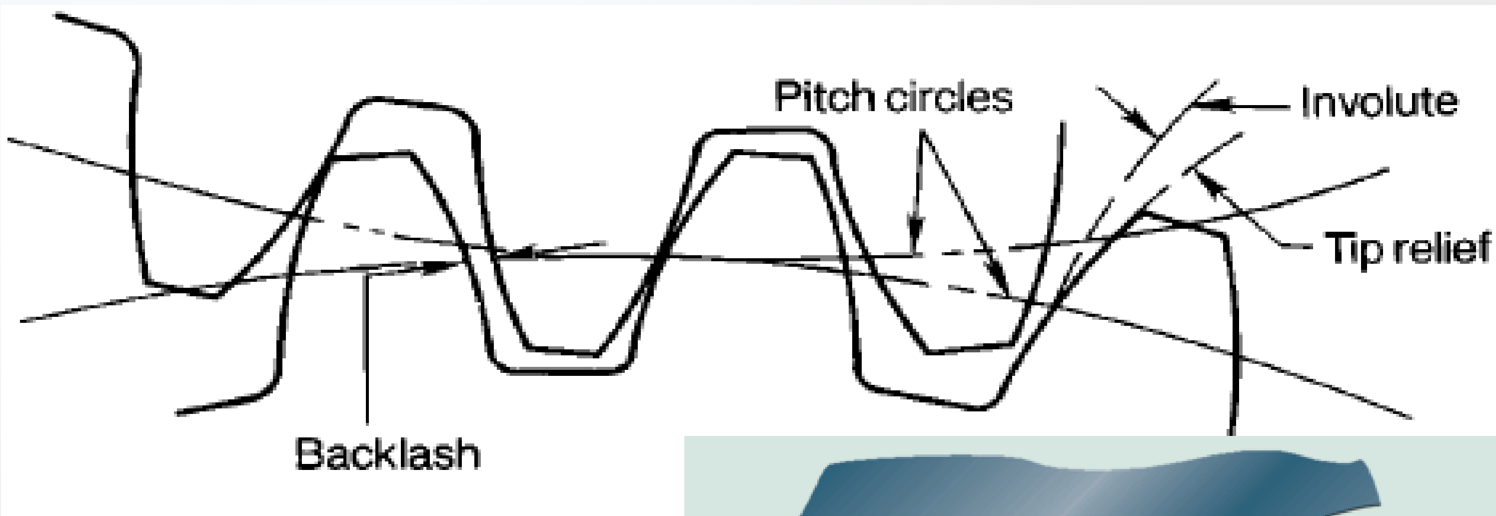




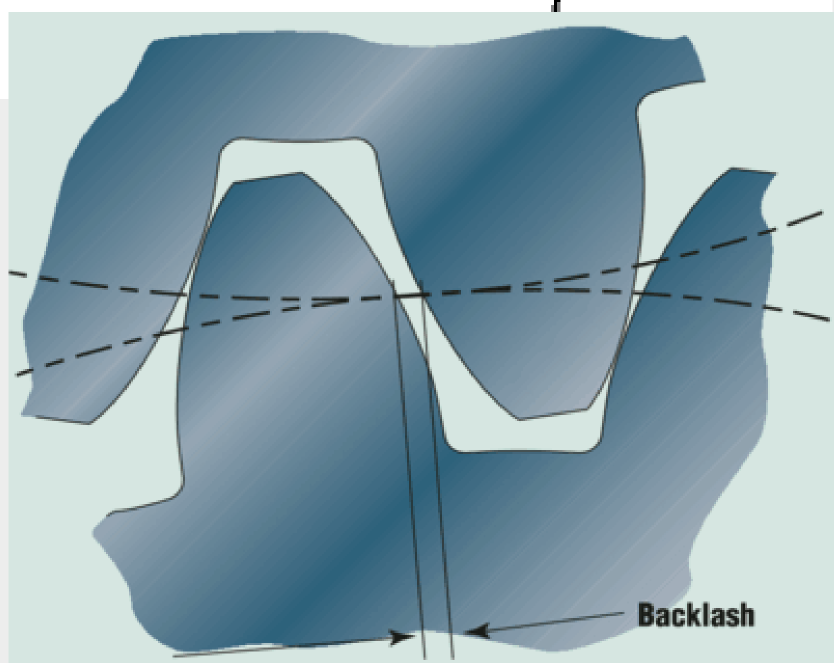




Dişli çarklardaki kontrol noktaları

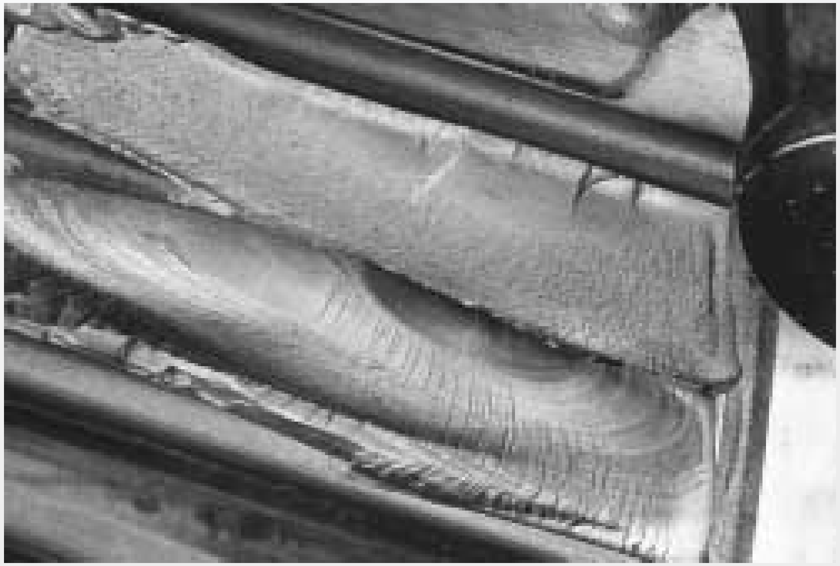


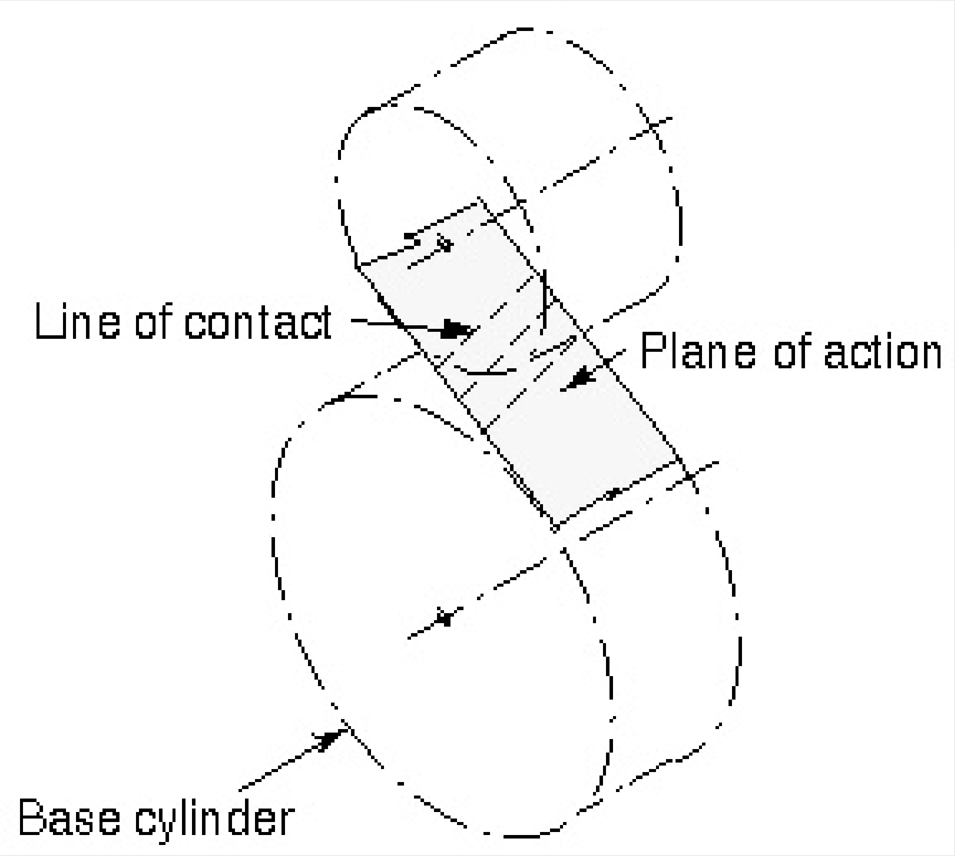
Dişler arası boşluk



Dişli çarklardaki kontrol noktaları

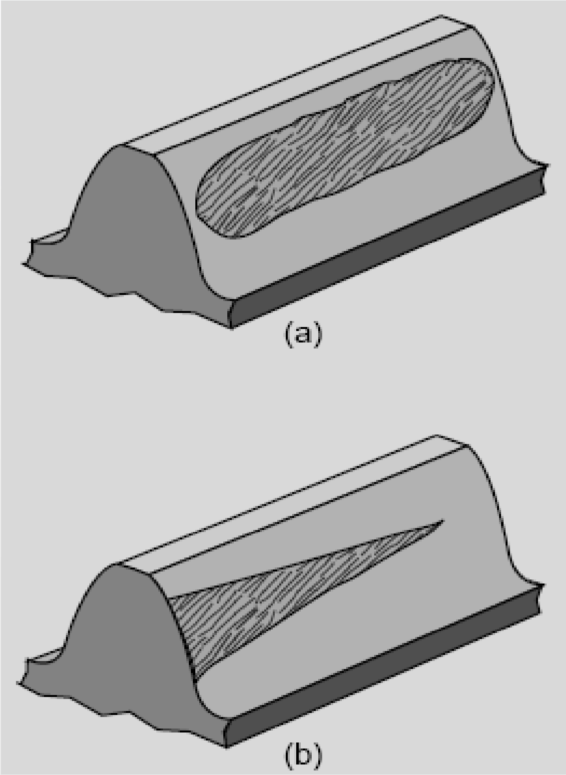
Eğilme yorulması





Dişli çarklardaki kontrol noktaları

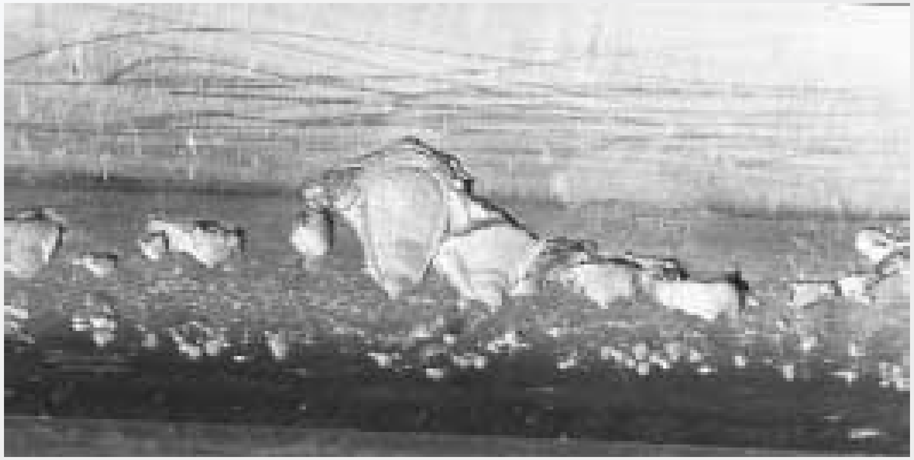
a) Düzgün b) Düzgün olmayan yükleme



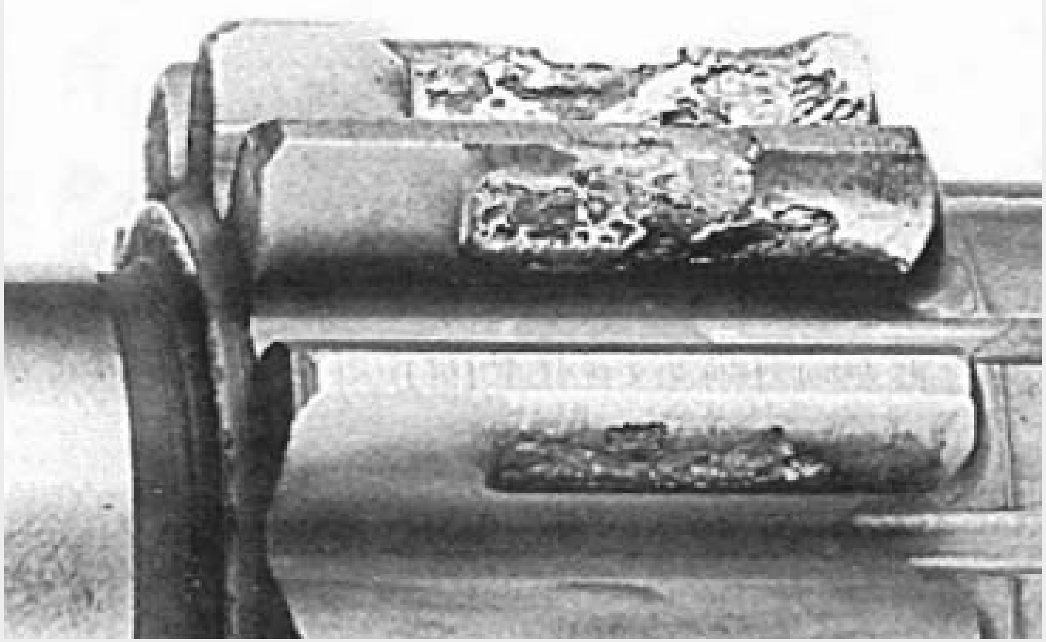
Diş dibi yuvarlatmaları neticesi yorulma

Dişli çarklardaki kontrol noktaları

Ezilme yorulması

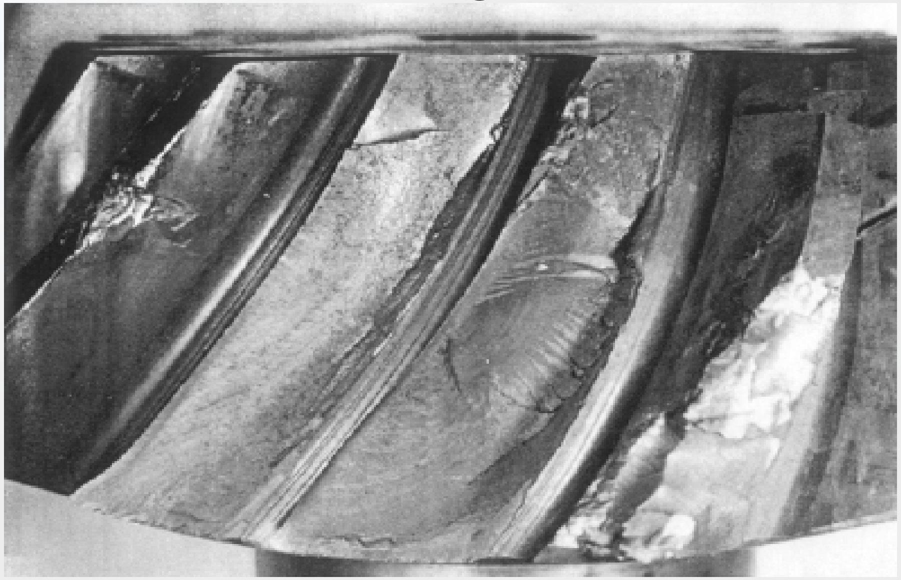


Ezilme yorulması

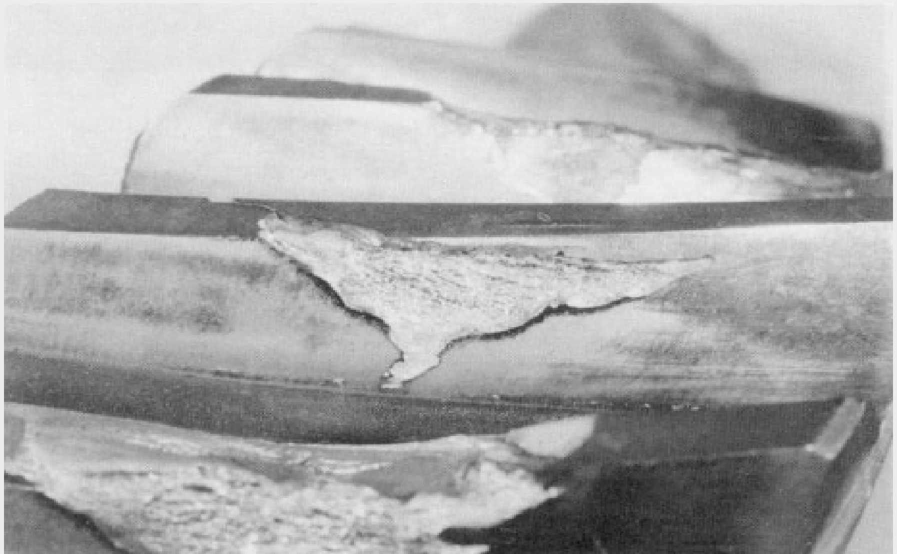


Dişli çarklardaki kırılma ve aşınma

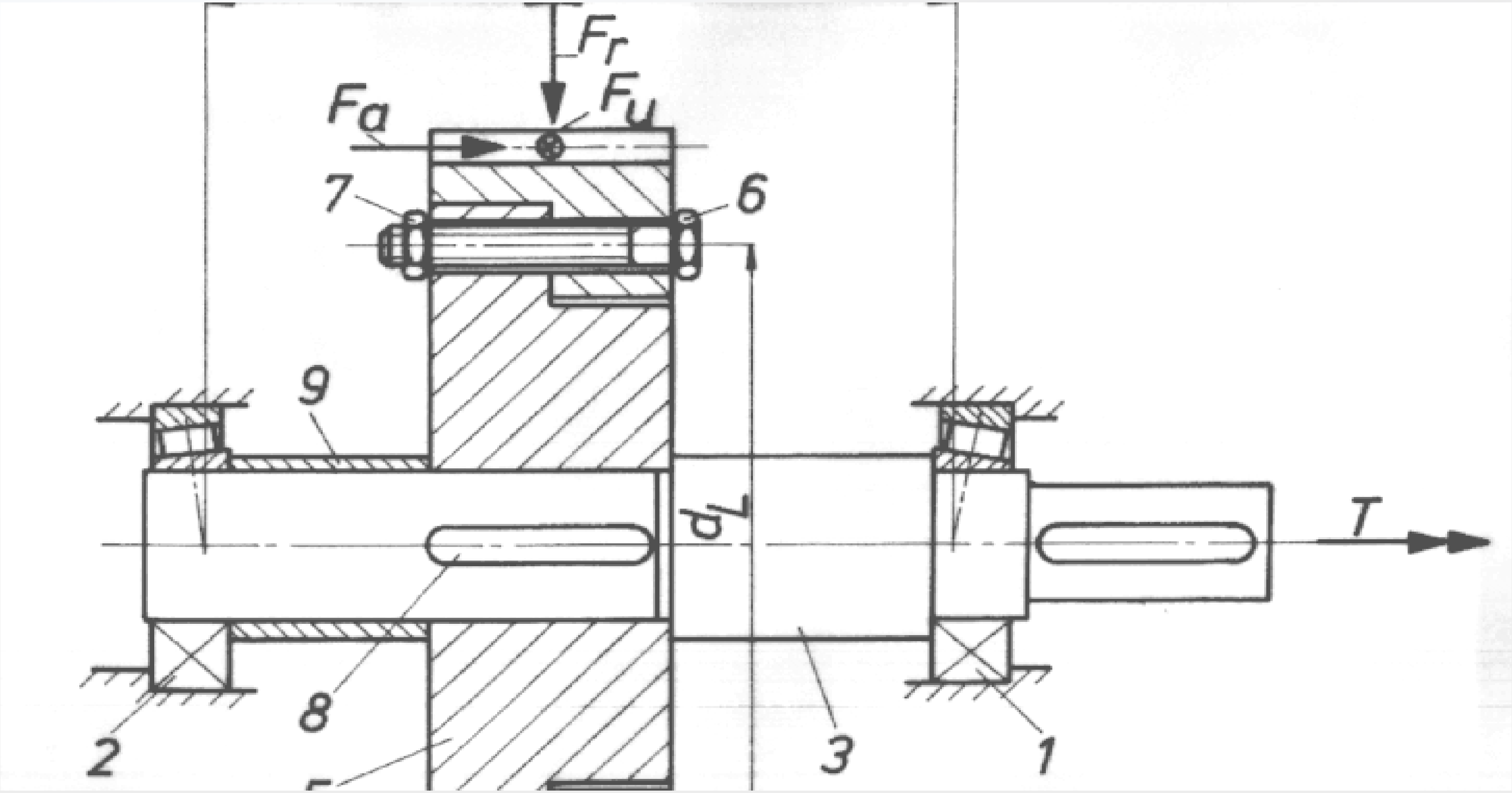
Eğilme yorulması



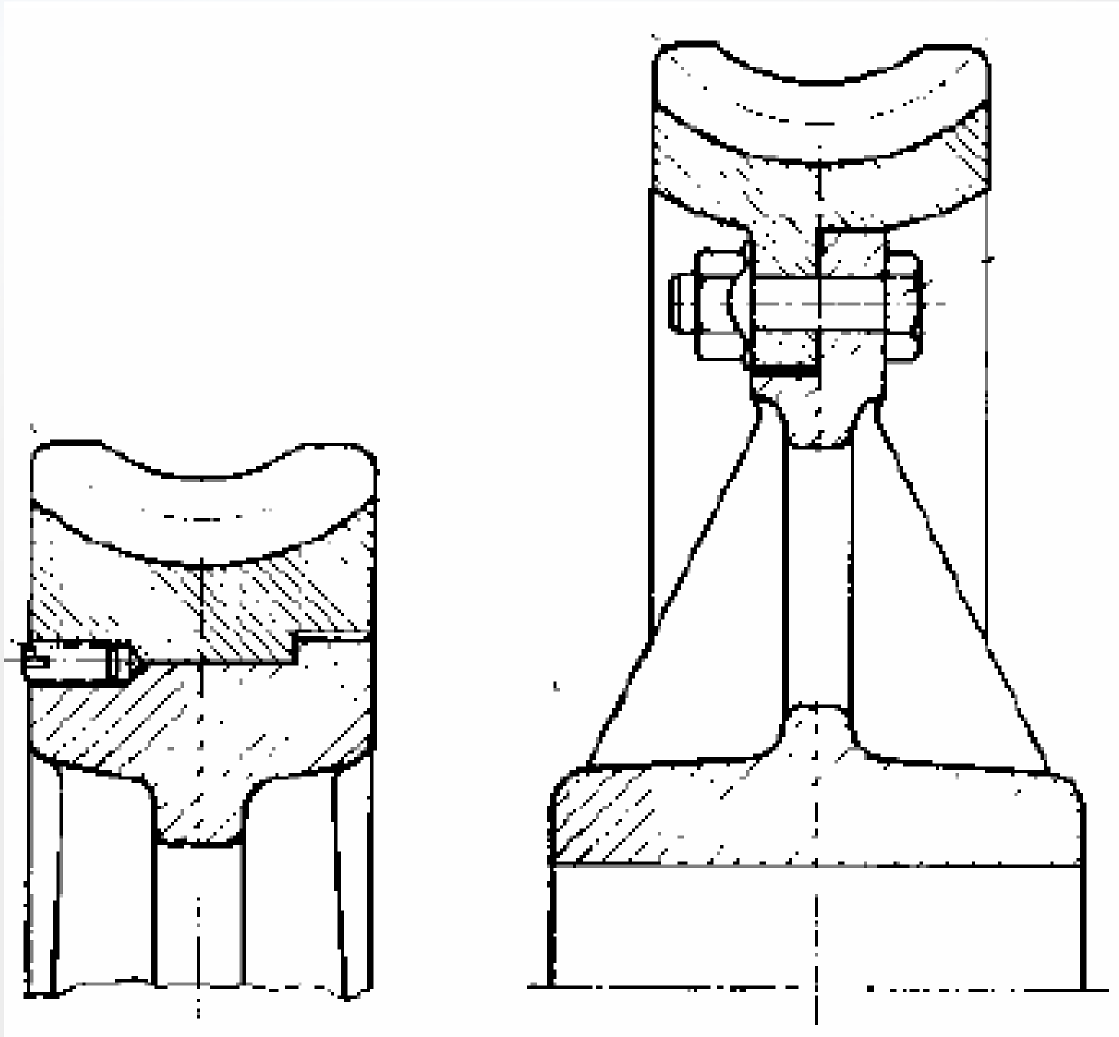
Yüzey yorulması sonucu aşınma



Dişleri değişebilir dişli çark



Çark-Göbek Bağlantıları



INTRODUCTION

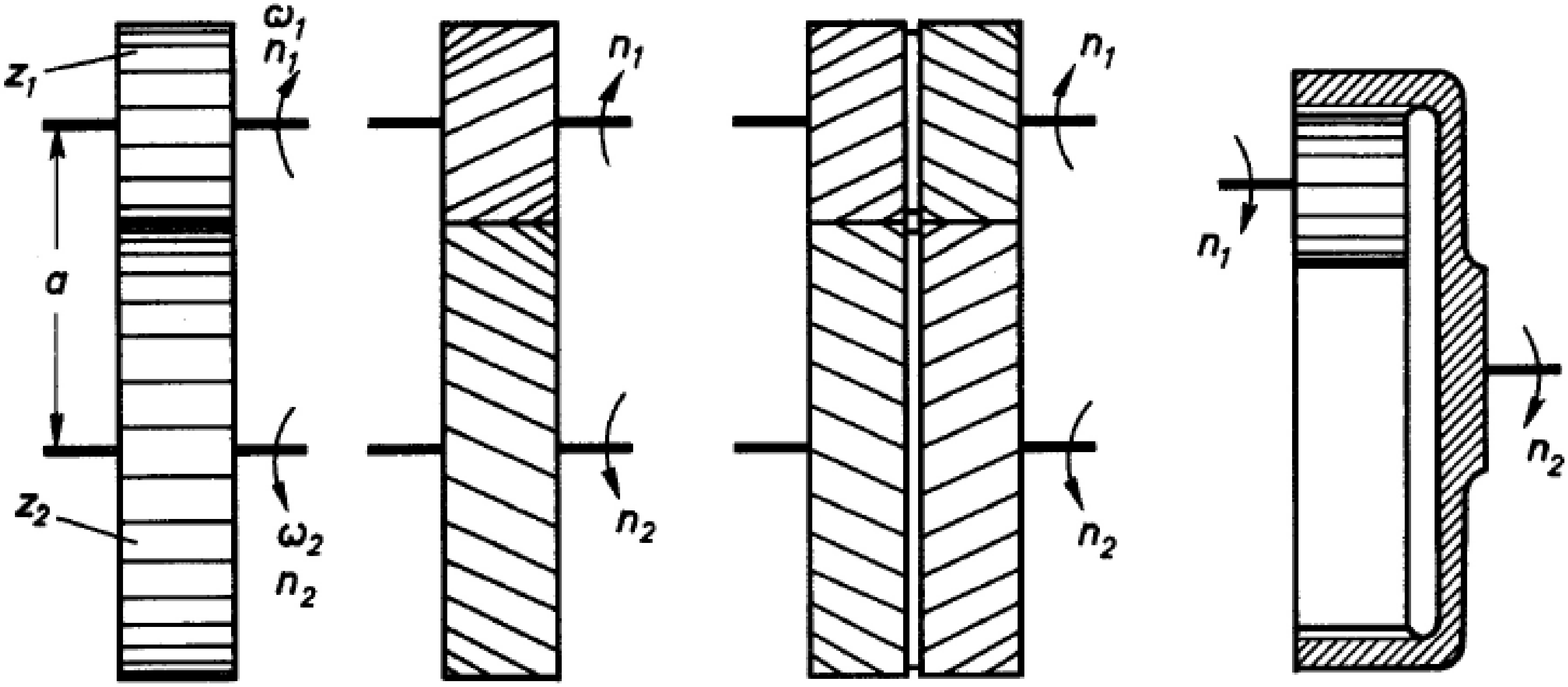
Gears are a means of changing the *rate of rotation* of a machinery shaft. This means the simultaneous *changing of torque*. They can also change the *direction of the axis of rotation* and can change *rotary motion to linear motion*.

These changes are to be made only with the power loss; the output power of a gear is less as the input one. The ratio of these quantities is the efficiency of the gear.

Gears are of several categories, and can be combined in a multitude of ways, some of which are illustrated in the following figures.

BASIC GEAR TYPES

Gears with parallel axes



Spur gears (external contact)

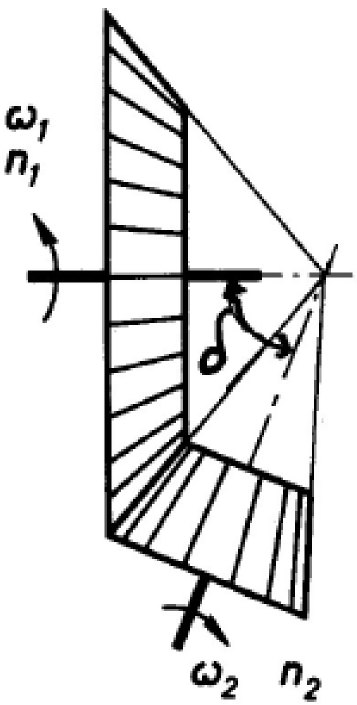
Helical gears

Double-helical gears *(Herringbone gears)*

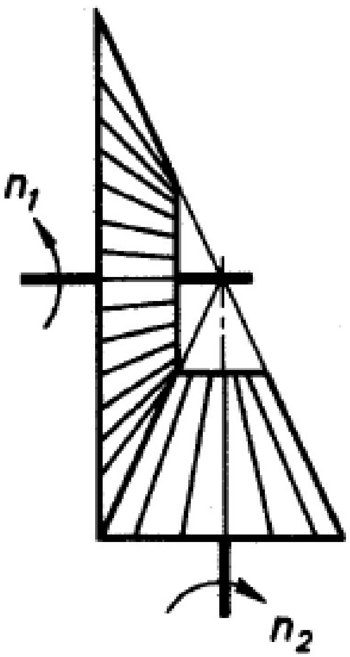
Spur gears (internal contact)

BASIC GEAR TYPES

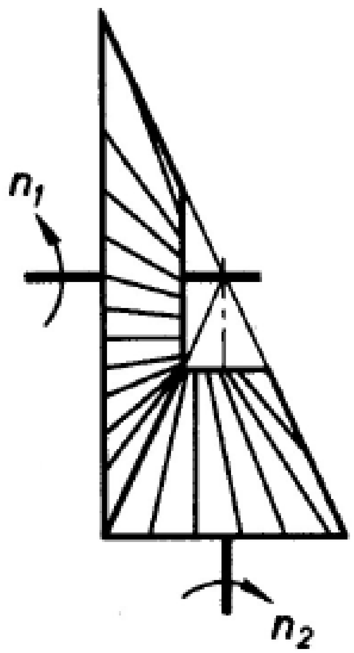
Gears with intersecting axes



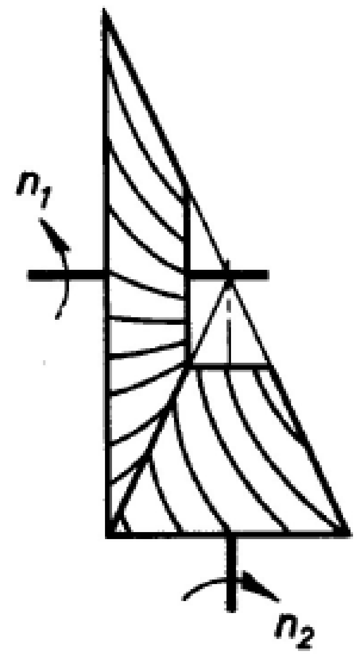
Straight bevel gears (axes at angle δ)



Straight bevel gears (perpendicular axes)



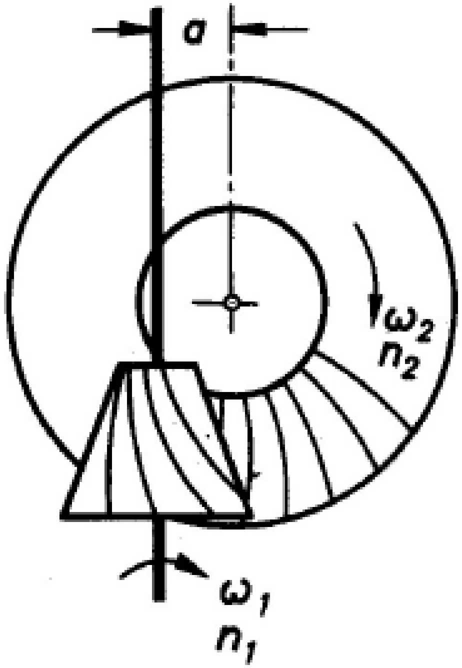
Helical bevel gears

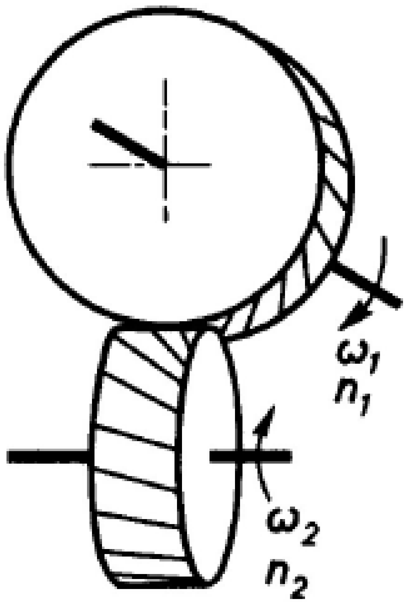


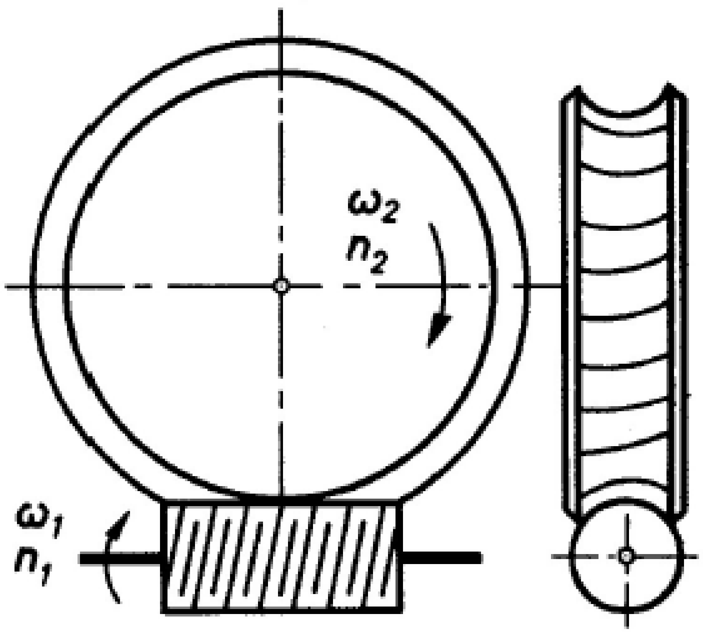
Spiral bevel gears (also involute and circular ones)

BASIC GEAR TYPES

Gears with non-intersecting axes







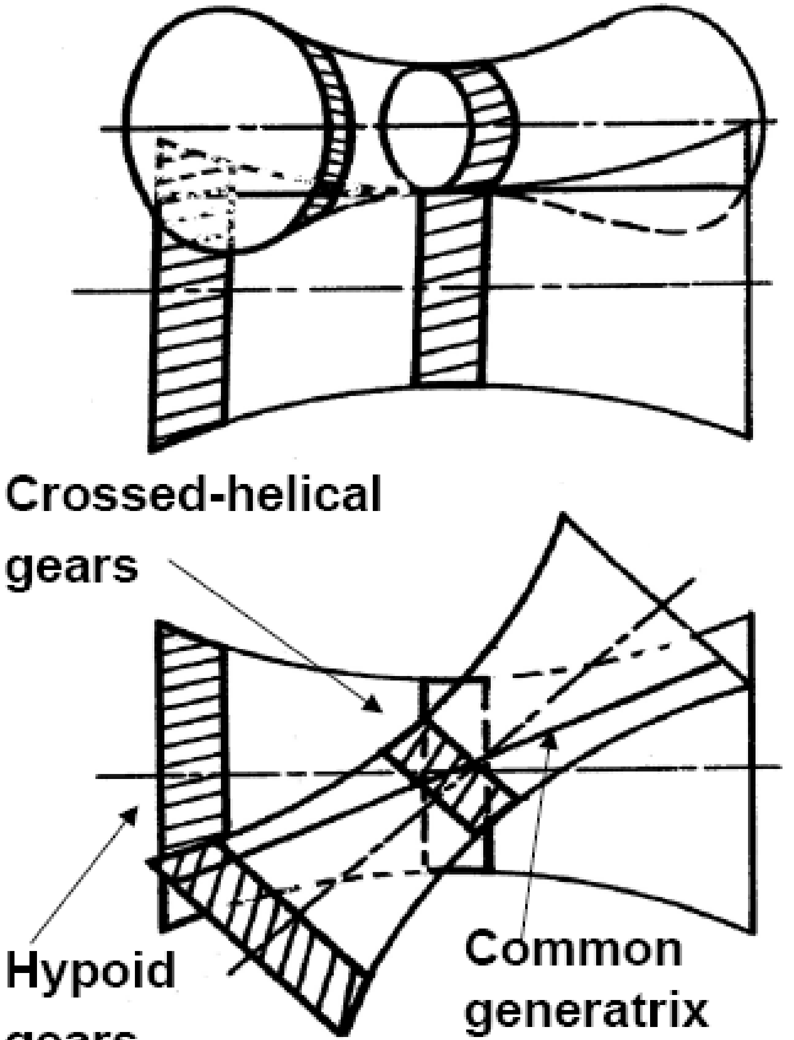
Hypoid gears

Crossed-helical gears

Worm gears

BASIC GEAR TYPES

Model for hypoid and crossed helical gears



Hyperboloids of one sheet

Hyperboloid of one sheet (quadric surface or quadric) contains straight lines

FUNDAMENTAL LAW OF GEAR-TOOTH ACTION

Essentially, the teeth on gears (wheels and pinions) are a series of cams. The motion imparted by one tooth to another in engagement with it depends on the respective profiles of the mating faces.

For a uniform transmission of rotary motion the ratio of angular velocities between the driving and driven gears must be constant through all operating positions of the two profiles, that is to say the two must be conjugate.

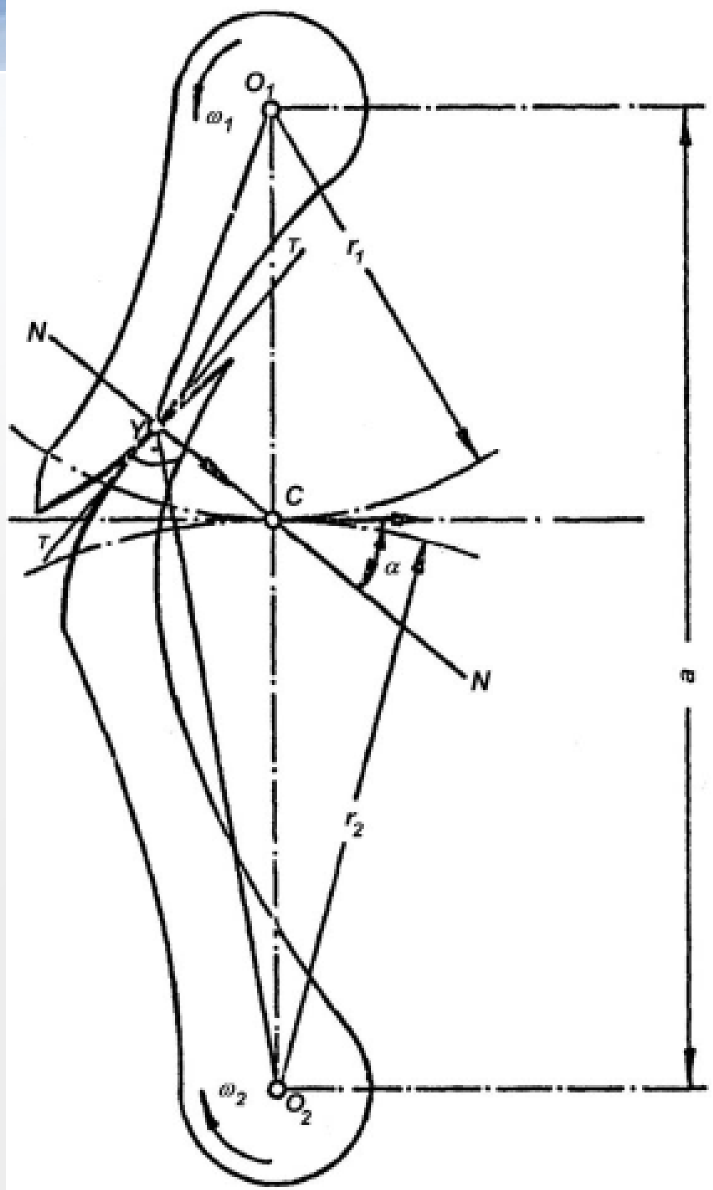
The requirements of conjugacy have been formulated by Willis in 18th century.

FUNDAMENTAL LAW OF GEAR-TOOTH ACTION

The path of contact between conjugate gear teeth is the locus of points of contact from the position of first engagement to that of disengagement. For any particular profile the path of contact is the same with a conjugate tooth of any pitch-circle diameter as it is with a conjugate rack; the length will, however, vary: the addendum circles of the two mating gears, i.e. the circles containing the crests of the teeth, limit the path of contact on either side of the line of centres.

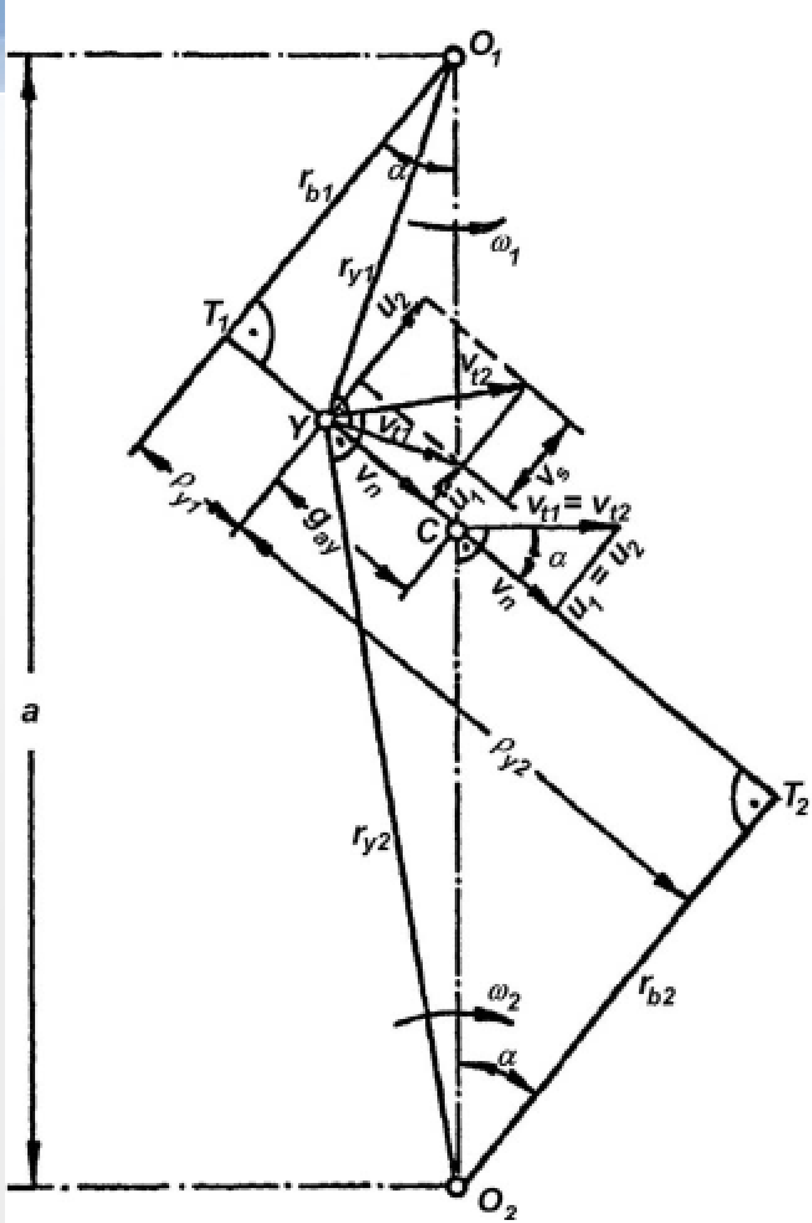
The pressure angle is the angle, at any point along the path of contact, between the common normal at the point of contact and the common tangent of the two pitch circles. It is not a condition of conjugacy that the pressure angle should be constant along the path of contact, and in fact it is a variable with all tooth forms except the involute.

FUNDAMENTAL LAW OF GEAR-TOOTH CTION



The basic condition for conjugacy (Willis principle) is represented by this figure, which shows the general situation. The two arms, one pivoting at O_1 and the other at O_2, have cam-like surfaces in engagement with each other. As in all cases of engagement of curved surfaces, the momentary line of action between them is the common normal (N-N) to the two curves at the point of tangency - point Y in the diagram. The point at which the common normal intersects the line of centres - point C on O_1-O_2 - is significant, since the momentary ratio of angular velocities between arm 1 and arm 2 is inversely proportional to the corresponding ratio O_1C to O_2C.

FUNDAMETAL LAW OF GEAR-TOOTH ACTION



The general kinematics of gearing:

it can easily be checked by inscribing vectors representing the absolute velocities of the points at Y on arms 1 and 2 respectively and resolving these into their tangential and normal components. For conjugate action the ratio O_1C to O_2C must remain constant, i.e. C must be fixed. When this is so, point C is referred to as the PITCH POINT and the distances O_1C and O_2C as the PITCH RADII. The pitch radii correspond with the radii of imaginary cylinders with parallel axes passing through the centres of rotation (once the centre distance has been fixed), the ratio of angular velocities, when the cylinders roll on each other without slip, being the reciprocal of the ratio of radii.

FUNDAMENTAL LAW OF GEAR-TOOTH ACTION

Velocities of driving (1) and driven (2) gears in point E are

$$v_{t1} = \omega_1 \cdot r_{y1}; \quad v_{t2} = \omega_2 \cdot r_{y2}$$

Components of these velocities along the common normal

$$v_{n1} = \omega_1 \cdot r_{b1} = \omega_1 \cdot r_1 \cdot \cos\alpha; \quad v_{n2} = \omega_2 \cdot r_{b2} = \omega_2 \cdot r_2 \cdot \cos\alpha$$

From obvious contact condition $v_n = v_{n1} = v_{n2}$ follows

$$\omega_1 \cdot r_{b1} = \omega_2 \cdot r_{b2} \quad or \quad \frac{\omega_1}{\omega_2} = \frac{r_{b2}}{r_{b1}} = u$$

Finally, the tooth ratio is

$$u = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

where r is pitch radius, d – pitch diameter and z – number of teeth.

FUNDAMENTAL LAW OF GEAR-TOOTH ACTION

The relative motion of spur gear teeth at the point of contact is a combination of rolling and sliding. Referring again to previous figure, it will be evident that the sweep velocities in point of contact (engagement) are

$$u_1 = \omega_1 \cdot r_{y1} = v \cdot (\sin\alpha - g_{ay} / r_1)$$

$$u_2 = \omega_2 \cdot r_{y2} = v \cdot (\sin\alpha + g_{ay} / r_2)$$

where v denotes the circumferential velocity

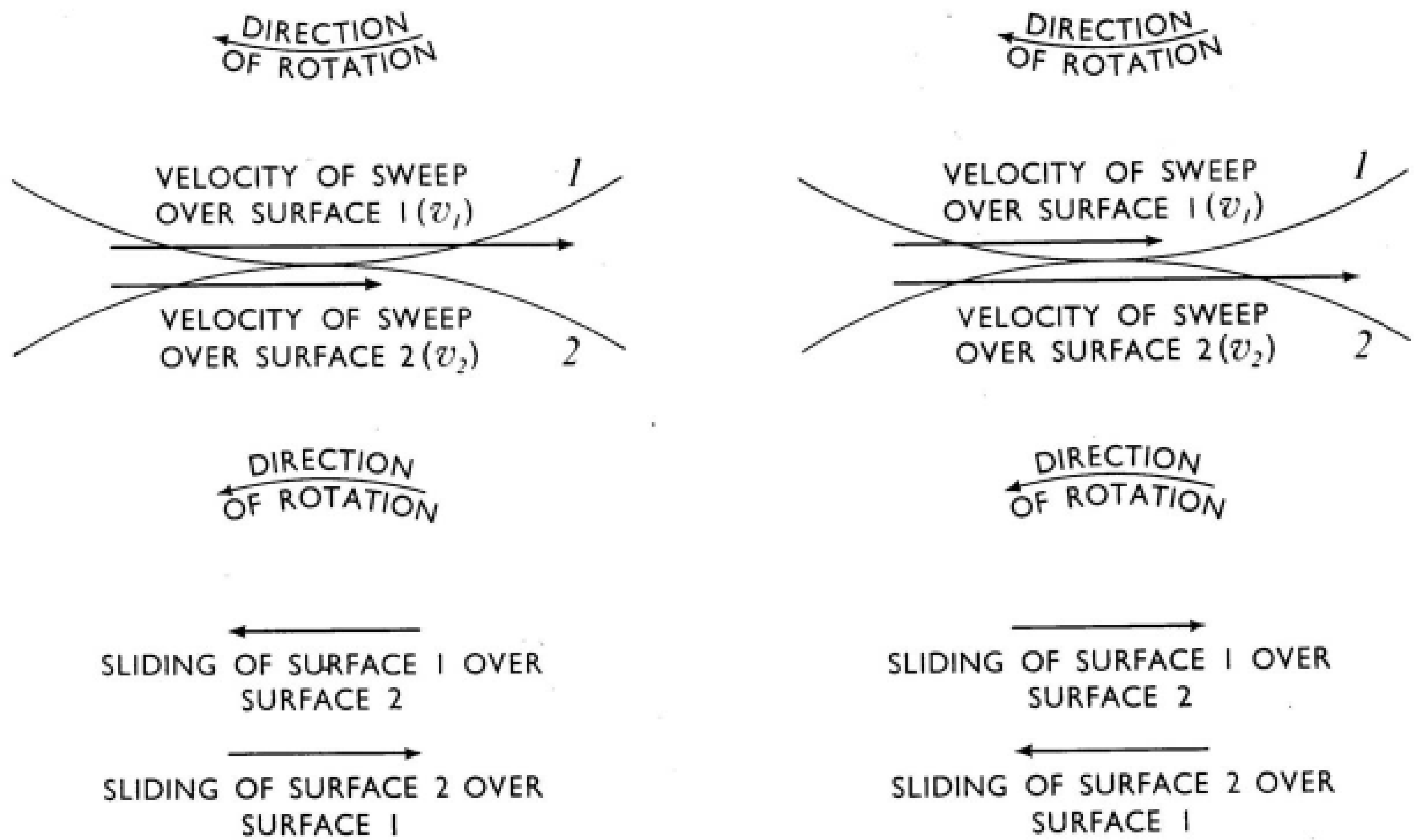
$$v = \omega_1 \cdot r_1 = \omega_2 \cdot r_2$$

Finally, the sliding velocity is

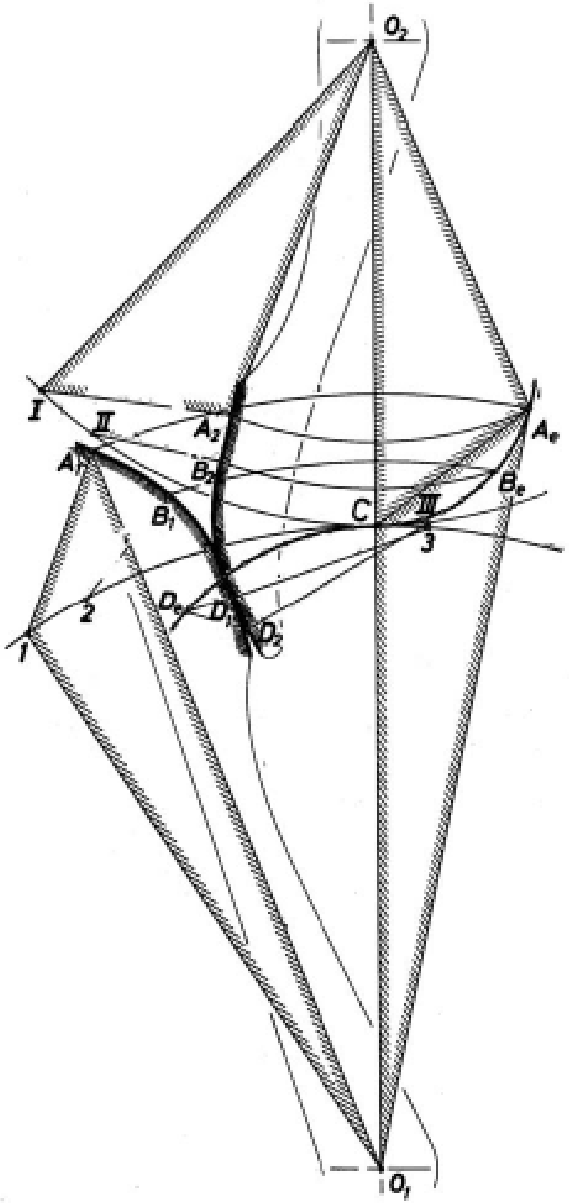
$$v_s = u_1 - u_2 = \pm v \cdot g_{ay} \cdot \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

FUNDAMENTAL LAW OF GEAR-TOOTH ACTION

The sliding velocity has a maximum value in one direction when the teeth first come into engagement, then diminishes towards the pitch point, at which it is zero; beyond the pitch point the direction of sliding is reversed and the sliding velocity increases progressively.



FUNDAMENTAL LAW OF GEAR-TOOTH ACTION

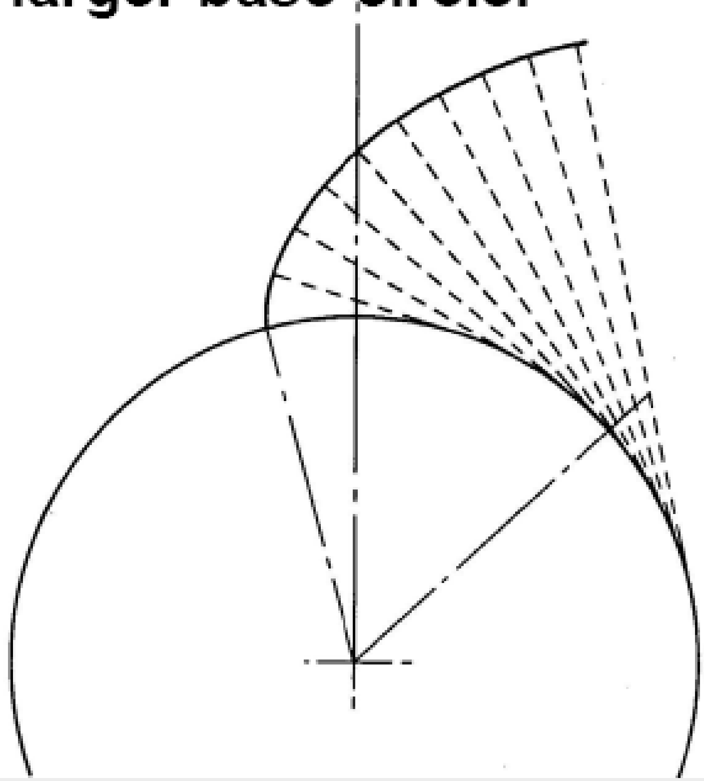


The geometrical procedure for developing of the form of the teeth: the points of the given flank are to be transformed into points of the mating flank that fulfil the condition of conjugacy.

At the present day the tooth forms used practically universally in spur, helical, bevel and worm gears are those of the involute family.

CHARACTERISTICS OF THE INVOLUTE

The involute is the curve formed by the end of a taut cord unwound from a base circle. The involute is a form of spiral, the curvature of which becomes straighter as it is drawn from a base circle and becomes a straight line at infinity. An involute drawn from a small base circle is more curved than one drawn from a larger base circle.



CHARACTERISTICS OF THE INVOLUTE

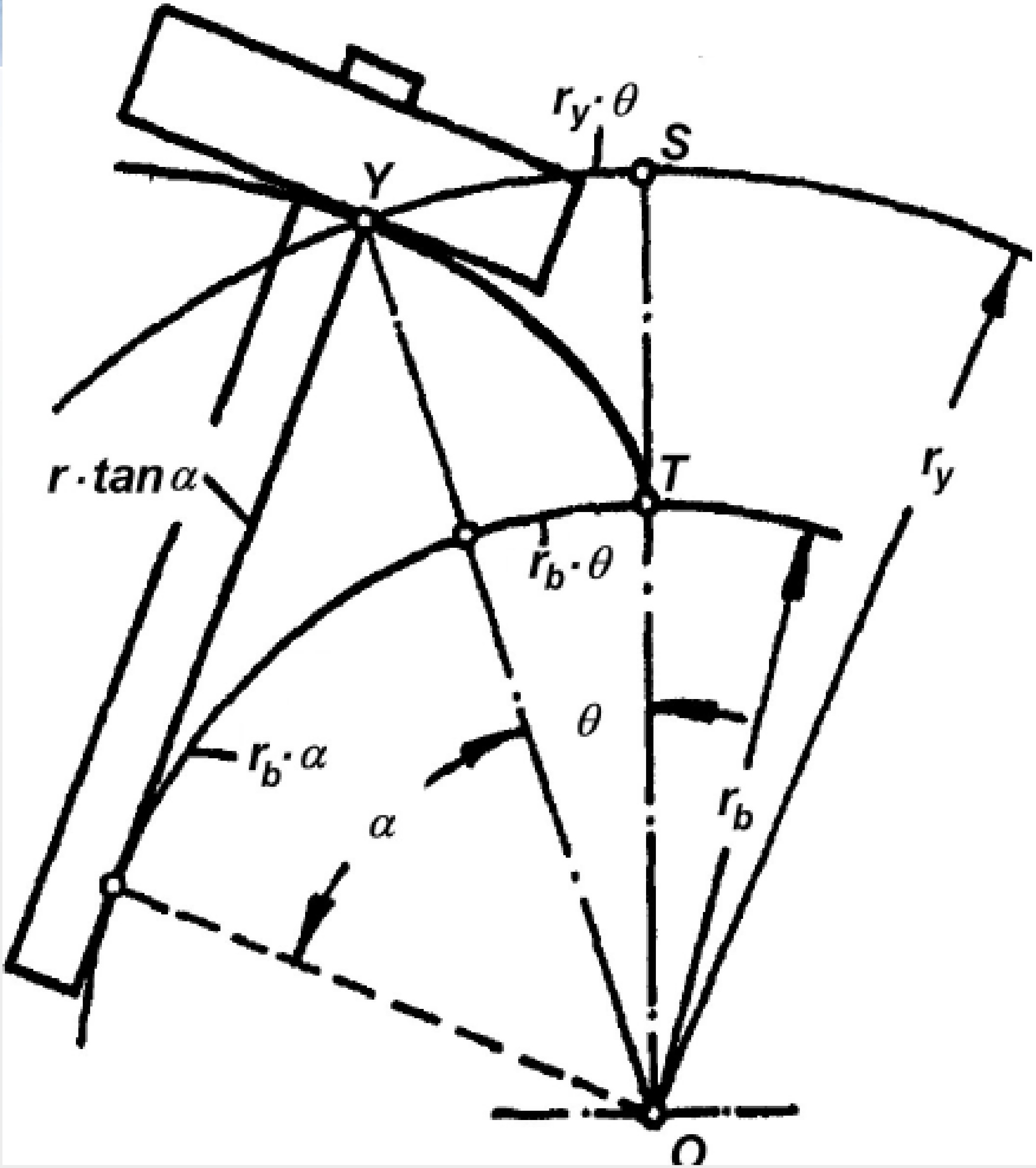
In the differential geometry of curves, we have a concept of roulette:

Take two curves. Fix some point, called the generator or pole, in relation to the first curve. Roll the first curve along the second, called the evolute; the generator traces out a curve. Such a curve is called roulette.

Thus, an involute is a roulette wherein the rolling curve is a straight line containing the pole and the original curve, that means evolute, is a circle.

This circle is called the base circle.

CHARACTERISTICS OF THE INVOLUTE



Point Y of T-square (drafting square) traces involute, for which we have

$$\alpha = \arccos(r_g / r_y)$$

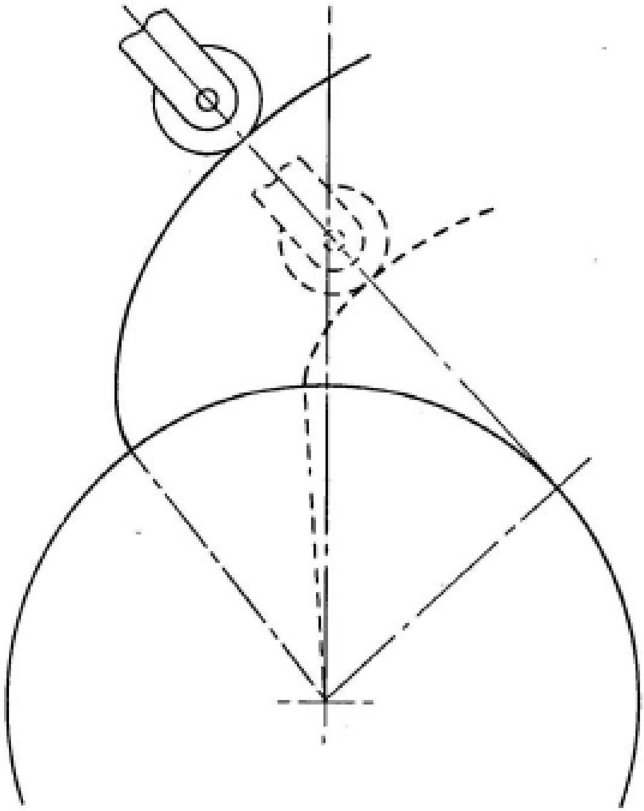
$$r_b \cdot \theta = r_b \cdot \tan\alpha - r_b \cdot \alpha$$

$$\theta = \tan\alpha - \alpha = inv\ \alpha$$

These equations define the involute curve in polar coordinates

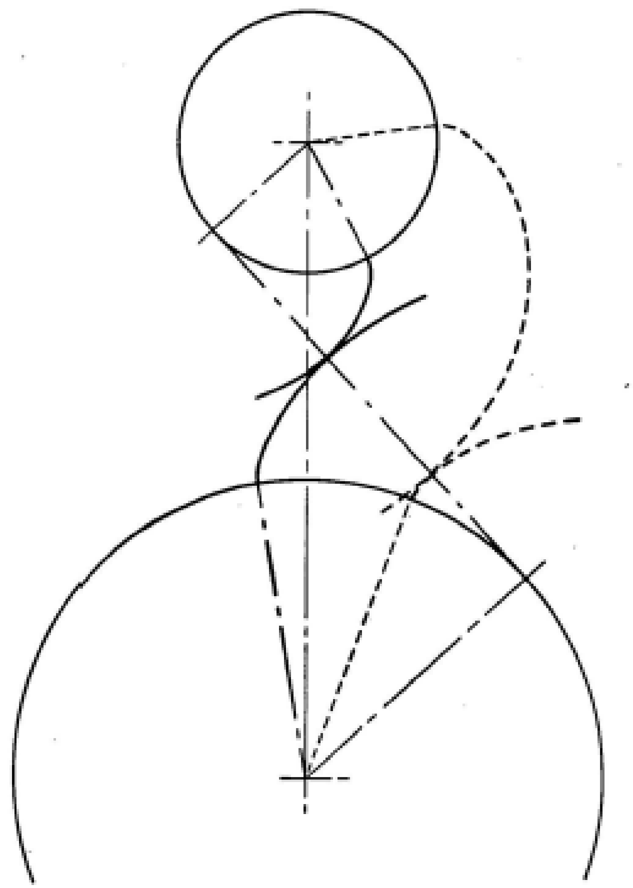
CHARACTERISTICS OF THE INVOLUTE

The involute, as a tooth form, has the property that rotation of the base circle at a uniform rate is associated with uniform displacement along a line tangent to the base circle: the effect is that of a uniform-rise cam, as shown in figure below.

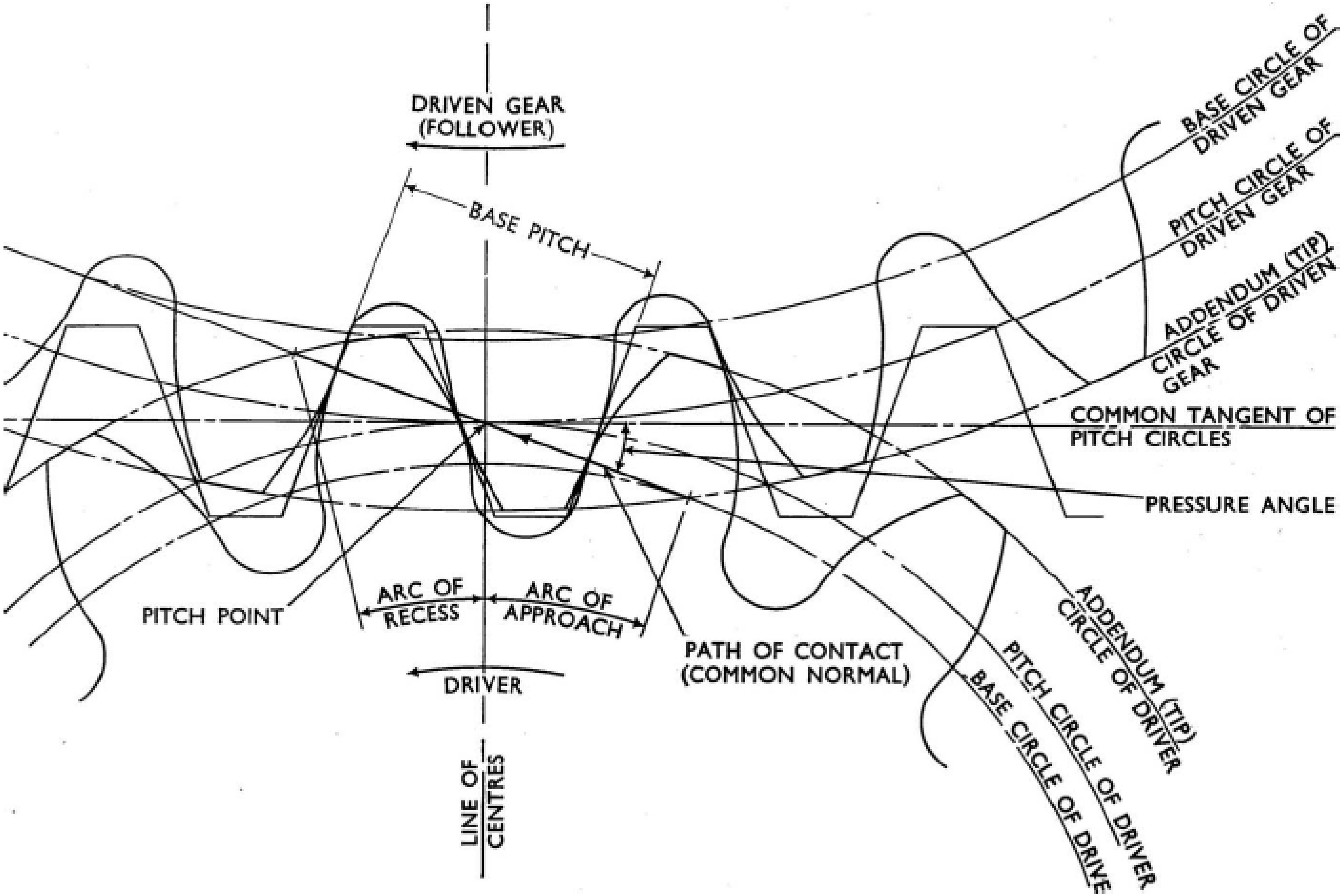


CHARACTERISTICS OF THE INVOLUTE

If the cam follower is replaced by another involute, of opposite hand, as shown in this figure, the action of the one on the other takes place along the line of the common tangent to the two base circles. As the line forming the involute curve is unwound from the base circle of the driving involute it is wound on to the base circle of the driven one, much as two pulleys might be connected by a crossed belt. Since the generating line of the involute being unwound is drawn out from one gear at the same rate as that at which the other is drawn in from the other gear, the ratio of angular velocities is inversely proportional to that of the base-circle diameters.

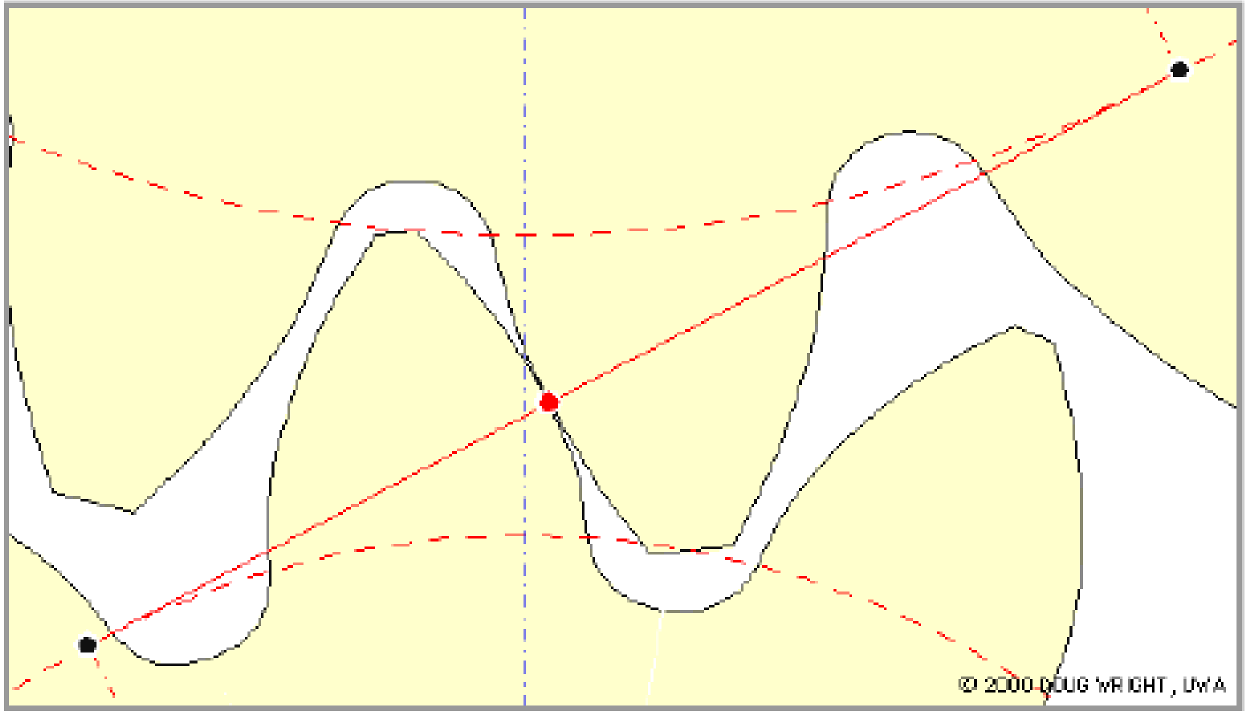


NOMENCLATURE FOR TEETH



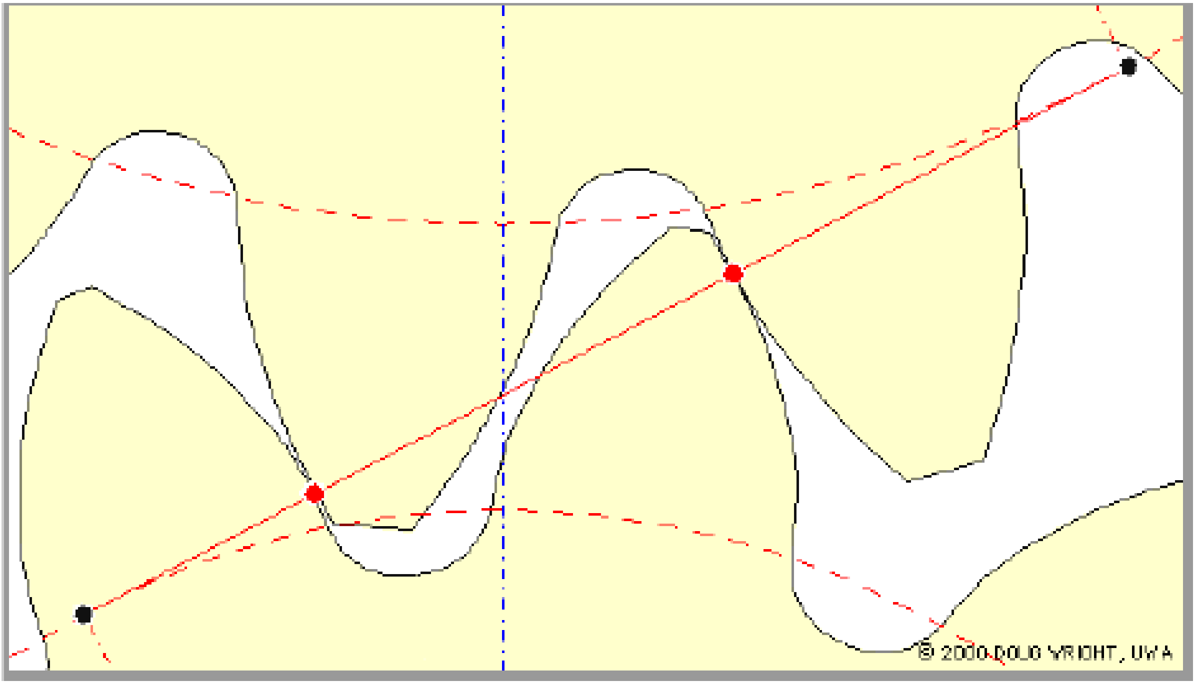
The advantage of the involute tooth form is a certain amount of tolerance in the distance between centres without ill-effects on the engagement.

CONTACT OF INVOLUTE TEETH



SINGLE CONTACT

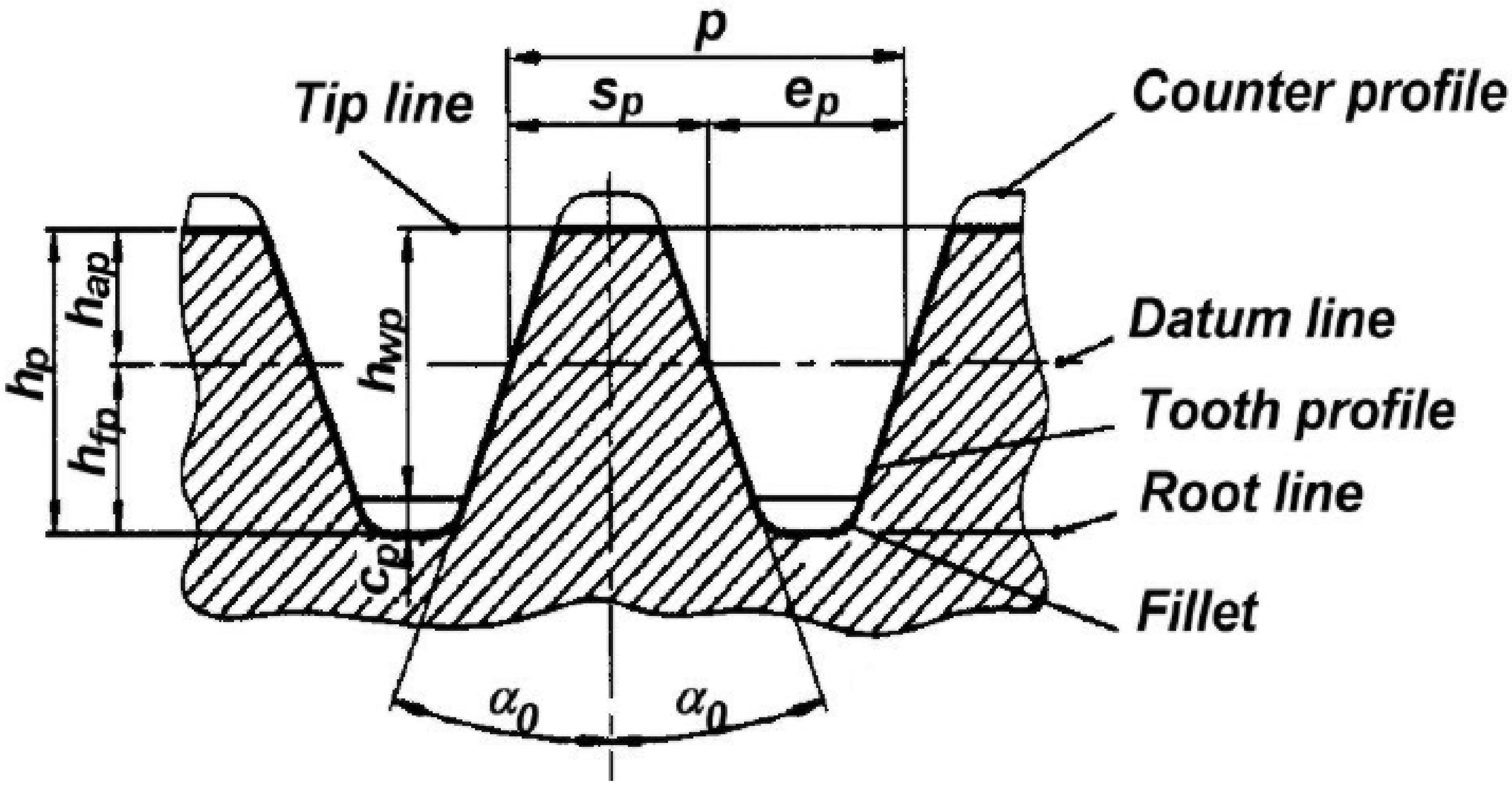
CONTACT OF INVOLUTE TEETH



DOUBLE CONTACT

(ANIMATION WILL BE SHOWN)

FEATURES OF INVOLUTE TEETH



The basic-rack form of involute gearing is characterised by straight working profiles.

FEATURES OF INVOLUTE TEETH

The diameter of the pitch circle is

$$d = p \cdot z / \pi$$

Introducing the conceptual <u>module</u>

$$m = p / \pi$$

one obtains

$$d = m \cdot z$$

The module is a standardised one – in order to limit the number of the required gear cutting tools. The table below shows series 1 (preferred) and 2 of standardised modules.

Series 1	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12	16	20	25	32
Series 2			1.75			3.5		4.5	7		9	14	18	22	28	

The Anglo-Saxon <u>diametral pitch</u> is the number of teeth per inch of diameter

$$DP = z / d$$

Diametral pitch is the reciprocal of the module (expressed in inch)

FEATURES OF INVOLUTE TEETH

Generation of teeth by rolling the pitch circle on the axis of symmetry of a basic rack produces gears in which the thickness of the teeth and the gap between each pair, measured round the pitch circle, are the same

$$s_p = e_p = p/2$$

If, with a particular basic rack of this form, the base-circle diameter and pressure angle are kept constant but the axis of symmetry of the rack is displaced from the pitch line on which the pitch circle of the gear is rolled, there will be a change in the length of the involute profile over which engagement will occur.

The addendum and dedendum are fixed by

$$h_{aP} = y \cdot m; \;\; h_{fP} = y \cdot m + c_P \;\; with \;\; y = 0.8 \;\dots\; \underline{1} \;\dots\; 1.2$$

The bottom clearance is

$$c_P = \Psi \cdot m \quad with \quad \Psi = 0.1 \;\dots\; \underline{0.2} \;\dots\; 0.4$$

Dişli çarklar

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