$$\Delta p = \left(\frac{64\nu}{\overline{V}d}\right)\left(\frac{L}{d}\right)\left(\frac{1}{2}\rho\overline{V}^2\right) \tag{6.70}$$

where d = 2R is the diameter of the circular tube. The first term on the right hand side of the above equation is the **friction factor** and may be given by

$$f = \frac{64\nu}{\overline{V}d} \tag{6.71}$$

Then Equation (6.70) takes the following form

$$\Delta p = f\left(\frac{L}{d}\right)\left(\frac{1}{2}\rho\overline{V}^2\right) \tag{6.72}$$

Now, defining the **Reynolds number**, which is based on the diameter of the circular tube as

$$Re_d = \frac{\overline{V}d}{\nu} \tag{6.73}$$

the friction factor in a circular tube is

$$f = \frac{64}{Re_d} \tag{6.74}$$

The second term on the right hand side of Equation (6.72) indicates that the pressure drop is proportional to the length of the circular tube, which is expressed in terms of the diameter of the circular tube.Finally, the last term on the right hand side of Equation (6.72) is the **dynamic pressure.**

6.3.4 Viscous Flow Between Two Concentric Cylinders Due to the Rotation of the Inner Cylinder

Consider the steady flow of an incompressible fluid with constant viscosity in the absence of body forces between two infinitely long concentric cylinders, which is caused by the rotation of the inner cylinder at a constant angular velocity of ω, as shown in Figure 6.6. In this case, the continuity equation (4.11a) and the Navier-Stokes equations (6.14) in the r, θ and z-directions may be given as follows:

$$\Delta p = \left(\frac{64\nu}{\bar{V}d}\right)\left(\frac{L}{d}\right)\left(\frac{1}{2}\rho\bar{V}^2\right) \tag{6.70}$$

where d = 2R is the diameter of the circular tube. The first term on the right hand side of the above equation is the **friction factor** and may be given by

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$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0 \tag{6.75}$$

$$V_r\frac{\partial V_r}{\partial r} + V_\theta\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$+ \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial z^2} - \frac{V_r}{r^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right] \tag{6.76}$$

$$V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_rV_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta}$$

$$+ \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial z^2} - \frac{V_\theta}{r^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right] \tag{6.77}$$

$$V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$+ \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right] \tag{6.78}$$

A careful investigation of Figure 6.6 indicates that the fluid is flowing in circular paths about the center, so that the velocity component in the r and z-directions are zero. Then Equation (6.75) reduces to

$$\frac{\partial V_\theta}{\partial \theta} = 0 \tag{6.79}$$

Since the flow in the annular space may assumed to be axisymmetric, then the other components of the velocity are also zero. One might also note that the cylinders have infinite length in the z-direction, so that the flow patterns are similar at all planes which are parallel to the rθ-plane. For this reason, the derivatives of the properties with respect to z may taken to be zero. Then Equations (6.76), (6.77) and (6.78) reduce to

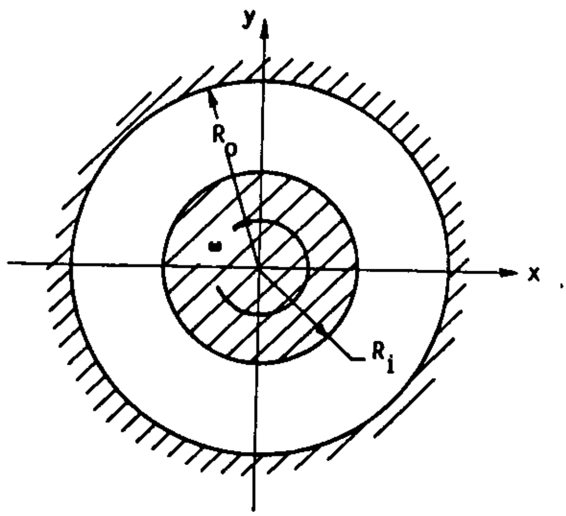

Figure 6.6 Viscous flow between two concentric cylinders due to the rotation of the inner cylinder

$$\frac{\partial p}{\partial r} = \frac{\rho}{r} V_\theta^2 \tag{6.80}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\theta}{\partial r}\right) - \frac{V_\theta}{r^2} = 0 \tag{6.81}$$

$$\frac{\partial p}{\partial z} = 0 \tag{6.82}$$

6.3.4.1 Velocity Distribution

As long as the velocity component in the θ-direction is not functions of θ and z, then it must only be a function of r. Therefore, the partial derivatives with respect to r in Equation (6.81) may be replaced with the total derivatives, so that

$$\frac{d}{dr}(r\ \frac{dV_\theta}{dr}) - \frac{V_\theta}{r} = 0$$

ay be rearranged to give

$$r^2\ \frac{d^2V_\theta}{dr^2} + r\frac{dV_\theta}{dr} - V_\theta = 0 \qquad (6.83)$$

point, one might introduce a new variable, ξ, such that

$$\xi = \ln\ r \qquad (6.84)$$

he partial derivatives of the tangential velocity with respect to me

$$\frac{dV_\theta}{dr} = \frac{dV_\theta}{d\xi}\ \frac{d\xi}{dr} = \frac{1}{r}\frac{dV_\theta}{d\xi}$$

$$\frac{d^2V_\theta}{dr^2} = \frac{1}{r}\frac{d^2V_\theta}{d\xi^2}\ \frac{d\xi}{dr} - \frac{1}{r^2}\frac{dV_\theta}{d\xi} = \frac{1}{r^2}\ (\frac{d^2V_\theta}{d\xi^2} - \frac{dV_\theta}{d\xi})$$

Equation (6.83) takes the following form:

$$\frac{d^2V_\theta}{d\xi^2} - V_\theta = 0$$

may be solved for the tangential velocity component as

$$V_\theta = Ae^{\xi} + Be^{-\xi}$$

e A and B are the constants to be determined from the boundary itions. Now, using equation (6.84), it is possible to obtain

(6.85)

$$V_\theta = \omega R_i \qquad \text{at} \qquad r = R_i \tag{6.86a}$$

$$V_\theta = 0 \qquad \text{at} \qquad r = R_o \tag{6.86b}$$

After the application of the boundary conditions (6.86)

$$A = \frac{\omega R_i^2}{R_i^2 - R_o^2}$$

$$B = -\frac{\omega R_i^2 R_o^2}{R_i^2 - R_o^2}$$

Hence the tangential component of the velocity is

$$V_\theta = \frac{\omega R_i^2}{R_i^2 - R_o^2}\left(r - \frac{R_o^2}{r}\right) \tag{6.87}$$

6.3.4.2 Pressure Distribution

Also, it is found that the pressure [illegible] functions of θ and z, then it must only be a function of r. Therefore, the partial derivative of the pressure in the r-direction [illegible] (6.8[illegible] be replaced by a total derivative. Then

$$\frac{dp}{dr} = \frac{\rho}{r} V_\theta^2$$

and with the aid of Equation (6.87) it is possible to obtain

$$\frac{dp}{dr} = \frac{\rho \omega^2 R_i^4}{(R_i^2 - R_o^2)^2}\left(r - \frac{2R_o^2}{r} + \frac{R_o^4}{r^3}\right)$$

The above equation may now be integrated to give

$$p = \frac{\rho\omega^2 R_i^4}{(R_i^2 - R_o^2)^2}\left(\frac{r^2}{2} - 2R_o^2 \ln r - \frac{R_o^4}{2r^2}\right) + A \qquad (6.88)$$

where A is a constant of integration. For the complete determination of the pressure field, one has to know a boundary condition for the pressure.

6.3.4.3 Shear Stress Distribution

The shear stress distribution within the incompressible fluid in the annular space between the two concentric cylinders may be obtained by using Equation (6.15a) and (6.87)

$$\tau_{r\theta} = \mu r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) = \frac{2\mu\omega R_o^2\, R_i^2}{(R_i^2 - R_o^2)r^2} \qquad (6.89)$$

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or for uniform flow through the exit area

$$\frac{\partial}{\partial t}(\rho V) + \rho_e V_e \int_{A_e} dA = 0$$

As long as the volume of the air in the control volume is constant, then the time rate of change of the density of the air in the tank is

$$\frac{\partial \rho}{\partial t} = -\frac{\rho_e V_e A_e}{V} = \frac{(4.73 kg/m^3)(300 m/s)(130 \times 10^{-6}\ m^2)}{(0.5\ m^3)} = \underline{-0.37\ kg/m^3 s}$$

The minus sign indicates that there is a decrease in the density of the air in the tank.

4.4 THE STREAM FUNCTION

It is convenient to have a tool to describe the form of any particular pattern of fluid flow. An adequate description should portray the shape of the streamlines and the magnitude of the velocity at all points of the flow field. A mathematical tool that serves this purpose is the stream function, and it is a relation between the principle of conservation of mass and the streamlines. It is possible to define a stream function either for a two-dimensional and incompressible flow or for a two-dimensional and steady flow.

4.4.1 The Stream Function for a Two-Dimensional Flow of an Incompressible Fluid

For a two-dimensional flow in the xy plane of the Cartesian coordinate systems, the continuity equation (4.11b) for an incompressible fluid reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4.21)$$

If a continuous function $\psi = \psi(x,y,t)$, which is known as the stream function, is defined such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \qquad (4.22)$$

hen the continuity equation (4.21) will exactly be satisfied, since

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

The equation of a streamline for a two-dimensional flow may be btained from Equation (3.24) as

$$udy - vdx = 0$$

ubstituting for the velocity components in terms of the stream function rom Equation (4.22) yields

$$\frac{\partial \psi}{\partial x}\,dx + \frac{\partial \psi}{\partial y}\,dy = 0$$

lthough the streamfunction is $\psi = \psi(x,y,t)$, at a certain instant of ime, t_0, it may be expressed as $\psi = \psi(x,y,t_0)$. At this instant, the tream function may be treated as though $\psi = \psi(x,y)$, so that the above quation becomes an exact or total differential, that is along an nstantaneous streamline. Therefore, the stream function is constant long an instantaneous streamline for a two-dimensional flow of an ncompressible fluid.

One should note that although the streamlines can be three-dimensional, the streamfunction can only be defined for two-dimensional flow fields.

$$d\psi = \frac{\partial \psi}{\partial x}\,dx + \frac{\partial \psi}{\partial y}\,dy = 0$$

If the flow is also steady, the stream function may be expressed as $\psi = \psi(x,y)$ and it is constant along a streamline at all times, since the streamlines do not change their position with time.

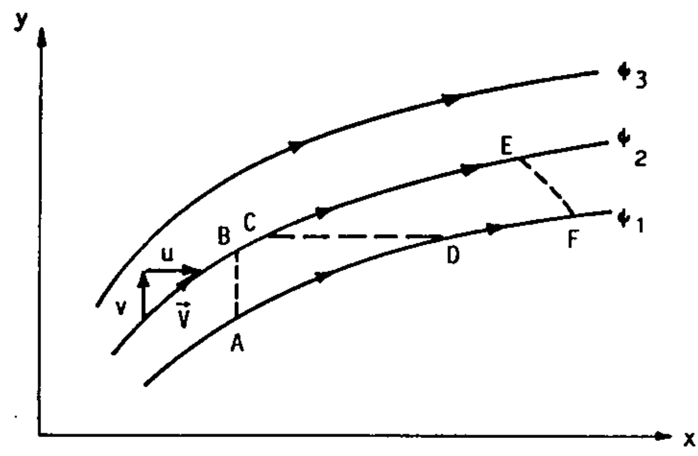

Figure 4.12 Instantaneous streamlines in a two-dimensional flow of an incompressible fluid

$$Q = \int_A \vec{V}.\hat{n}\, dA = \int_A (u\hat{i} + v\hat{j}).(\hat{i})dA = \int_A udA$$

But $u = \partial\phi/\partial y$ and $dA = bdy$ with b being the depth of the flow field, then the volumetric flow rate per unit depth is

$$q = \frac{Q}{b} = \int_{y_1}^{y_2} \frac{\partial\phi}{\partial y}\, dy$$

Along line AB, x is constant, so that $d\phi = (\partial\phi/\partial y)dy$. Therefore

$$q = \int_{\phi_1}^{\phi_2} d\phi = \phi_2 - \phi_1$$

Similarly over area CD, $\vec{V} = u\hat{i} + v\hat{j}$ and $\hat{n} = \hat{j}$, then the volumetric flow rate is

$$Q = \int_A \vec{V}.\hat{n}\, dA = \int_A (u\hat{i} + v\hat{j})(\hat{j})dA = \int_A vdA$$

The volumetric flow rate per unit depth may be expressed in the following form, since $v = -\partial\phi/\partial x$ and $dA = bdx$

$$q = \frac{Q}{b} = - \int_{x_1}^{x_2} \frac{\partial \psi}{\partial x}\, dx$$

· along line CD y is constant, so that $d\psi = (\partial\psi/\partial x)dx$. Therefore,

$$q = - \int_{\psi_2}^{\psi_1} d\psi = \psi_2 - \psi_1$$

.s the volumetric flow rate per unit depth between any two streamlines, and ψ_1, can be expressed as the numerical difference between the nstant values of the stream function defining these two streamlines, at is

$$q = \psi_2 - \psi_1 \tag{4.23}$$

In the $r\theta$ plane of the cylindrical coordinate system, the in- mpressible continuity equation (4.11c) reduces to

$$\frac{\partial (rV_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta} = 0 \tag{4.24}$$

en it is possible to define a stream function, $\psi = \psi(r,\theta,t)$, such that

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad V_\theta = - \frac{\partial \psi}{\partial r} \tag{4.25}$$

ich satisfies the continuity equation (4.24) exactly. The flow in the plane of the cylindrical coordinate system, which is also referred as e **axisymmetric flow**, is characterized with identical flow patterns on ery constant θ plane. In this case, the incompressible continuity uation (4.11c) reduces to

$$\frac{1}{r}\frac{\partial (rV_r)}{\partial r} + \frac{\partial V_z}{\partial z} = 0 \tag{4.26}$$

4.4.2 The Stream Function for a Two-Dimensional and Steady Flow

For a two-dimensional and steady flow in the xy plane of the Cartesian coordinate system, the continuity equation (4.10b) reduces to

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{4.28}$$

Then it is possible to define a continuous stream function, $\phi = \phi(x,y)$, such that

$$\rho u = \frac{\partial \phi}{\partial y} \quad \text{and} \quad \rho v = -\frac{\partial \phi}{\partial x} \tag{4.29}$$

which satisfies the continuity equation (4.28) exactly as

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right) = 0$$

For a two-dimensional flow, the equation of a streamline, Equation (3.24), reduces to

$$u\,dy - v\,dx = 0$$

If the velocity components in the above equation are expressed in terms of the stream function with the aid of Equation (4.29), then the equation of a streamline becomes

$$\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

Since $\phi = \phi(x,y)$, the above equation is an exact differential, that is $d\phi = 0$ along a streamline. Therefore, **the stream function is constant along a streamline for two-dimensional and steady flow.**

By using the analysis, which is presented in the previous section, one may prove that the mass flow rate per unit depth between any two streamlines, ϕ_2 and ϕ_1, can be expressed as the numerical difference between the constant values of the stream function defining these two streamlines, that is

$$\frac{\dot{m}}{b} = \psi_2 - \psi_1 \tag{4.30}$$

n the rθ plane of the cylindrical coordinate system, the continuity on (4.10c) for a steady flow reduces to

$$\frac{\partial(\rho r V_r)}{\partial r} + \frac{\partial(\rho V_\theta)}{\partial \theta} = 0 \tag{4.31}$$

t it is possible to define a stream function, $\psi = \psi(r,\theta)$, such that

$$\rho V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad \rho V_\theta = -\frac{\partial \psi}{\partial r} \tag{4.32}$$

satisfies the continuity equation (4.31) exactly. However, in the of the axisymmetric flow in the rz plane of the cylindrical nate system the continuity equation (4.10c) for a steady flow may ressed as

$$\frac{1}{r}\frac{\partial(\rho r V_r)}{\partial r} + \frac{\partial(\rho V_z)}{\partial z} = 0 \tag{4.33}$$

stream function, $\psi = \psi(r,z)$ may be defined as

$$\rho V_r = \frac{1}{r}\frac{\partial \psi}{\partial z} \quad \text{and} \quad \rho V_z = -\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{4.34}$$

t the continuity equation (4.33) is satisfied exactly.

Example 4.10

he velocity field for the steady flow of an incompressible fluid en by $\vec{V} = ax\hat{i} - ay\hat{j}$ with $a = 2s^{-1}$.

) Determine the stream function that will describe this flow field.
) Sketch the streamlines for $\psi = 0$, $\psi = +2$ and $\psi = \pm 8$, and

Solution

a) The velocity field in the Cartesian coordinate system is specified as $\vec{V} = ax\hat{i} - ay\hat{j}$, so that the components of the velocity vector are $u = ax$, $v = -ay$ and $w = 0$. In this case, the components of the velocity are functions of two space coordinates, x and y, and also the velocity component in the z-direction is zero. Therefore the flow field is two-dimensional. Before proceeding further, one has to check whether the continuity equation (4.21) is satisfied by the given flow field or not. Then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial(ax)}{\partial x} + \frac{\partial(-ay)}{\partial y} = a - a = 0$$

As long as the given two-dimensional flow of an incompressible fluid satisfies the continuity equation, then it is possible to define a stream function. Hence, using Equation (4.22)

$$u = \frac{\partial\psi}{\partial y} = ax$$

Integration with respect to y yields

$$\psi = \int \frac{\partial\psi}{\partial y}\,dy = \int ax\,dy = axy + f(x)$$

where f(x) is a function of x due to the partial integration in the y-direction. The velocity component in the y-direction may now be obtained via Equation (4.22) as

$$v = -\frac{\partial\psi}{\partial x} = -ay - \frac{df(x)}{dx}$$

But the velocity component in the y-direction is $v = -ay$, then

$$\frac{df(x)}{dx} = 0$$

and integration with respect to x yields

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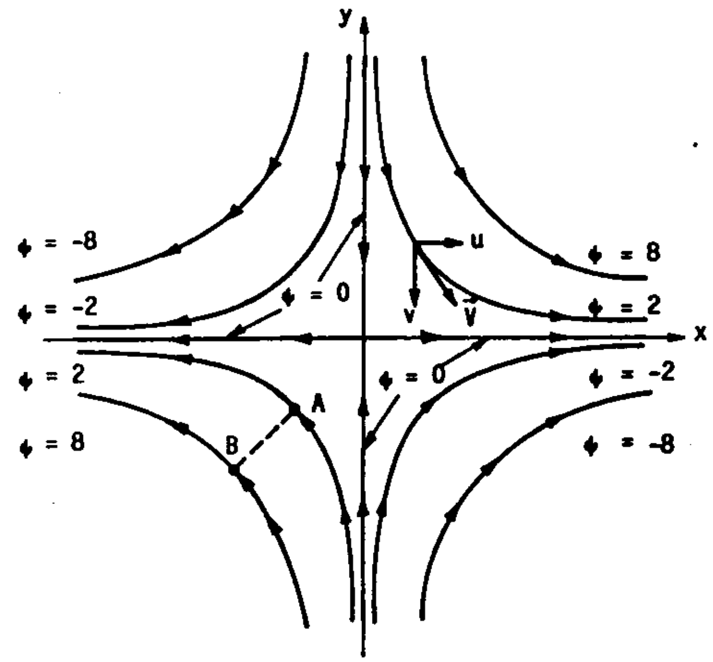

Figure 4.13 Sketch of the streamlines for Example 4.10

$$f(x) = C$$

with C being the constant of integration. Then the stream function is

$$\phi = axy + C$$

setting this arbitrary integration constant, C, does not change the shape of the streamlines, but only changes the constants which define these streamlines. Hence

$$\phi = axy = 2xy$$

b) Equation of streamlines $\phi = 0$, $\phi = \pm 2$ and $\phi = \pm 8$ are $xy = 0$, $xy = \pm 1$ and $xy = \pm 4$ respectively. They represent inverted hyperbolas in the xy plane as shown in Figure 4.13. The direction of the fluid flow on these streamlines may now be found by considering the first quadrant in the xy plane where x and y are greater than zero. Therefore, u is positive and v is negative, and the direction of the fluid flow on these streamlines are shown in Figure 4.13.

c) In order to determine the volumetric flow rate per unit depth between the streamlines passing through points A and B, the constant numerical values of the stream function on these streamlines must be evaluated. Then

$$\psi_A = ax_Ay_A = (2s^{-1})(-1\ m)(-1\ m) = 2\ m^2/s$$

and

$$\psi_B = ax_By_B = (2s^{-1})(-2\ m)(-2\ m) = 8\ m^2/s$$

The volumetric flow rate per unit depth may be obtained by using Equation (4.23) as

$$q = \psi_B - \psi_A = 8\ m^2/s - 2\ m^2/s = \underline{6\ m^2/s}$$

Example 4.11

The velocity field for the steady flow of an incompressible fluid is given as $V_r = 0$ and $V_\theta = ar$ with $a = -2\ s^{-1}$

a) Determine the stream function that will describe this flow field

b) Sketch the streamlines, $\psi = 1$ and $\psi = 4$, and determine the direction of fluid flow on these streamlines.

c) Determine the magnitude of the volumetric flow rate per unit depth between streamlines passing through points $A(1,\pi/4)$ and $B(2,\pi/4)$.

Solution

a) For a steady and two-dimensional flow of an incompressible fluid, Equation (4.25) may be used as

$$V_\theta = -\frac{\partial\phi}{\partial r} = ar$$

egration with respect to r yields

$$\phi = \int \frac{\partial\phi}{\partial r}\, dr = -\int ardr = -\frac{ar^2}{2} + f(\theta)$$

re $f(\theta)$ is a function of θ due to the partial integration in the r ection. The velocity component in the r direction may now be obtained Equation (4.25) as

$$V_r = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{df(\theta)}{d\theta}$$

as long as the velocity component in the r-direction is zero, then

$$\frac{df(\theta)}{d\theta} = 0$$

integration with respect to r yields

$$f(\theta) = C$$

h C being the constant of integration. Then the stream function is

$$\phi = -ar^2/2 + C$$

ting this arbitrary integration constant, C,does not change the shape the streamlines, but only changes the constants which define these eamlines. Hence

$$\phi = -ar^2/2 = \underline{r^2}$$

b) Equations of streamlines, $\phi = 1$ and $\phi = 4$ may then be given as 1 and r = 2 respectively. They represent circles of radii 1 and 2 in rθ plane, as shown in Figure 4.14. The direction of the fluid flow these streamlines is clockwise since V_θ assumes a negative value for values of r, which is always positive.

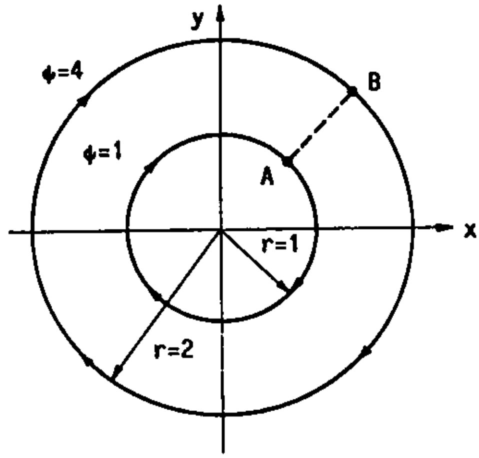

Figure 4.14 Sketch of the streamlines for Example 4.11

values of the stream function on these streamlines must be known. Then

$$\psi_A = -ar_A^2/2 = -(-2s^{-1})(1\ m)^2/2 = 1\ m^2/s$$

and

$$\psi_B = -ar_B^2/2 = -(-2s^{-1})(2\ m)^2/2 = 4\ m^2/s$$

The volumetric flow rate per unit depth may now be evaluated via Equation (4.23) as

$$q = \psi_B - \psi_A = 4\ m^2/s - 1\ m^2/s = \underline{3\ m^2/s}$$