Operations Research Modelling Examples

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Basic OR Concepts

- Basic concepts required to define for mathematical modeling:
  - Variables
  - Constraints
  - Objective

Linear Programming

- all variables continuous (i.e. can take fractional values)
- a single objective (minimize or maximize)
- the objective and constraints are linear i.e. any term is either a constant or a constant multiplied by an unknown.

- LP's are important - this is because:
  - many practical problems can be formulated as LP's
  - there exists an algorithm (called the simplex algorithm) which enables us to solve LP's numerically relatively easily

Oil Blending

- Sunco Oil manufactures three types of gasoline (gas 1, gas 2, and gas 3). Each type is produced by blending three types of crude oil (crude 1, crude 2, and crude 3). Sunco can purchase up to 5,000 barrels of each type of crude oil daily. The three types of gasoline differ in their octane rating and sulfur content
  - Gas 1 > average octan rating at least 10, sulfur content at most %1
  - Gas 2 > average octan rating at least 8, sulfur content at most %2
  - Gas 3 > average octan rating at least 6, sulfur content at most %1
- It costs $4 to transform one barrel of oil into one barrel of gasoline, and Sunco's refinery can produce up to 14,000 barrels of gasoline daily.
- Demands (barrels) - gas 1: 3000, gas 2: 2000, gas 3: 1000
- Each dollar spent daily in advertising a particular type of gas increases the daily demand for that type of gas by 10 barrels.
- Formulate an LP that will enable Sunco to maximize daily profits

Crude	Octane rating	Sulfur content (%)
1	12	0,5
2	6	2
3	8	3

Gas	Sales Price	Crude	Purchase price
1	70	1	45
2	60	2	35
3	50	3	25

Allocation Problem - Investment

- The investor has \$300,000 that can be invested. In addition to the money at hand, it is possible to borrow up to \$100,000 at 12% interest. The investor has narrowed down the choices to six alternatives (see table).
- the decision maker faces the following constraints
  - The expected value of assets (exclusive interest) at the end of the planning period should be at least 7% higher than at the beginning,
  - invest at least 50% of all the money invested in stocks and bonds combined,
  - invest no more than 20% of total amount available (excluding the amount borrowed) in real estate and silver combined, and
  - the average risk of the portfolio should not exceed 10.
- Formulate an LP to maximize the expected value of the assets at the end of the planning period

Investment type	Expected annual interest/dividend	Expected annual increase in value	Average risk per dollar
Real estate	0%	18%	20
Silver	0%	10%	12
Savings account	2%	0	1
Blue chip stocks	3%	6%	7
Bonds	4%	0%	3
Hi-tech stocks	0%	20%	30

Production Process

- We produce six products. Each unit of processed raw material yields four units of product 1, two units of product 2, and one unit of product 3. we can sell at most 1200 units of product 1 and 300 units of product 2.
- Product 1 can also be processed to one units of product 4. Demands of product 3 and 4 are unlimited.
- Product 2 can also be processed to 0,8 units of product 5 and 0,3 units of product 6. demands of product 5 and 6 are – at most- 1000 and 800 respectively.
- We can provide at 3000 units of raw material at a cost of TL 6 per unit.
- Leftover units of products 5 and 6 must be destroyed. It costs $4 to destroy each leftover unit of product 5 and $3 to destroy each leftover unit of product 6.
- Formulate an LP whose solution will yield a profit maximizing production schedule.

Product	Sales price (TL)	Production cost (TL)
1	7	4
2	6	4
3	4	2
4	3	1
5	20	5
6	35	5

Workforce Scheduling

- Each year, Ayakco Shoes faces demands (which must be met on time) for pairs of shoes as shown in the following Table. During a month in which a worker works, he or she can produce up to 50 pairs of shoes. Each worker is paid $500 per month. At the end of each month, a holding cost of $50 per pair of shoes is assessed. Workers will work four months and receive one month off in the following 5 months.
- Formulate an LP that can be used to minimize the cost per year (labor + holding) of meeting the demands for shoes.

Jan	Feb	Mar	Apr	May
600	300	800	100	500

Multi-Period Planning

- You own a wheat warehouse with a capacity of 20,000 bushels. At the beginning of month 1, you have 6,000 bushels of wheat. Each month, wheat can be bought and sold at the price per 1000 bushels given in the table.
- The sequence of events during each month is as follows:
  - You observe your initial stock of wheat.
  - You can sell any amount of wheat up to your initial stock at the current month's selling price.
  - You can buy (at the current month's buying price) as much wheat as you want, subject to the warehouse size limitation.
- Your goal is to formulate an LP that can be used to determine how to maximize the profit earned over the next 10 months.

Month	Sales Price $	Purchase Price $
1	3	8
2	6	8
3	7	2
4	1	3
5	4	4
6	5	3
7	5	3
8	1	2
9	3	5
10	2	5

Production Planning

- ATK-White manufactures three products *P1*, *P2*, and *P3* on two machines *M1* and *M2*. Each of the products must be processed on both machines in arbitrary order. Operators are required to operate the machines. 0.5 hours of operator hour is required to operate a machine for one hour. The unit profits of the products are 8TL, 11TL, and 7TL, respectively, the machine capacities are 40 and 30 hours per planning period, and 30 operator hours available per planning period. The following Table indicates *how many units of the products can be made each hour*.
- In addition, it is required that at least fifteen units of the second product are made. Formulate a profit-maximizing linear programming problem.

	P1	P2	P3
M1	4	6	9
M2	7	3	13

Cash Flow Problem

- ITU Cafeteria enterprise $1,000 in available cash. At the beginning of each of the next six months, they will receive revenues and pay bills as shown in the table. It is clear that the firm will have a short-term cash flow problem until the opening of the university. To solve this problem, ITU Cafe must borrow money.
  - At the beginning of May, ITU Cafe may take out a six-month loan. Any money borrowed for a six-month period must be paid back at the end of October along with 9% interest (early payback does not reduce the interest cost of the loan).
  - ITU Cafe may also meet cash needs through month-to-month borrowing. Any money borrowed for a one-month period incurs an interest cost of 4% per month.
- Use linear programming to determine how ITU Cafe can minimize the cost of paying its bills on time.

Month	Revenues ($)	Bills ($)
May	1000	5000
June	2000	5000
July	2000	6000
August	4000	2000
September	7000	2000
November	9000	1000

Fuel replenishment

- Transeast Airlines flies planes on the following route: L.A.–Houston–N.Y.–Miami–L.A. The length (in miles) of each segment of this trip is as follows: L.A.–Houston, 1,500 miles; Houston–N.Y., 1,700 miles; N.Y.–Miami, 1,300 miles; Miami–L.A., 2,700 miles. At each stop, the plane may purchase up to 10,000 gallons of fuel. The price of fuel at each city is as follows: L.A., 88¢; Houston, 15¢; N.Y., $1.05; Miami, 95¢. The plane's fuel tank can hold at most 12,000 gallons. To allow for the possibility of circling over a landing site, we require that the ending fuel level for each leg of the flight be at least 600 gallons. The number of gallons used per mile on each leg of the flight is
  - 1 + (average fuel level on leg of flight/2,000)
- To simplify matters, assume that the average fuel level on any leg of the flight is
  - [(Fuel level at start of leg) + (fuel level at end of leg)] / 2
- Formulate an LP that can be used to minimize the fuel cost incurred in completing the schedule.

Maximum of the numbers

- Suppose a_1, a_2,..., a_n, are decision variables in a LP model.
- If the objective is to minimize the maximum of these variables, how could you formulate the LP model?