Operations Research
Graphical Solution

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Solving LPs

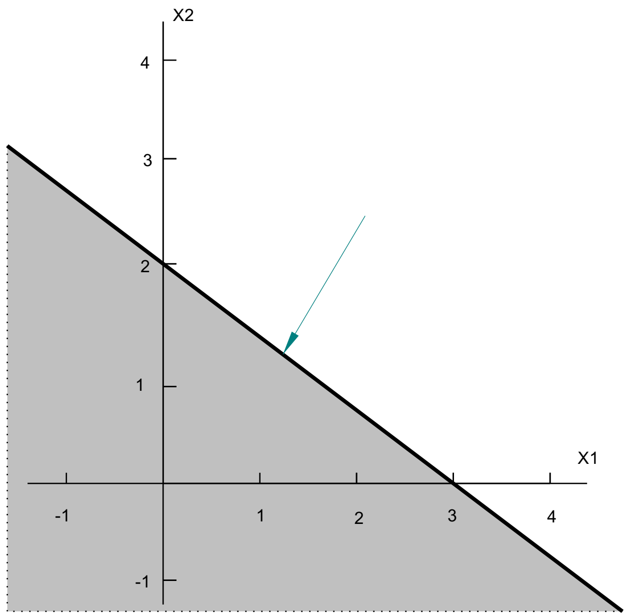
- The Graphical Solution
- The Simplex Algorithm
- Using Software

LP Solutions: Four Cases

- The LP has a unique optimal solution.
- The LP has alternative (multiple) optimal solutions.
  - It has more than one (actually an infinite number of) optimal solutions.
- The LP is infeasible.
  - It has no feasible solutions. The feasible region contains no points.
- The LP is unbounded.
  - In the feasible region there are points with arbitrarily large (in a max problem) objective function values.

The Graphical Solution

- Any LP with only two variables can be solved graphically.



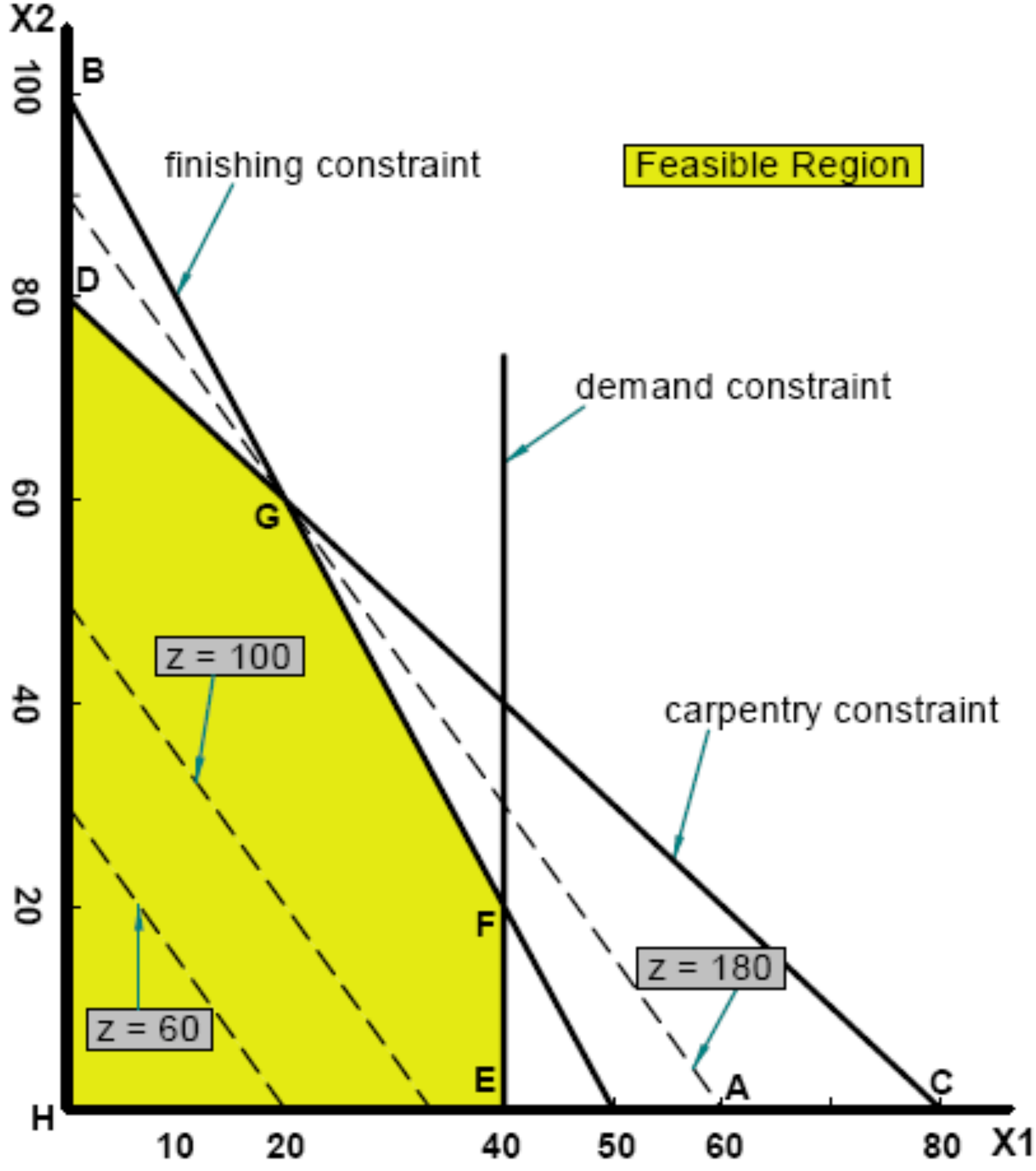
The Graphical Solution
Giapetto Example

- Giapetto Example
- Since the Giapetto LP has two variables, it may be solved graphically.
- The feasible region is the set of all points satisfying the constraints.

max $z = 3x_1 + 2x_2$

s.t.	$2x_1 + x_2 \leq 100$	(Finishing constraint)
	$x_1 + x_2 \leq 80$	(Carpentry constraint)
	$x_1 \leq 40$	(Demand constraint)
	$x_1, x_2 \geq 0$	(Sign restrictions)

The set of points satisfying the LP is bounded by the five sided polygon DGFEH. Any point **on** or **in** the interior of this polygon (the shade area) is in the **feasible region**.



Isoprofit / Isocost

- Having identified the feasible region for the LP, a search can begin for the **optimal solution** which will be the point in the feasible region with the *largest* z-value (maximization problem).

- To find the optimal solution, graph a line on which the points have the same z-value. In a max problem, such a line is called an **isoprofit** line while in a min problem, this is called the **isocost** line

- (*The figure shows the isoprofit lines for z = 60, z = 100, and z = 180*)

Binding

- A constraint is ***binding*** (active, tight) if the left-hand and right-hand side of the constraint are equal when the optimal values of the decision variables are substituted into the constraint.
  - The finishing and carpentry constraints are binding.

- A constraint is ***nonbinding*** (inactive) if the left-hand side and the right-hand side of the constraint are unequal when the optimal values of the decision variables are substituted into the constraint.
  - The demand constraint for wooden soldiers is nonbinding since at the optimal solution $x_1 < 40$ ($x_1 = 20$).

Advertisement Example

(Winston 3.2, p.61)

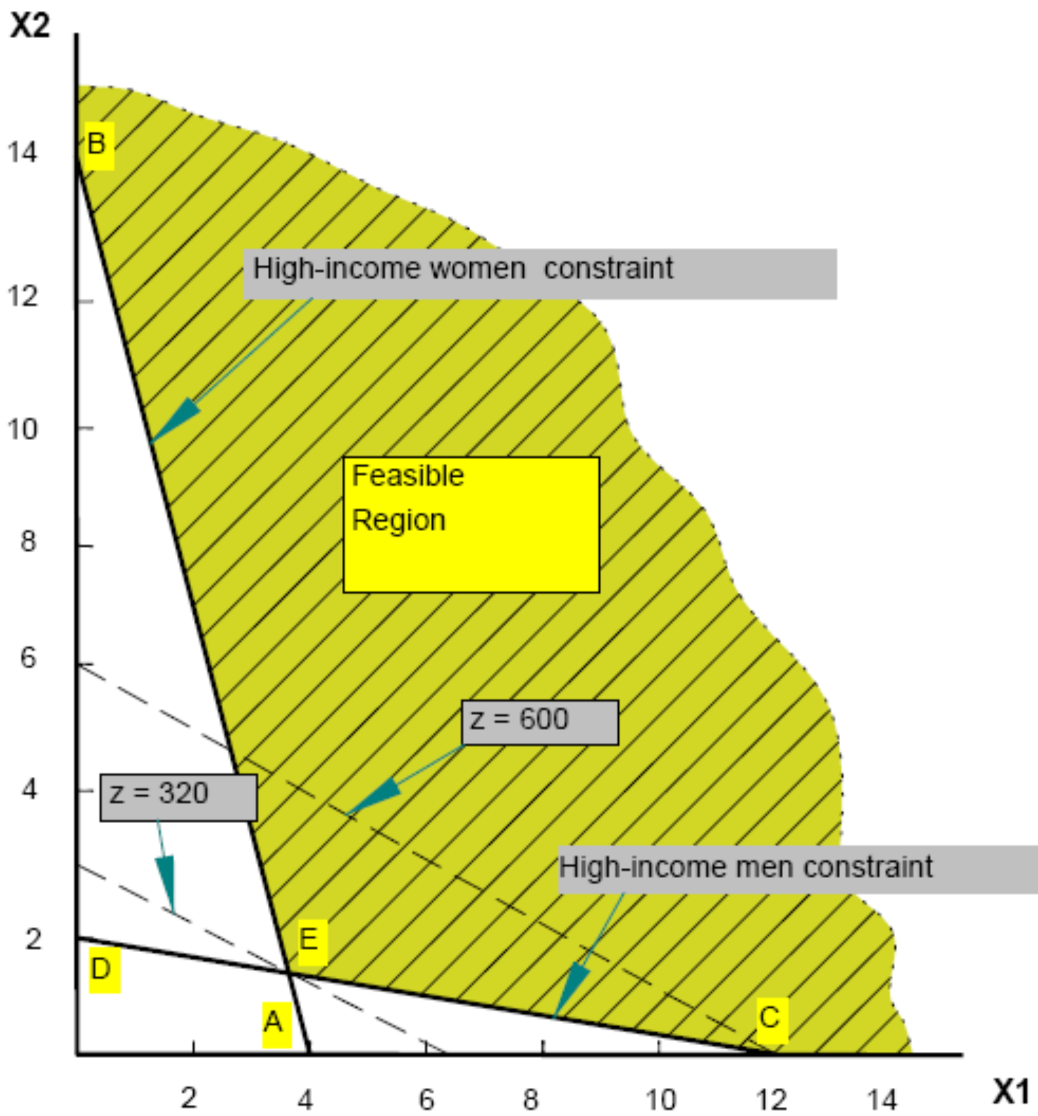
- Since the Advertisement LP has two variables, it may be solved graphically.
- The feasible region is the set of all points satisfying the constraints.

min z =$50x_1 + 100x_2$

s.t. $7x_1 + 2x_2 \geq 28$ (high income women)

$2x_1 + 12x_2 \geq 24$ (high income men)

$x_1, x_2 \geq 0$



X2
14
12
10
8
6
4
2
B
High-income women constraint
Feasible
Region
z = 600
z = 320
High-income men constraint
E
D
A
C
2
4
6
8
10
12
14
X1

Advertisement Example Solution

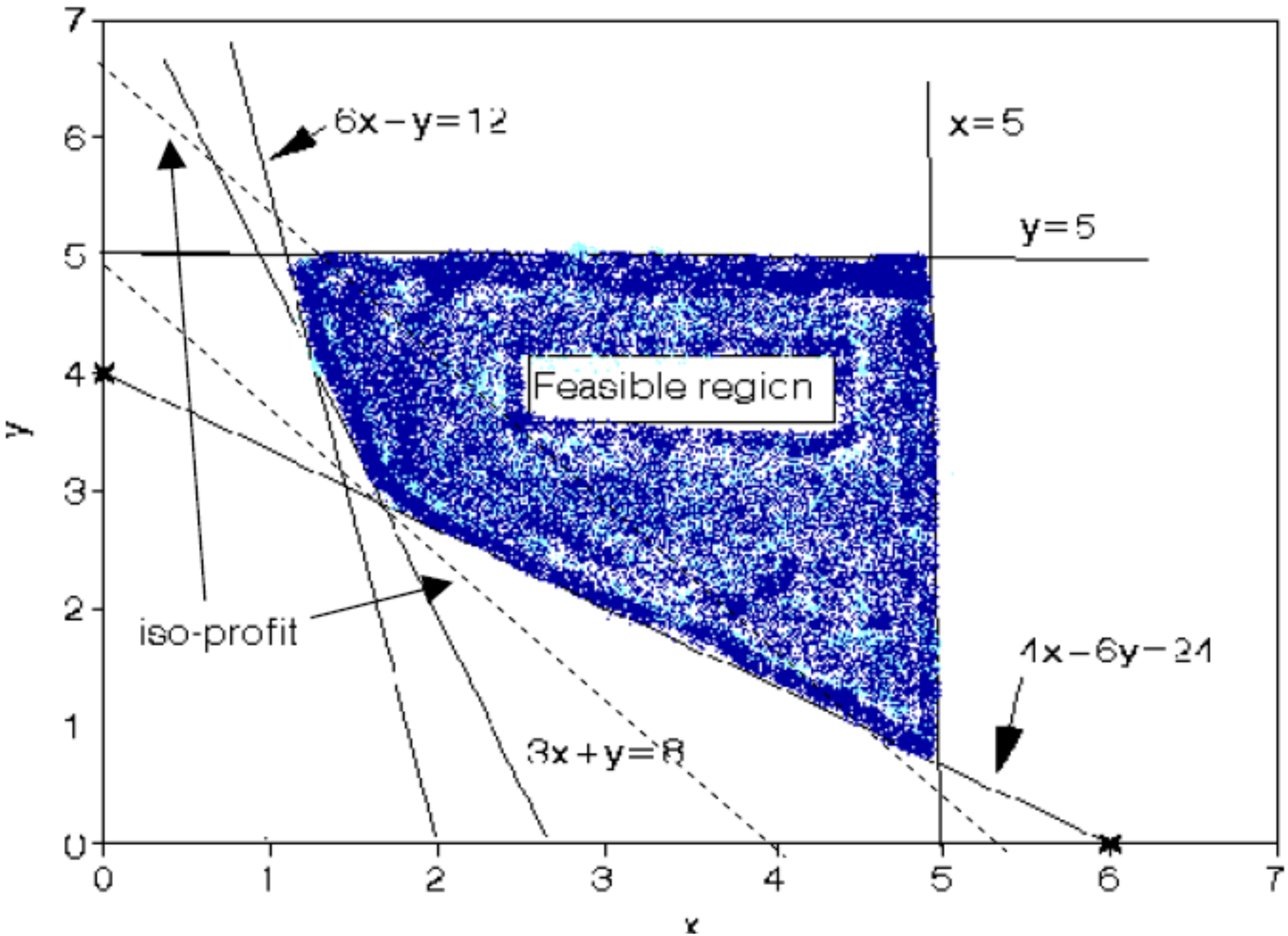
- Since Dorian wants to minimize total advertising costs, the optimal solution to the problem is the point in the feasible region with the *smallest* z value.
- An isocost line with the smallest z value passes through point *E* and is the optimal solution at

  $x_1 = 3.6$ and $x_2 = 1.4$ giving $z = 320$

- Both the high-income women and high-income men constraints are satisfied, both constraints are binding.

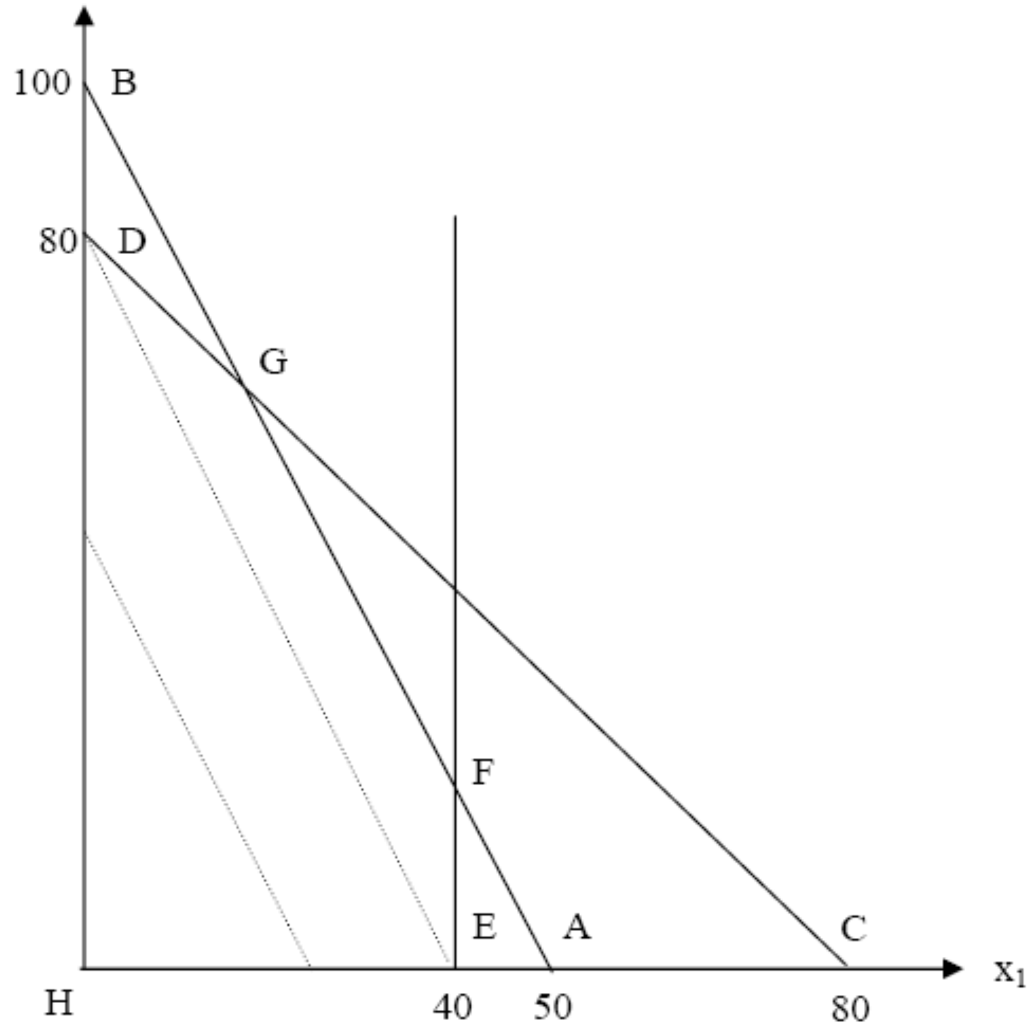
Two Mines Example

- min $180x + 160y$
- S.t.
  - $6x + y >= 12$
  - $3x + y >= 8$
  - $4x + 6y >= 24$
  - $x <= 5$
  - $y <= 5$
  - $x,y >= 0$


6x − y = 12
x = 5
y = 5
Feasible region
iso-profit
4x − 6y − 24
3x + y = 8
x
y
0
1
2
3
4
5
6
7

Modified Giapetto (1)

- maks $z = 4x1 + 2x2$
- s.t.
  - $2x1 + x2 \leq 100$ *(Finishing constraint)*
  - $x1 + x2 \leq 80$ *(Carpentry constraint)*
  - $x1 \leq 40$ *(Demand constraint)*
  - $x1, x2 \geq 0$ *(Sign restrictions)*



Points on the line between points G (20, 60) and F (40, 20) are the **alternative optimal solutions**. (z=200)

Thus, for $0 \le c \le 1$,

c [20 60] + (1-c) [40 20]

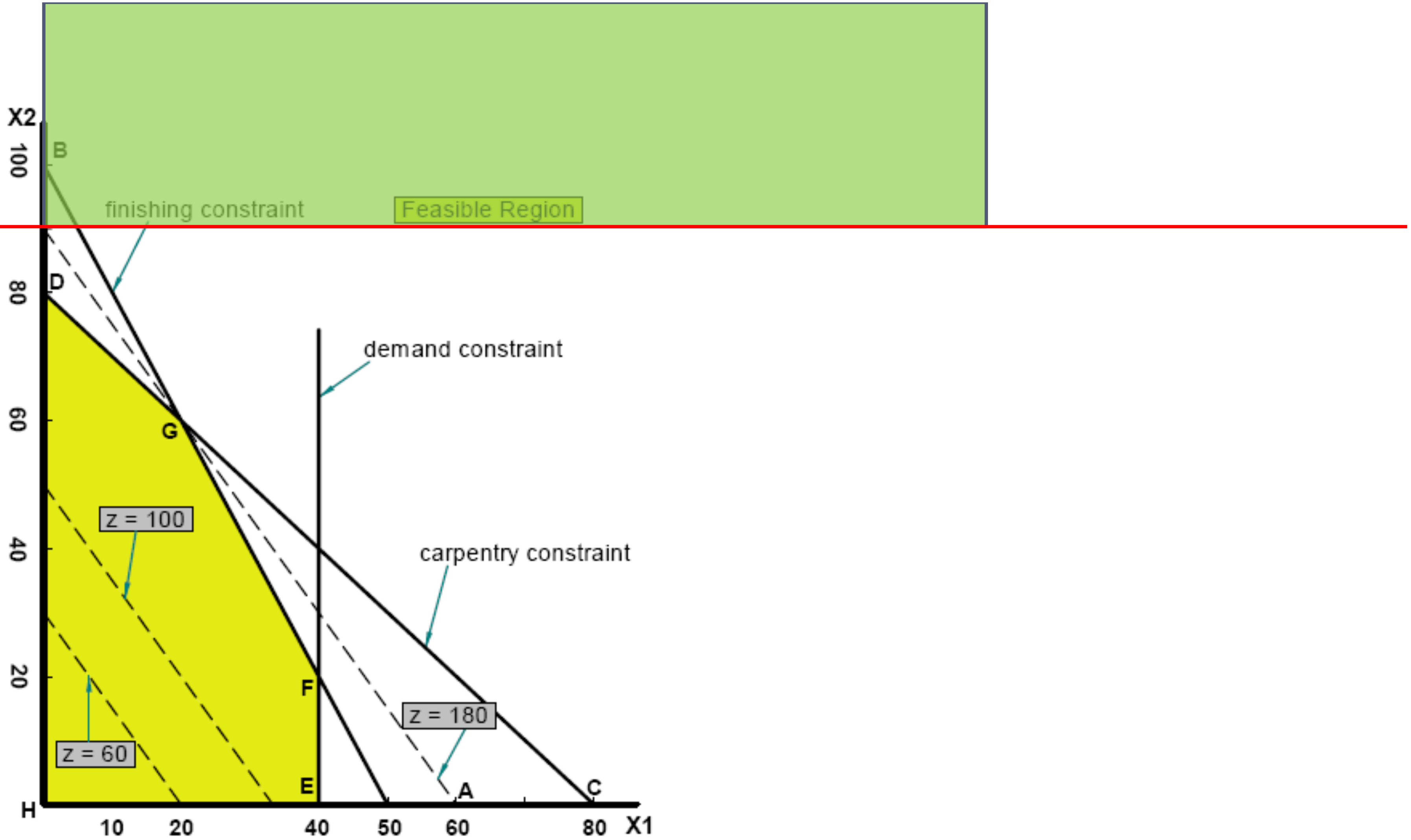
= [40-20c, 20+40c]

will be optimal

Modified Giapetto (2)

- max $z = 3x1 + 2x2$
- s.t.
  - $2x1 + x2 \leq 100$ *(Finishing constraint)*
  - $x1 + x2 \leq 80$ *(Carpentry constraint)*
  - $x1 \leq 40$ *(Soldier Demand constraint)*
  - $x2 \geq 90$ *(Train demand constraint)*

  - $x1, x2 \geq 0$ *(Sign restrictions)*



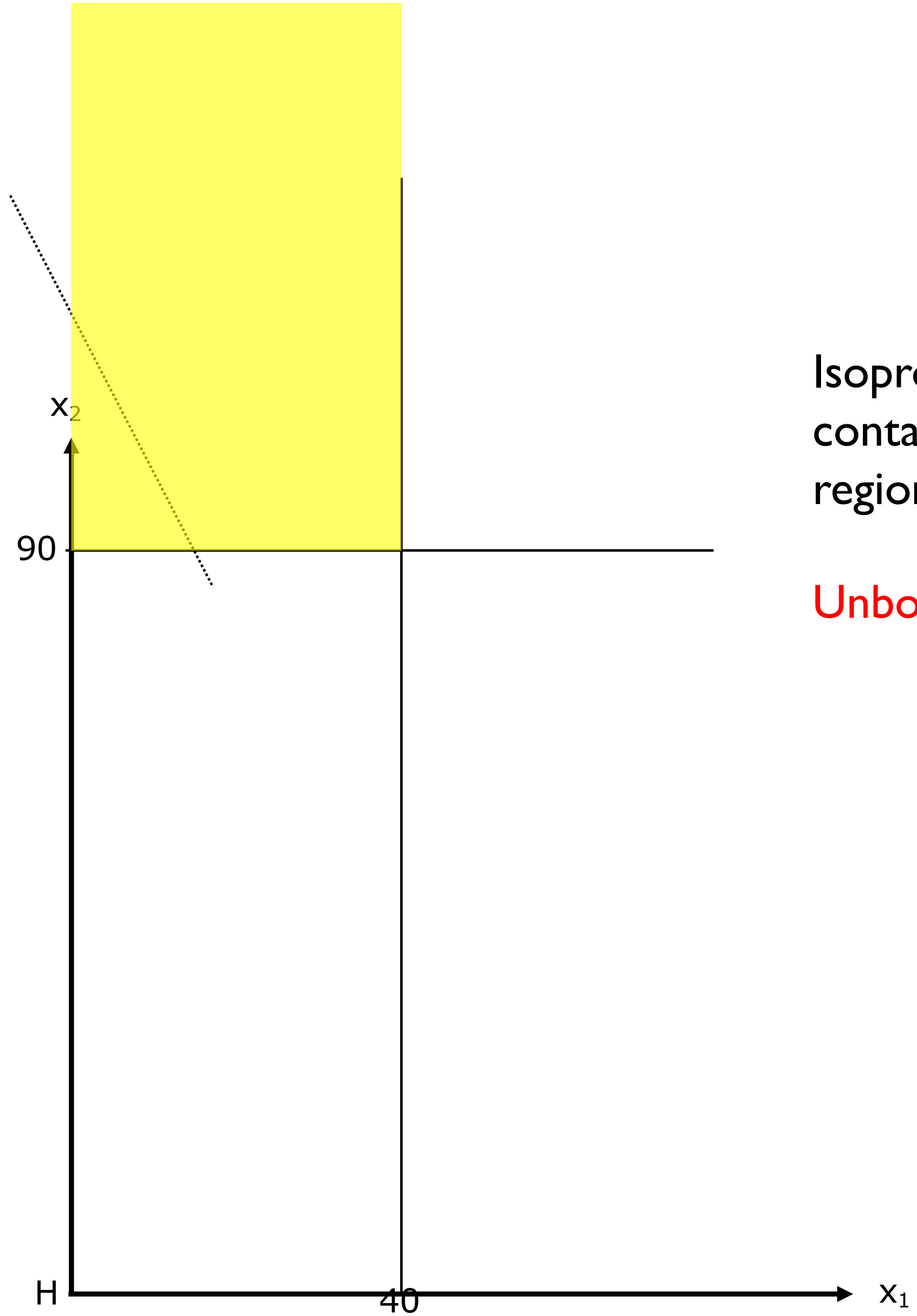
No points satisfy all constraints

Infeasible LP

Modified Giapetto (3)

- max $z = 3x1 + 2x2$
- s.t.
  - ~~$2x1 + x2 \leq 100$ (Finishing constraint)~~
  - ~~$x1 + x2 \leq 80$ (Carpentry constraint)~~
  - $x1 \leq 40$ *(Soldier Demand constraint)*
  - $x2 \geq 90$ *(Train demand constraint)*

  - $x1, x2 \geq 0$ *(Sign restrictions)*



Isoprofit line never lose contact with the feasible region:

Unbounded LP!