# Operations Research Software Packages

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# Software Packages

- Lindo (https://www.lindo.com/index.php/ls-downloads)
  - Download classic Lindo

- Open Solver (https://opensolver.org/ )

- Download them from the link:
  - http://web.itu.edu.tr/~topcuil/ya/yonyaz.htm

# Capital Budgeting

- A company has nine projects under consideration. The NPV added by each project and the capital required by each project during the next two years is given in the following Table. All figures are in millions.
  - E.g. Project 1 will add $14 million in NPV and require expenditures of $12 million during year 1 and $3 million during year 2.
- $50 million is available for projects during year 1
- $20 million is available during year 2.
- Assuming we may undertake a fraction of each project, how can we maximize NPV?

| | Project | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Year 1 Outflow | 12 | 54 | 6 | 6 | 30 | 6 | 48 | 36 | 18 |
| Year 2 Outflow | 3 | 7 | 6 | 2 | 35 | 6 | 4 | 3 | 3 |
| NPV | 14 | 17 | 17 | 15 | 40 | 12 | 14 | 10 | 12 |

# Capital Budgeting

- Decision Variables:
- 
- $x_i$ : investment ratio to project $i$ ($i$= 1,2,…,9.)
- 
- Parameters:
- $N_i$ : NPV of project $i$.
- $H_{ji}$: outflow for project $i$. in year j. $j$ = 1,2.
- $B_j$ : budget available in year $j$.
- 
- Objective Function
- Max Z = $\sum_i^9 N_i x_i$
- 
- Subject to:
- $\sum_i^9 H_{ji} x_i \leq B_j$ $j$ = 1,2.
- $x_i \leq 1$ $i$= 1,2,…,9.
- $x_i \geq 0$ $i$= 1,2,…,9.

# Lindo Model

```
max 14x1+ 17x2 + 17x3 + 15x4 + 40x5 + 12x6 + 14x7 + 10x8 + 12x9
st
12x1 + 54x2 + 6x3 + 6x4 + 30x5 + 6x6 + 48x7 + 36x8 + 18x9  <50
3x1 + 7x2 + 6x3 + 2x4 + 35x5 + 6x6 + 4x7 + 3x8 + 3x9  <20
x1<1
x2<1
x3<1
x4<1
x5<1
x6<1
x7<1
x8<1
x9<1
end
```

# Lindo

- MAX or Min {objective function}
- St
- {constraint lefthand side} <, = or > {righthand side}
- END

- Unrestricted in sign variables:
  - Free X1
- Integer variables:
  - GIN X2
- 0-1 Binary variables:
  - INT x2

- MAX 20 X + 14 Y + 13 Z
- SUBJECT TO
- 2) 17 X + 13 Y + 11 Z <= 93
- END

- FREE <*Variable*>
  - Free x
  - Free y

- INTEGER <*Variable* | *NumberOfVars*>
- GIN <*Variable* | *NumberOfVars*>
  - **gin x** **inte x**
  - **gin y** **int y**
  - **gin z** **integer z**
- **or**
  - **gin 3** **inte 3**

# Open Solver

- The key to solving an LP on a spreadsheet is to set up a spreadsheet that tracks everything of interest (costs or profits, resource usage, etc.).
- Next, identify the cells of interest that can be varied. These are called **variable cells.**
- After defining the changing cells, identify the cell that contains your objective function as the **objective cell.**
- Next, we identify and insert our constraints.
- Finally, tell the OpenSolver to Solve the problem.
- At this point, the optimal solution to our problem will be placed in the spreadsheet.

# OpenSolver – Example 1
# Capital Budgeting

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# OpenSolver – Example 2
# Dakota Furniture

- Let $x_1$, $x_2$, $x_3$ be the number of desks, tables and chairs produced.
- Let the weekly profit be $z.

$$
\begin{aligned}
\max z &= 60x_1 + 30x_2 + 20x_3 \\
\text{s.t.} \quad & 8x_1 + 6x_2 + x_3 \leq 48 \\
& 4x_1 + 2x_2 + 1.5x_3 \leq 20 \\
& 2x_1 + 1.5x_2 + .5x_3 \leq 8 \\
& x_2 \leq 5 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

# Additional Example 1

- ATK-White manufactures three products P1, P2, and P3 on two machines M1 and M2. Each of the products must be processed on both machines in arbitrary order. Operators are required to operate the machines. 0.5 hours of operator hour is required to operate a machine for one hour. The unit profits of the products are 8TL, 11TL, and 7TL, respectively, the machine capacities are 40 and 30 hours per planning period, and 30 operator hours available per planning period. The following table indicates how many units of the products can be made each hour.
- In addition, it is required that at least fifteen units of the second product are made. Formulate a profit-maximizing LP model.

| | P1 | P2 | P3 |
|---|---|---|---|
| M1 | 4 | 6 | 9 |
| M2 | 7 | 3 | 13 |

# Additional Example 1 (LP formulation)

- Decision variables
  - $x_1$, $x_2$, and $x_3$ as the quantities of the products to be manufactured
-
- Objective is to maximize the total profit
  - Max $8x_1 + 11\ x_2 + 7x_3$
-
- Subject to
  - $x_1/4 + x_2/6 + x_3/9 \leq 40$ (machine 1 capacity)
  - $x_1/7 + x_2/3 + x_3/13 \leq 30$ (machine 2 capacity)
  - $[(x_1/4 + x_2/6 + x_3/9) + (x_1/7 + x_2/3 + x_3/13)]*0.5 \leq 30$ (available operator hours)
  - $x_2 \geq 15$ (at least 15 units of product 2)
  - $x_1, x_2, x_3 \geq 0$ (sign restrictions)

# Additional Example 2

- AEK-Pink Shoe store has $5,000 in available cash. At the beginning of each of the next seven months, AEK will receive revenues and pay bills as shown in the following table. It is clear that AEK will have a short-term cash flow problem until the store receives revenues from the spring season. To solve this problem, AEK must borrow money.
- At the beginning of November 2017, AEK may take out a seven-month loan. Any money borrowed for a seven-month period must be paid back at the end of May 2018 along with 10% interest (early payback does not reduce the interest cost of the loan). AEK may also meet cash needs through month-to-month borrowing. Any money borrowed for a one-month period incurs an interest cost of 4% per month. Formulate a linear program to determine how AEK can minimize the cost of paying its bills on time.

| | Revenue ($) | Bills ($) |
|---|---|---|
| November 2017 | 5,000 | 30,000 |
| December 2017 | 12,000 | 40,000 |
| January 2018 | 16,000 | 43,000 |
| February 2018 | 20,000 | 9,000 |
| March 2018 | 35,000 | 22,000 |
| April 2018 | 45,000 | 7,000 |
| May 2018 | 50,000 | 11,000 |

# Additional Example 2 (LP formulation)

- <u>Define decision variables:</u>
- *y*: amount of seven-month loan barrowed at the beginning of November 2017
- $x_i$: amount of one month loan barrowed at the beginning of month *i* (*i*=1, 2,3,4,5,6,7)
- $k_i$: cash available at the end of month *i* (*i*=1, 2,3,4,5,6,7)

- Define $r_i$ and $b_i$ as parameters indicating revenue and bills in month i, respectively.

- Min $0.10y + 0.04\sum_{i=1}^{7} x_i$

- Subject to
- $5 + y + x_1 + 5 = b_1 + k_1$ (month 1 cash flow)
- $k_{i-1} + x_i + r_i = b_i + 1.04x_{i-1} + k_i$ for i = 2,..,7 (month 2 to 7)
- $k_7 \geq 1.1y + 1.04x_7$ (end of month 7)

- All variables ≥ 0

# Additional Example 3

- Larry Edison is the director of the Computer Center for Buckly College. He now needs to schedule the staffing of the center. It is open from 8 A.M. until midnight. Larry has monitored the usage of the center at various times of the day, and determined that the following number of computer consultants are required:

| Time of Day | Minimum number of consultants required to be on duty |
| --- | --- |
| 8 A.M. – noon | 4 |
| Noon – 4 P.M. | 8 |
| 4 P.M. – 8 P.M. | 10 |
| 8P.M. – midnight | 6 |

- Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for 8 consecutive hours in any of the following shifts: morning (8 A.M.–4 P.M.), afternoon (noon–8 P.M.), and evening (4 P.M.–midnight). Full-time consultants are paid $14 per hour.
- Part-time consultants can be hired to work any of the four shifts listed in the above table. Part-time consultants are paid $12 per hour.
- An additional requirement is that during every time period, there must be at least 2 full-time consultants on duty for every parttime consultant on duty.
- Larry would like to determine how many full-time and how many part-time workers should work each shift to meet the above requirements at the minimum possible cost.
- Formulate a linear programming model for this problem.

# Additional Example 3 (LP formulation)

- Decision variables:
- $f_i$ : fulltime consultants in shift $i$, $i$ = 1 (8 A.M.–4 P.M.), 2 (noon–8 P.M.), 3 (4 P.M.–midnight).
- $p_j$ : part time consultants in shift $j$, $j$ = 1 (8 A.M. – noon), 2 (Noon – 4 P.M.), 3(4 P.M. – 8 P.M.), 4(8P.M. – midnight)
-
- objective function:
- Min Z = $14 * 8 * \sum_{i=1}^{3} f_i + 12 * 4 * \sum_{j=1}^{4} p_i$
- Constrains:
- $p_1 + f_1 \geq 4$ $\quad f_1 \geq 2\, p_1$
- $p_2 + f_1 + f_2 \geq 8\; f_1 + f_2 \geq 2\, p_2$
- $p_3 + f_2 + f_3 \geq 10$ $\quad f_2 + f_3 \geq 2\, p_3$
- $p_4 + f_3 \geq 6$ $\quad f_3 \geq 2\, p_4$
- $f_i, p_j \geq 0.\ \forall i, \forall j.$