Lecture 11

Premixed Turbulent Combustion: The Regime Diagram



Regimes in Premixed Turbulent Combustion

Diagrams defining regimes of premixed turbulent combustion in terms of velocity and length scale ratios have been proposed by Borghi (1985), Peters (1986), Abdel-Gayed and Brandley (1989), Poinsot et al. (1990) and many others.

For scaling purposes it is useful to assume equal diffusivities for all reactive scalars, a Schmidt number of unity

$$Sc = \nu/D \stackrel{!}{=} 1$$

and to define the flame thickness and the flame time as

$$\ell_F = \frac{D}{s_L}, \quad t_F = \frac{D}{s_L^2}$$



Then, using v = D, the turbulent intensity and the turbulent length scale introduced in Lecture 10, we define the turbulent Reynolds number as

$$\operatorname{Re} = \frac{v'\ell}{s_L\ell_F}$$

and the turbulent Damköhler number

$$\mathsf{Da} = \frac{s_L \ell}{v' \ell_F}$$

Furthermore, with the Kolmogorov time, length, and velocity scales defined in Lecture 10, we introduce two turbulent Karlovitz numbers.

The first one defined as

$$\mathsf{Ka} = \frac{t_F}{t_\eta} = \frac{\ell_F^2}{\eta^2} = \frac{v_\eta^2}{s_L^2}$$

measures the ratios of the flame scales in terms of the Kolmogorov scales.



Using the definitions

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}, \quad t_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2}, \quad v_\eta = (\nu\varepsilon)^{1/4}$$

with v = D and

$$\varepsilon \sim \frac{v_n^2}{t_n} \sim \frac{v_n^3}{\ell_n} \sim \frac{\ell_n^2}{t_n^3}$$

taken as equality it is seen that

$$\operatorname{Re} = \frac{v'\ell}{s_L\ell_F}, \quad \operatorname{Da} = \frac{s_L\ell}{v'\ell_F}, \quad \operatorname{Ka} = \frac{t_F}{t_\eta} = \frac{\ell_F^2}{\eta^2} = \frac{v_\eta^2}{s_L^2}$$

can be combined to show that

$$Re = Da^2 Ka^2$$



Referring to the discussion about the appropriate reaction zone thickness δ in twosteps premixed flames in Lecture 6, a second Karlovitz number Ka_{δ} may be introduced as

$$\mathsf{Ka}_{\delta} = \frac{\ell_{\delta}^2}{\eta^2} = \delta^2 \mathsf{Ka}$$

where for the reaction zone thickness

$$\ell_{\delta} = \delta \, \ell_F$$

has been used.



 $\log(v'/s_L)$ versus $\log(\ell/\ell_F)$

Using

$$\mathsf{Ka} = \frac{\ell_F^2}{\eta^2} \qquad \text{and} \qquad \mathsf{Re} = \frac{v'\ell}{s_L\ell_F}$$

where for scaling purposes we have set $\varepsilon \equiv \frac{v'^3}{\ell}$, such that the

Kolmogorov length scale squared becomes $\eta^2 = \left(\frac{\nu^3}{\varepsilon}\right)^{1/2} = \left(s_L^3 \ell_F^3 \frac{\ell}{\nu'^3}\right)^{1/2}$

the ratios v'/s_L , ℓ/ℓ_F may be expressed in terms of Re and Ka as

$$\frac{v'}{s_L} = \operatorname{Re}\left(\frac{\ell}{\ell_F}\right)^{-1} = \operatorname{Ka}^{2/3}\left(\frac{\ell}{\ell_F}\right)^{1/3}$$



Using these relations the lines Re = 1, Ka = 1 represent boundaries between different regimes of premixed turbulent combustion.

Other boundaries of interest are the line

$$v'/s_L = 1$$

which separates the

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wrinkled flamelets from the corrugated flamelets, and the line denoted by

 $Ka_{\delta}=1$, which separates thin reaction zones from broken reaction zones.

The line Re = 1 separates all turbulent flame regimes Characterized by Re > 1from the regime of laminar flames (Re < 1), which is situated in the lower-left corner of the diagram. We will consider

turbulent combustion for

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large Reynolds numbers, which corresponds to a region sufficiently removed from

the line Re = 1 towards the upper r.h.s.



We will not consider the wrinkled flamelet regime, because it is not of much practical interest.

In that regime, where $v' < s_L$, the turn-over velocity v' of even the large eddies is not large enough to compete with the advancement of the



flame front with the laminar burning velocity s_L . Laminar flame propagation therefore is dominating over flame front corrugations by turbulence.

We will also not consider the broken reaction zones Regime in any detail for reasons to be discussed at the end of this lecture.

Among the remaining two regimes, the corrugated flamelets regime is characterized by the inequalities $v' > s_L$, Re > 1 and Ka_{δ} < 1.

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The corrugated flamelet regime



$$\mathsf{Ka} = \frac{t_F}{t_\eta} = \frac{\ell_F^2}{\eta^2} = \frac{v_\eta^2}{s_L^2}$$

the Ka < 1 indicates that

$$\ell_F < r$$

which means that the entire reactive-diffusive flame structure

of thickness ℓ_F is embedded within eddies of the size of the Kolmogorov scale, where the flow is quasi-laminar.





The corrugated flamelet regime

Therefore the flame structure is not perturbed by turbulent fluctuations and remains quasi-steady.

The boundary of the corrugated flamelets regime to the thin reaction zones regime is given by Ka = 1, which, according to

$$\ell_F \equiv \eta$$

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is equivalent to the condition that the flame thickness is equal to the Kolmogorov length scale. This is called the Klimov-Williams criterion.

The thin reaction zones regime is characterized by Re > 1, $\text{Ka}_{\delta} < 1$, and Ka > 1.

Ka >1 indicates that the smallest eddies of size η can enter into the reactivediffusive flame structure since

 $\eta < \ell_F$

These small eddies are still larger than the inner layer thickness

 $\eta > \ell_{\delta} = \delta \ell_F$

and can therefore not penetrate into that layer.



The non-dimensional thickness δ of the inner layer in a premixed flame is typically one tenth.

Therefore the inner layer thickness is one tenth of the preheat zone thickness, which is of the same order of magnitude as the flame thickness.

Using

$$\mathsf{Ka}_{\delta} = \frac{\ell_{\delta}^2}{\eta^2} = \delta^2 \mathsf{Ka}$$

we see that the line $Ka_{\delta} = 1$ corresponds with $\delta = 0.1$ to Ka=100.



This value is used for the upper limit of the thin reaction zones regime.

It seems roughly to agree with the flamelet boundary obtained in numerical studies, where 2D interactions between a laminar premixed flame front and a vortex pair were analyzed Poinsot (1991).

These simulations correspond to Ka=180 for cases without heat loss

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and Ka=25 with small heat loss. The authors argued that since quenching by vortices occurs only for larger Karlovitz numbers, the region below the limiting value of the Karlovitz number should correspond to the flamelet regime.

We will now enter into a more detailed discussion of the two flamelet regimes.

In the regime of corrugated flamelets there is a kinematic interaction between turbulent eddies and the advancing quasi-laminar flame structure.

With Ka < 1 we have: $v' \ge s_L \ge v_\eta$

To determine the size of the eddy that interacts locally with the flame front, we set the turn-over velocity $v_n = s_L$ in

$$\varepsilon \sim \frac{v_n^2}{t_n} \sim \frac{v_n^3}{\ell_n} \sim \frac{\ell_n^2}{t_n^3}$$

This determines the the Gibson scale (cf. Peters ,1986) as $\ell_G = \frac{s_L^3}{c}$



Eddies of the size of the Gibson scale

$$\ell_G = \frac{s_L^3}{\varepsilon}$$

which have a turnover velocity $v_n = s_L$ can interact with the flame front.

Since the turn-over velocity of the

large eddies is larger than the

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laminar burning velocity, these eddies will push

the flame front around, causing a substantial corrugation.

Smaller eddies of size $\ell_n < \ell_G$ having a turnover velocity smaller than s_L will not even be able to wrinkle the flame front.

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With $\varepsilon \sim v'^3/\ell$

one may also write

$$\frac{\ell_G}{\ell} = \left(\frac{s_L}{v'}\right)^3$$

The Gibson scale may be illustrated graphically within the inertial range.

Here, following Kolmogorov scaling in the inertial range given by

$$\varepsilon \sim \frac{v_n^3}{\ell_n}$$



the logarithm of the velocity v_n is plotted over the logarithm of the length scale ℓ_n .



- We assume v' and ℓ and thereby ε , and also v and thereby v_{η} and η to be fixed.
- If one enters on the vertical axis with the burning velocity $s_L = v_n$ into the diagram, one obtains the Gibson scale as the corresponding length scale on the horizontal axis.



The laminar flame thickness ℓ_F , which is smaller than the Kolmogorov scale η in the corrugated flamelets regime is also shown.



This diagram illustrates the limiting values of the Gibson scale:

If the burning velocity is equal to v', the Gibson scale is equal to the integral length scale:

$$s_l = v' \quad \Rightarrow \quad \ell_G = \ell$$

This case corresponds to the borderline between corrugated and wrinkled flamelets in the regime diagram.



Conversely, if the burning velocity is equal to the Kolmogorov velocity, the Gibson scale is equal to the Kolmogorov scale,

$$s_l = v_\eta \quad \Rightarrow \quad \ell_G = \eta$$

which corresponds to the line Ka = 1 in the regime diagram.



It has been shown by Peters (1992) that the Gibson scale is the lower cut-off scale of the scalar spectrum function in the corrugated flamelets regime.

At that cut-off there is only a weak change of slope in the scalar spectrum function.

This is the reason why the Gibson scale is difficult to measure.

The stronger diffusive cut-off occurs at the Obukhov-Corrsin scale defined by

$$\ell_C = \left(\frac{D^3}{\varepsilon}\right)^{1/4}$$

Since we have assumed D = v this scale is equal to the Kolmogorov scale.

$$D = \nu \quad \Rightarrow \quad \ell_C = \eta$$



The next flamelet regime in the regime diagram is the regime of thin reaction zones.

As noted earlier, in the thin reaction zone regime

 $\eta < \ell_F$

small eddies can enter into the preheat zone and increase scalar mixing,

However these eddies cannot penetrate into the inner layer since

 $\eta > \ell_\delta$

The burning velocity is smaller than the Kolmogorov velocity which would lead to a Gibson scale that is smaller than the Kolmogorov scale.



A time scale, however, can be used in the thin reaction zones regime to define a characteristic length scale using Kolmogorov scaling in the inertial range.

That time scale should represent the response of the thin reaction zone and the surrounding diffusive layer to unsteady perturbations.

The appropriate time is the as the flame time t_F .

Combining the flame time with the diffusivity D, the resulting diffusion thickness

$$\ell_D = \sqrt{D t_F}$$

is then of the order of the flame thickness:

$$\ell_D = \sqrt{D t_F} = \mathcal{O}(\ell_F)$$



By setting $t_n = t_F$ and $\ell_m = \ell_n$ in

$$\varepsilon \sim \frac{\ell_n^2}{t_n^3}$$

one obtains the length scale

$$\ell_m = (\varepsilon t_F^3)^{1/2}$$

An appropriate interpretation is that of a mixing length scale, which has been advocated based on the concept of thin reaction zones by Peters (1999).

It is the size of an eddy within the inertial range which has a turnover time equal to the time needed to diffuse scalars over a distance equal to the diffusion thickness.

$$t_m = t_D$$



During its turnover time an eddy of size ℓ_m will interact with the advancing reaction front and will be able to transport preheated fluid from a region of thickness ℓ_D in front of the reaction zone over a distance corresponding to its own size.



broadened flame structure at scales larger than ℓ_m .

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The physical interpretation of ℓ_m is therefore that of the maximum distance that preheated fluid can be transported ahead of the flame.

As a mixing length scale ℓ_m had already been identified by Zimont (1979).

Differently from the Gibson length scale the mixing length scale can be observed experimentally.

Changes of the instantaneous flame structure with increasing Karlovitz numbers have been measured by Buschmann et al. (1996) who used 2D-Rayleigh thermometry combined with 2D laser-induced fluorescence on a turbulent premixed Bunsen flame.

They varied the Karlovitz number between 0.03 and 13.6 and observed at Ka > 5 thermal thicknesses that largely exceed the size of the smallest eddies in the flow.

The mixing scale may be illustrated in a log-log plot of t_n over ℓ_n .

If one enters the time axis at $t_F = t_n$, the mixing length scale ℓ_m on the length scale axis is obtained.

If t_F is equal to the Kolmogorov time t_{η} , the mixing length is equal to the Kolmogorov scale.



In this case, one obtains $\ell_m = \ell_D \approx \ell_F$

at the border between the thin reaction zones regime and the corrugated flamelets regime.



Similarly, if the flame time is equal to the integral time

$$t_F = \tau = k/\varepsilon \approx \ell/v'$$

the mixing length is equal to the integral length scale.

This corresponds to Da = 1, which Borghi (1985) interpreted as the $\ell_G \eta = \ell_G \eta$ borderline between two regimes in turbulent combustion.

However, it merely sets a limit for the mixing scale which cannot increase beyond the integral scale.





The diffusion thickness

$$\eta < \ell_D < \ell_m$$

is also indicated.

There also appears the Obukhov-Corrsin scale, which is the lower cut-off scale of the scalar spectrum in the thin reaction zones regime.



Since we have assumed v = D, the Obukhov-Corrsin scale is equal to the Kolmogorov length scale:

$$\ell_C = \eta$$



As a final remark related to the corrugated flamelets regime and the thin reaction zones regime, it is important to realize that turbulence in high Reynolds number turbulence is intermittent and the dissipation ε has a statistical distribution.

This refinement of Kolmogorov's theory has led to the notion of intermittency or "spottiness" of the activity of turbulence in a flow field, Monin and Yaglom (1975).

This may have important consequences on the physical appearance of turbulent flames at sufficiently large Reynolds numbers.

One may expect that the flame front shows manifestations of strong local mixing by small eddies as in the thin reaction zones regime as well as rather smooth regions where corrugated flamelets appear. The two regimes discussed above may therefore both be apparent in the same experimentally observed turbulent flame.



Beyond the line $Ka_{\delta} = 1$ there is a regime called the broken reaction zones regime where Kolmogorov eddies are smaller than the inner layer thickness ℓ_{δ} .

They may therefore enter into the inner layer and perturb it with the consequence that chemistry breaks down locally due to enhanced heat loss to the preheat zone followed by temperature decrease and the loss of radicals.

When this happens the flame will extinguish and fuel and oxidizer will interdiffuse and mix at lower temperatures where combustion reactions have ceased.

In a series of papers Mansour et al. (1992), Chen et al. (1996), Chen and Mansour (1997) and Mansour et al. (1998) have investigated highly stretched premixed flames on a Bunsen burner which were surrounded by a large pilot.



Simultaneous temperature and CH measurements





They found a thin reaction zone, as deduced from the CH profile, and steep temperature gradients in the vicinity of that zone.

There also was evidence of occasional extinction of the reaction zone.

This corresponds to instantaneous shots where the CH profile was absent as in the picture on the upper r.h.s. Such extinction events do not occur in the flame F3 which has a Karlovitz number of 23 and is located in the middle of the thin reaction zones regime.

It can be expected that local extinction events would appear more frequently, if the exit velocity is increased and the flame enters into the broken reaction zones regime.



Local extinction events will occur at an exit velocity close to 75m/s so frequently that the entire flame extinguishes.

Therefore one may conclude that in the broken reaction zones regime a premixed flame is unable to survive.

The measurements also show strong perturbations of the temperature profile on the unburnt side of the reaction zone.

This is most evident in the picture on the lower l.h.s., where the temperature reaches more than 1100 K but falls back to 800 K again.

This seems to be due to small eddies that enter into the preheat zone and confirms the concept of the thin reaction zones regime.



Regimes in Premixed Combustion LES

A similar regime diagram can be constructed for LES using the filter size Δ as the length scale and the subfilter velocity fluctuation v'_{Δ} as the velocity scale.

Such a representation introduces both physical and modeling parameters into the diagram.

A change in the filter size, however, also leads to a change in the subfilter velocity fluctuation. This implies that the effect of the filter size, which is a numerical or model parameter, cannot be studied independently.

In response to this issue, an LES regime diagram for characterizing subfilter turbulence/flame interactions in premixed turbulent combustion was proposed by Pitsch & Duchamp de Lageneste (2002), and was recently extended by Pitsch (2005).





In contrast to the RANS regime diagrams, Δ/ℓ_F and the Karlovitz number Ka are used as the axes of the diagram.

The Karlovitz number, defined as the ratio of the Kolmogorov timescale to the chemical timescale, describes the physical interaction of flow and combustion on the smallest turbulent scales.

It is defined solely on the basis of physical quantities, and is hence independent of the filter size.

The subfilter Reynolds and Damköhler numbers and the Karlovitz number relevant in the diagram are defined as

$$\operatorname{Re}_{\Delta} = \frac{v_{\Delta}' \Delta}{s_L \ell_F}, \quad \operatorname{Da}_{\Delta} = \frac{s_L \Delta}{v_{\Delta}' \ell_F}, \quad \text{and} \quad \operatorname{Ka} = \frac{\ell_F^2}{\eta^2} = \left(\frac{v_{\Delta}'^3 \ell_F}{s_L^3 \Delta}\right)^{1/2}$$



In LES, the Karlovitz number is a fluctuating quantity, but for a given flow field and chemistry it is fixed.

The effect of changes in filter size can therefore easily be assessed at constant Ka number.

An additional benefit of this regime diagram is that it can be used equally well for DNS if Δ is associated with the mesh size.

In the following, the physical regimes are briefly reviewed and relevant issues for LES are discussed.



The effect of changing the LES filter width can be assessed by starting from any one of these regimes at large ratios

$$\Delta/\ell_F\gg 1$$

As the filter width is decreased, the subfilter Reynolds number, Re_{Δ} , eventually becomes smaller than one.

Then the filter size is smaller than the Kolmogorov scale, and no subfilter modeling for the turbulence is required.

However, the entire flame including the reaction zone is only resolved if $\Delta < \delta$.



In the corrugated flamelets regime, if the filter is decreased below the Gibson scale,

$$\Delta < \ell_G$$

which is the smallest scale of the subfilter flame-front wrinkling, the flame-front wrinkling is completely resolved.



It is apparent that in the corrugated flamelet regime, where the flame structure is laminar, the entire flame remains on the subfilter scale, if

$$\Delta/\ell_F>1$$

This is always the case for LES.



In the thin reaction zones regime, the preheat region is broadened by the turbulence. Peters (1999) estimated the broadened flame thickness from the assumption that the timescale of the turbulent transport in the preheat zone has to be equal to the chemical timescale, which for laminar flames leads to the burning velocity scaling given in the beginning of this section.

From this, the ratio of the broadened flame thickness ℓ_m and the filter size can be estimated as (Pitsch (2006)

$$\frac{\ell_m}{\Delta} = \left(\frac{v'_{\Delta}\ell_F}{s_L\Delta}\right)^{3/2} = \mathrm{Ka}\frac{\ell_F}{\Delta} = \mathrm{Da}_{\Delta}^{-3/2}.$$

Hence, the flame is entirely on the subfilter scale as long as $Da_{\Delta} > 1$, and is partly resolved otherwise.



It is important to realize that the turbulence quantities, especially v'_{Δ} , and hence most of the nondimensional numbers used to characterize the flame/turbulence interactions, are fluctuating quantities and can significantly change in space and time.

To give an example, the variation of these quantities from a specific turbulent stoichiometric premixed methane/air flame simulation is shown in the regime diagram.

This simulation was done for an experimental configuration with a nominal Karlowitz number of Ka = 11, based on experimentally observed integral scales.

The simulated conditions correspond to flame F3 of Chen et al. (1996), and details of the simulation can be found in Pitsch & Duchamp de Lageneste (2002).







Appendix



Therefore the flame structure is not perturbed by turbulent fluctuations and remains quasi-steady.

The boundary of the corrugated flamelets regime to the thin reaction zones regime is given by Ka = 1, which, according to

$$\mathsf{Ka} = \frac{t_F}{t_\eta} = \frac{\ell_F^2}{\eta^2} = \frac{v_\eta^2}{s_L^2}$$

is equivalent to the condition that the

flame thickness is equal to the Kolmogorov length scale: $\ell_F \equiv \eta$ This is called the Klimov-Williams criterion.

For Ka = 1 the flame time is equal to the Kolmogorov time and the burning velocity is equal to the Kolmogorov velocity.



Among the flames F1, F2 and F3 that were investigated, the flame F1 with an exit velocity of 65 m/s was close to total flame extinction which occured on this burner at 75 m/s.

A photograph of the flame is shown in Chen et al. (1996).

Mansour (1999) has reviewed the recent results obtained from laser-diagnostics applied to turbulent premixed and partially premixed flames.

Mansour et al. (1998) have shown that the flame F1 is on the borderline to the broken flamelets regime having a Karlovitz number of Ka=91.



The three regimes with essentially different interactions of turbulence and chemistry are the corrugated flamelet regime, the thin reaction zones regime, and the broken reaction zones regime.

In the corrugated flamelet regime, the laminar flame thickness is smaller than the Kolmogorov scale, and hence Ka < 1.

Turbulence will therefore wrinkle the flame, but will not disturb the laminar flame structure.

In the thin reaction zones regime, the Kolmorogov scale becomes smaller than the flame thickness, which implies Ka > 1.

Turbulence then increases the transport within the chemically inert preheat region.



In thin reaction zones regime, the reaction zone thickness is still smaller than the Kolmogorov scale:

 $\ell_\delta < \eta$

Because the reaction zone, which appears as a thin layer within the flame, can be estimated to be an order of magnitude smaller than the flame thickness, the transition to the broken reaction zones regime occurs at approximately

 $\mathrm{Ka}\approx 100$

The thin reaction zone retains a laminar structure in the thin reaction zones regime, whereas the preheat region is governed by turbulent mixing, which enhances the burning velocity.



In the broken reaction zones regime, the Kolmogorov scale becomes smaller than the reaction zone thickness:

 $\eta < \ell_{\delta}$

This implies that the Karlovitz number Ka_{δ} , based on the reaction zone thickness, becomes larger than one.

 ${\sf Ka}_{\delta}>1$

Most technical combustion devices are operated in the thin reaction zones regime, because mixing is enhanced at higher Ka numbers, which leads to higher volumetric

heat release and shorter combustion times.

The broken reaction zones regime is usually avoided in fully premixed systems.



In the broken reaction zones regime, mixing is faster than the chemistry, which leads to local extinction.

This can cause noise, instabilities, and possibly global extinction.

However, the broken reaction zones regime is significant, for instance, in partially premixed systems.

In a lifted jet diffusion flame, the stabilization occurs by partially premixed flame fronts, which burn fastest at conditions close to stoichiometric mixture.

Away from the stoichiometric surface toward the center of the jet, the mixture is typically very rich and the chemistry slow. Hence, the Ka number becomes large. This behavior has been found in the analysis of DNS results of a lifted hydrogen/air diffusion flame (Mizobuchi et al. 2002).



For a given point in time, the Ka number has been evaluated using appropriate subfilter models for all points on the flame surface.

Because of the spatially varying filter size, but also because of heat losses to the burner, which locally lead to changes in the flame thickness, there is a small scatter of the dimensionless filter width.

Although the flame is mostly in the thin reaction zones regime, there is a strong variation in Ka number, ranging from the corrugated to the broken reaction zones regime.

